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Interpretations of Quantum Mechanics
A General Purview

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Quantum mechanics revolutionized physics in early 20$^{\text{th}}$ century and lead to one of the two field theories, the standard model, which is a crowing achievement of our contemporary times. However, paradoxes within the quantum mechanics, were recognized early on. These paradoxes have paved way to multiple interpretations of quantum mechanics over the century. These interpretations do not particularly affect the validity of the empirically established observations and measurements. We will attempt to introduce a few major interpretations of quantum mechanics and present their advantages and disadvantages in a very limited fashion. Though most interpretations do not tend themselves to experimental tests, it is possible that many of the interpretations are mere differing versions of the same objective truth, while some interpretations that do tend themselves to experimental test, beg further exploration of the foundations of quantum mechanics.

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1 Introduction: Origins of Quantum Mechanics

Descartes [1] had initially proposed that light was made up of particles in 1637, later elaborated by Newton [2] in 1704 and called corpuscles. Newton theorized that various colors of light arise from different particles, each carrying a color. On the other hand Hooke [3] and Huygens [4] had both proposed the mechanical wave theory of light as early as in late 17^{th} century. It was Huygens's idea of there being a hypothetical medium called luminiferous aether, through which the (longitudinal) mechanical waves of light could propagate, like sound waves though air or waves on water. Both the particle and wave theories of light could explain the phenomena of reflection (bouncing of light off surfaces like mirrors), refraction (bending of light when passing a material boundary, like air and glass), and dispersion (splitting of light into its constituent colors).

These wave theories of light were later strongly supported by the now famous Young's "double slit experiment" [5] in early 19^{th} century. In the original double slit experiment, in actuality, there were no two slits, but a beam of sun-light was split into two with the help of a paper card, and the image formed on a screen (onto which the two beams were incident), displayed an alternating dark and light bands called fringes. These dark and light bands were explained by Young using interference properties of waves, much like when two water waves collide. The wave theory was further formalized by the electromagnetic field theory of Maxwell [6], and in fact the unification of electricity and magnetism into a unified theory of electromagnetism itself deserves a separate discussion. The early 20^{th} century saw further experimental glimpses into the nature of light, such as the Compton effect [7] (where light bounced off [electrons in] materials), and photo-electric effect [8] (where light incident on materials ejected electrons). The observations of (mechanical) scattering phenomena brought the particle nature of light back to the fore-front.

Faraday's observation of electric discharge between two charged plates placed in vacuum [9], paved way for eventual formal study of "cathode rays" by Goldstein [10] and others. Particularly, J. J. Thomson [11] showed that these discharge were made up of tiny particles with a specific ratio between their electric charge and mass, leading to the idea that there could in fact be various other particles other than light. We today refer to these cathode rays as electrons. Just like light, the wave nature of electrons (and matter in a broader perspective) were theorized by de Broglie [12] in 1924 and only recently observed in a Young double slit type setup, but with electrons instead of light [13]. Generally, in modern times, this particle-wave duality in both massive materials and in mass-less light, is well accepted. We will further discuss this later in this article.

The photoelectric effect, observed by Hertz, showed very characteristic behavior where the electrons were only ejected when the wavelength of light was sufficiently low. This behavior was explained with the help of the idea, that light comes in "quanta" or fixed energy packets, by Einstein [14] in 1905, and he called these packets, photons. Actually, the idea of "quanta" was introduced by Max Planck in his famous 1901 paper [15] which explained the radiation from hot (black) bodies. Similarly, "quantum" ideas were also superbly successful in describing atomic transitions, as laid out by Bohr [16]. The dawn of 20^{th} is generally accepted as the genesis of Quantum sciences. But, the discovery of light "quanta" does not yet bring us to Quantum mechanics. Quantum mechanics is the theory of motion in the framework that the energy comes in packets.

Note that Newtonian mechanics [17] consists of continuous phenomena. But in the "quantum" realm, the energies of bodies come in packets, making it consist of discrete phenomena. Though clearly, Planck, Einstein, and Bohr, among others, had used this idea to explain many of the experimentally observed phenomena, the idea of applying these ideas to a general theory of quantum mechanics only took form with the works of Heisenberg [19] and Schrödinger [18], in 1925 and 1926, respectively.

The mechanics governing motion of tiny objects, where the particle-wave duality is most manifest, shows unusual behavior, often never seen in the realm of human experience. This has lead to a body of paradoxes, some only seemingly, while others remain challenging to reconcile with, even today. We will begin this article with a series of paradoxes that have been reconciled with, and move on to others that remain open to interpretation. We will elaborate on the key aspects of quantum mechanics (or any other required bit of concept) as and when the need arises. These paradoxes will directly guide us to the many ways in which one can reconcile with the bizarre quantum realm, giving rise to the many interpretations of formulation of quantum mechanics. *Even though we will arrive at various interpretations of quantum mechanics, each seemingly able to* *rationalize scientific measurements and observations from its own unique perspective, there in fact is no [known universal] way to rank the perspectives in order of merit, or generalize to a grander interpretation encompassing the many.* Nonetheless, we will also present the philosophical means to compare the interpretations.

This article is meant to be reachable to a general audience, so it may over-simplify certain nuances, and sidestep others. Furthermore, it is meant to present the story in a historical context, which might mean taking a look at the origins of a particular thought, even if the text reads more like a popular science article. Also, the text is meant as a stand alone read. So, for the most part, each discovery or development has been referenced with the primary source (if an expert wanted to review the narrative here), alas, clearly, a few are stated as is.

2 Paradoxes in Quantum Mechanics

Rohrlich [20] defines a paradox “to be an apparent contradiction that follows from apparently acceptable assumptions via apparently valid deductions.” A logical progression admits no contradictions, ergo, either the assumptions or the deductions, or both, are not acceptable. Paradoxes, even apparent ones, can help bring logical gaps to light, if they exist, or, at the very least, provide a new perspective of understanding the arguments.

Here is a good time to briefly introduce the basic tenets of quantum mechanics. A measurement is the act of quantifying a state or characteristic of a system. Characteristic descriptors of a physical object, like a penny, are its mass and dimensions (and so forth), so the act of assigning a numerical value to any of those descriptors would be measurement. Quantum mechanics is intimately tied to the act of measurement, as we shall see. Measurement is the first part of the Baconian scientific method [21], before one can derive reasonable deductions though logic. Note that measurement can involve assigning a continuous numerical value, like when an event occurred in terms of time, which could be any finite value. The point being that, there are an infinite choices with regard to what time we assign an event. The choice of where one starts the clock, after which the event is measured is arbitrary. Then, there are clearly discrete measurements, for example, the outcome of a coil flip, which can only be heads or tails. It is the later kind of measurement we are more concerned with in quantum mechanics.

Given the possible outcomes of a measurement, quantum mechanics assigns a probability to each of these outcomes. Note that, the probabilities may evolve over time; probability assigned to one of the outcomes may interact with the probabilities assigned to one or more other outcomes; and of course the act of measuring itself may change the relevant probabilities. All of these aspects of quantum mechanics are nuanced and have tangible affect(s) on the act of measurement.

That brings us back to Rohrlich, who further goes on to say that paradoxes may often arise from thought experiments, experiments that are only imagined, in order to understand the flow of logic, or gain a new perspective. For example, there is a way to test Aristotelian idea, that heavier objects fall to the ground faster than the lighter ones if dropped from the same height, with the help of a thought experiment. Let us first assume that Aristotle is indeed true that heavier objects fall to the ground faster than the lighter ones if dropped from the same height. Then, let us imagine that we tied the two stones together, and dropped them from the same height as in prior cases. One could think that the resulting time that this new composite stone takes to drop is between the time that each of the stones took to drop on their own. Or in contradiction, one could also imagine that the composite stone is heavier than each of the individual ones independently, and therefore, according to Aristotle, may take less time than the time that the heaviest stone took to drop. We now know that it takes the same amount of time for a body to fall through a height, regardless of their mass, accounting for air drag. The reality is in fact between the two outcomes of the thought experiment, where in one case the heavier stone was retarded when tied together with a lighter one, and in another case accelerated.

Sometimes, further investigations may lead to a resolution of a paradox, which then makes the original paradox only an apparent one. On the other hand, a paradox may in fact be a true paradox. A true paradox does not necessarily mean that there is a gap in our understanding, it may in fact be a matter of perspective. Nonetheless, we may not have gotten to a point in our understanding where one could resolve an apparent paradox. Many of the paradoxes discussed here arise from thought experiments. With our state of the understanding today, Rohrlich distinguishes between true and apparent paradoxes, and in keeping with the source, we will discuss seven paradoxes, the first three including apparent ones, and the last four being open to interpretation:

1. Atom Collapse
2. Complementarity: Position and Momentum
3. Complementarity: Energy and Time
4. Quantum Zeno's Paradox
5. EPR Paradox
6. Wave Function Collapse
7. Schrödinger's Cat

We will now discuss each of the paradoxes, courtesy Aharonov and Rohrlich [20].

2.1 Atom Collapse

Rutherford and his colleagues (Geiger and Marsden) had pioneered scattering experiments to study the structure of atom and materials in general [22]. In this paper, they scattered alpha particles coming from an isotope of Radium, ^{226}Ra, off a gold foil. Note that the atomic masses of alpha particles, 4, and gold atoms 197, was well known by 1900s. We now know that Alpha particles are Helium atoms whose electrons have been stripped, making them a doubly charged species. Though, it was not yet known by early 1910s, when these experiments were carried out, that charges come in multiples of charge on the electron [23], the ratios between mass and charge was already well measured, thanks to work of Thomson and others. So, talking about charge of the alpha particles, does make sense, in a way, historically.

The results of these experiments were puzzling, and indicated a dense positively charged core, now called the nucleus, yet the atoms (and thereby the material) were neutral, so a rare negatively charged cloud must surround the dense core. Maxwell's electromagnetism also was well known by 1900s, and upon application of electromagnetism to such an atom, with a dense positive core and surrounding rare negative cloud, indicated that the negative cloud would collapse onto the positive core thereby collapsing the atom. This would nullify the existence of any matter at all, indicating a paradox.

The atom collapse paradox was resolved by Bohr [16], who proposed that if the energy of the negatively charged cloud come in a particular series (in multiples of $1^2,\ 2^2,\ 3^2,\ \ldots$), then it would solve the atom collapse paradox. Bohr's seemingly magic ratios, are the first signs of quantization of energy in atoms. These specific energy levels balance out the inward attraction between the negative cloud and positive core exactly with the orbital outward force (centrifugal force, the same force that makes one feel the push inside a car when the car rounds a corner).

2.2 Complementarity: Position and Momentum

Einstein had challenged Bohr's new atomic theory, with a thought experiment [24], in which Einstein proposed a Young's double slit experiment type experiment, where not only does one try to measure the dark and light fringes, but also tries to pin-point the slit light passed through (by installing a coil to detect the light passing through). The fringes caused by the interference indicate the wave nature of light, whereas if one were to pinpoint the location of the light, then it would demonstrate the particle nature of light. In a formal sense, the fringes are a measure of the wavelength or the momentum of light, whereas the act of pinpointing the location of the light particle is the measurement of its position.

Bohr had proposed the idea of complementarity [24], that an experiment either measures the wave properties or the particle properties, or in other words, the momentum (associated with the mass and speed of the particle) or the position of light, but never the two together. But since the outcome of Einstein's thought experiment wasn't known in early 1900s, there was Einstein's experiment which could measure both position (particle nature) and momentum (wave nature) of light on one hand, and on the other hand was Bohr's complementarity, which stood in opposition to Einstein's thought experiment.

Today the outcome of such an experiment is in fact well known and understood [25]. It turns out that as long as one does not attempt to pin-point the slit through which the light passes, a standard interference pattern is formed demonstrating the wave like nature of light. But as soon as one measures the position, or the slit through which the light passes before falling on the screen, the interference pattern is destroyed. This validates Bohr's complementarity. Moreover, complementarity is also consistent with the mathematical formulation of the same phenomena which was laid out by Heisenberg, and is famously called the uncertainty principle [26]. It also states that one can never measure the two "complementary" aspects of momentum and position at once, precisely.

2.3 Complementarity: Energy and Time

The trajectory of a charged particle curves when passing through a magnetic (or electric) field. This follows the Maxwell's laws and does not require quantum mechanics to understand, in a classical sense. From Maxwell's laws, one can associate the kinetic energy of a charged particle to the amount it curves in a magnetic field.

The same set of Maxwell's laws also predicts that a charged particle moving along a curved path emits photons, or light, losing energy. So one can expect that the kinetic energy with which the charged particle entered a magnetic field, is always greater than any point in time spent in the field. Another way to look at it is that, the kinetic energy of the charged particle moving under the influence of a magnetic field at a given time is always greater than at any instant after the given time, owing to having lost energy to emission of light.

Landau and Peierls [27] noted that if one measures the kinetic energy of the charged particle within a small enough time span, the energy associated with the charged particle is uncertain. The magnitude of this uncertainty could be thought of as comparable to the "quanta" of light, since there can be no smaller amount of energy that the photon can remove out of the charged particle. As there is a smallest energy associated with the energy difference, so there would be a smallest time difference over which measuring the kinetic energy would make sense.

This leads us to an analog of the momentum-position complementarity paradox we discussed in the previous sub-section, but this time, between energy and time. Bohr's complementarity can be extended to measurements of energy and time. Heisenberg and others showed that an experiment can only measure one of the two aspects of a particle: time or its energy, precisely. Like position and momentum were "complementary" aspects, similarly, energy and time are also "complementary" properties.

2.4 Dichotomy: Quantum Zeno's Paradox

The Greek philospher, Zeno of Elea (circa. 490-430 BC), laid out a set of philosophical problems which challenges ones senses, and proposed that motion or change is just an illusion. For the scope of this article, we shall consider only one of Zeno's nine paradoxes, the "Dichotomy paradox" [28]. As recounted by Aristotle in Physics VI:9, 239b10 [29], "That which is in locomotion must arrive at the half-way stage before it arrives at the goal."

Note that Zeno was from a time before the works of Archimedes (circa. 287-212 BC), who first studied infinite series, and showed that infinite series can sum to finite numbers. In order to reach a destination from the origin, by motion, *for the sake of simplicity, let us assume along a straight line*, one would need to get to a point which is half way to the destination, and in order to get to the half way point, one would need to get to the quarter way point, and so forth infinitely down to an infinitesimal. This forms an infinite series of movements, which Zeno argued was not possible, presenting our paradox. It also means that the next step is indeterminable as there is a point between the origin and any given next step. The dichotomy here is the act of splitting any distance into two. Ergo, Zeno argued that motion or change just had to be imagined, and is not real.

Spin is usually an intrinsic property of a particle in quantum mechanics. Another way to think about spin is to simply associate it with the magnetic moment (like a bar magnet) of the particle. Generally, spin is defined to be either along an axis or against the axis, giving us only two measurable states. Just like two magnets feel each others influence, a particle with a spin also feels a magnetic field (which could be generated by another particle with non-zero spin). Classically one could apply a magnetic field to a particle whose spin is aligned along an axis in a such a way that the spin precesses, much like an imbalanced top when spun, and may end up in state where the spin is anti-aligned with the axis. There is a maximum energy associated with the process of spin-flip. The energy difference in the case of a spin flip of an imbalanced top is the gravitational potential energy difference obtained from the mass of the top and change in height of its center of gravity, as it flips itself.

An analog of classical Zeno's paradox exists within quantum mechanics [30], *i.e.* the quantum Zeno's paradox. If we start out with a particle with positive spin, and allow it to evolve under the influence of a magnetic field, after some time, there is a non-zero probability that the spin, when measured, turns out to be negative. The quantum Zeno paradox posits that, we could in practice frequently measure the above system, in order to check its spin state. But if we improve upon the precision with which we measure when the spin flipped, from the energy-time Heisenberg's uncertainty relationship, the uncertainty on the energy blows up to infinity, allowing extremely high energies being associated with the system. This seems paradoxically in disagreement with the maximum energy difference associated with the process of spin-flip.

Quantum Zeno effect has now been experimentally tested, rigorously [31]. The observations from these experiments are that, when a system is measured at a high frequency, it tends to stay in its initial state, and neglects the evolution. This way there is no change in the system, and thereby the energy associated with the process of spin-flip blowing up to infinity in the prior case is sidestepped.

2.5 EPR Paradox

Positrons, particles similar to electrons but with an opposite charge, had been discovered in 1932 [32]. Further investigations in similar experiments showed that, an electron and a positron may simultaneously originate at a point, together (from some third source of energy like photons), or annihilate each other at a point (to produce energy as photons). This also implied that if the electron was measured with a positive spin, then the positron had to have been measured with a negative spin, and vice-versa, such that the sum is always zero. Such pairs are now called "entangled pairs". In such entangled pairs, the momentum or position of these particles are similarly correlated, *i.e.* given the position or momentum of one particle, one can infer the same for the other particle.

Einstein, Podolsky, and Rosen proposed a thought experiment, which has now come to be known as the EPR paradox [33], where they proposed measuring the position of one of the particles in an entangled pair, and momentum of the other particle in the entangled pair. This way, one does not measure the complementary aspects of position and momentum in the same particle. Furthermore, this also implies that the momentum and position of the other particle in the entangled pair may be inferred. Meaning, we could paradoxically get a precise measurement of both position and momentum, disallowed in quantum mechanics.

Another way to think is in terms of spin. Both electron and positron pair produced may have either negative or positive spin, but their sum is always zero. Let us say that one measures the spin of the positron, then this fixes the spin of the electron as well, without even having measured the electron's spin. This implies that the information about the spin of the electron, which still could have been positive or negative, was influenced instantaneously at the time of measurement of the spin of the positron. The instantaneous "action at a distance", or faster than speed of light, transmission of information, is disallowed by Einstein's special theory of relativity [34], again demonstrating our paradox here [*].

It was later demonstrated by John S. Bell [35] that measurements in entangled pairs always follow certain inequalities such that the uncertainty principle is always respected. So, one in fact cannot measure the position and momentum of a particle precisely in an entangled pair. This has now been further experimentally verified, quite extensively, with the first experiments demonstrating Bell's inequalities in entangled photons [36], as early as in 1970s. Another way to reconcile with the paradox is recognizing that the entangled pair were in fact at the same location at some point in the past, so there is no instantaneous transmission of information.

2.6 Wave Function Collapse

One can already see in the previous sub-section that the electron, until the point of measurement, could have had either positive spin or negative spin. Only upon the measurement, does one know the spin state, and moreover, we only measure a single spin state of the two possibilities. This is the start of the idea of wave function collapse.

Wave function in quantum mechanics consists of the probabilities associated with the outcome of the measurement. Assigning probabilities to the outcomes was one of the key techniques of how

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[*] This made Einstein defend his earlier "God does not play dice" remark about quantum mechanics, and in his view demonstrated the incompleteness of the formulation of quantum mechanics.

quantum mechanics worked, in contrast to the deterministic Newtonian mechanics. For example in the previous case, regarding the measurement of the spin of an electron, the quantum mechanical wave function consists of two parts, one each corresponding to the two outcomes. But after measurement, just one of the parts remains, indicating a collapse from two parts to just one part of the wave function.

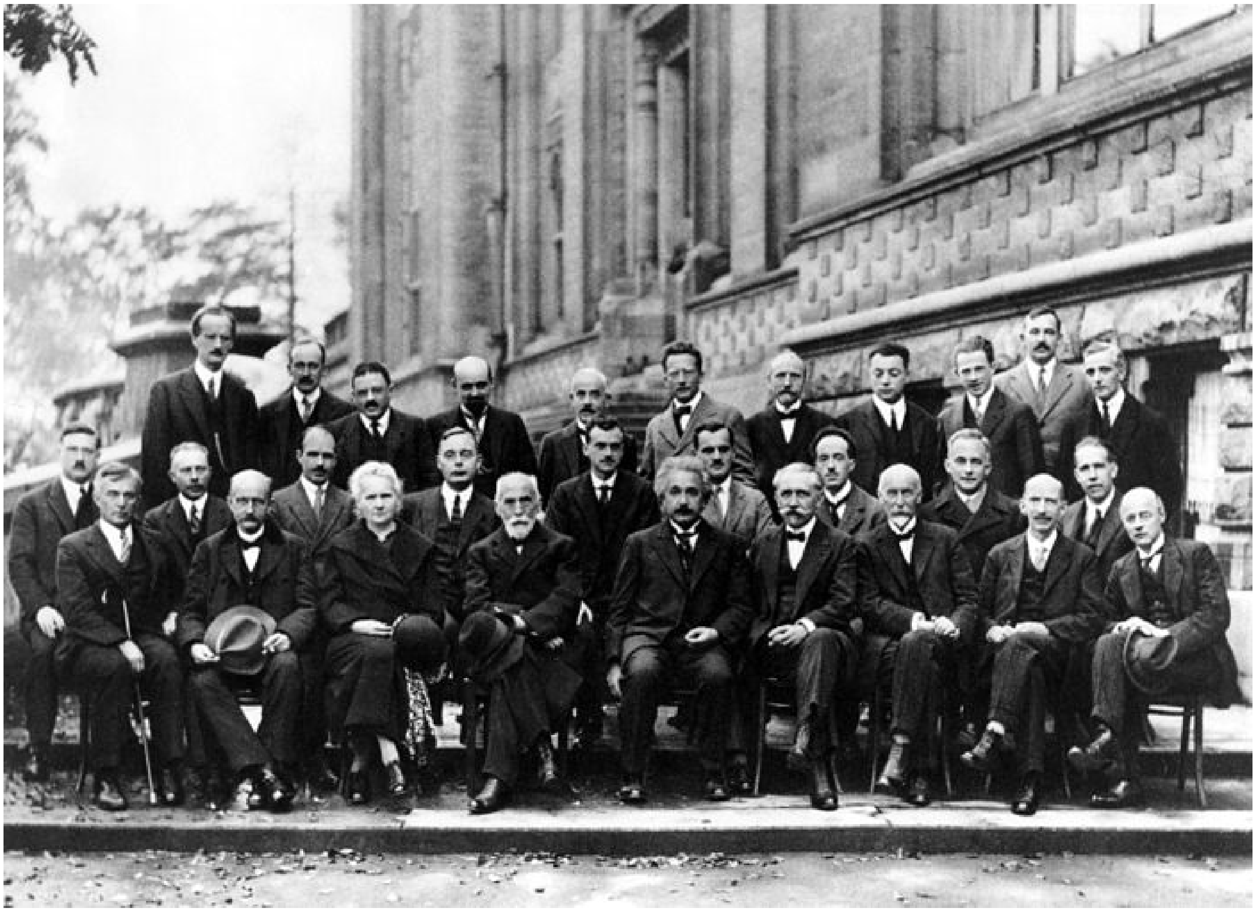

Figure 1: In rows, top to bottom, left to right: A. Piccard, E. Henriot, P. Ehrenfest, E. Herzen, Th. de Donder, E. Schrödinger, J. E. Verschaffelt, W. Pauli, W. Heisenberg, R. H. Fowler, L. Brillouin; P. Debye, M. Knudsen, W.L. Bragg, H. A. Kramers, P. A. M. Dirac, A. H. Compton, L. de Broglie, M. Born, N. Bohr; I. Langmuir, M. Planck, M. Curie, H. A. Lorentz, A. Einstein, P. Langevin, Ch.-E. Guye, C. T. R. Wilson, O. W. Richardson. Fifth conference participants, 1927. *Courtesy* Institut International de Physique Solvay in Leopold Park.

In 1927 Solvay conference, Einstein illustrated the concept of wave function collapse with a thought experiment, of light incident on the screen. Light being a wave, is not localized in space but spread out, with a small but uniform probability of being anywhere. The locations where the photon could be, would be the parts of the wave function in this case with equal weights to indicate the uniform probability distribution. But, he recognized that, as soon as one measures the location of the photon, say with the help of a photographic plate, it has been localized. Then, the probability that one may find the photon any where else drops to zero. The instantaneous drop in probability is inconsistent with Einstein's special theory of relativity, and leads to the issue of wave function collapse paradox.

The concept of wave function collapsing remains open to interpretation. If one were to repeat the experiment of measuring the spin of the electron (measured by E) and positron (measured by P) in an entangled pair, many times over, the results will indeed convince both E and P, that they are performing measurements on an entangled system by correlating their results. But to E, P's measurement might have collapsed E's wave function, or to P, E's measurement might have collapsed P's wave function. So the observers may disagree on how the wave function collapsed, but they will not disagree on the results. Furthermore, collapse has been shown to obey the basic uncertainty principles (or Bell's inequalities) by Aharanov and others [37].

2.7 Schrödinger's Cat

Perhaps the most popular of the quantum mechanical paradoxes is the Schrödinger's Cat. Schrödinger, one of the chief architects of quantum mechanics, himself was perplexed by the indeterminacy inherent to quantum mechanics. In this thought experiment, Schrödinger imagined a cat in a box, with a vial of poisonous gas which could be cracked open when the measurement of the spin of an electron is positive. Since the event governing whether the vial has been broken open is random, one could imagine that no-one could tell with certainty if the cat has died or is still alive until the box is opened, *i.e.* until a measurement has been performed.

In the simplest version, we are dealing with two systems here, the electron and the cat, both of each can only be in two states: the electron having positive or negative spin, and the cat being dead or alive. The possibilities in this thought experiment are that: (i) the electron's spin is positive, and the cat is dead, (ii) the electron's spin is negative, and the cat is alive, (iii) the electron's spin is negative, and the cat is dead, or (iv) the electron's spin is negative, and the cat is alive. But we know that situations (ii) and (iii) cannot ever be the outcome. It is as though the measurement of electron's spin is entangled with the state of the cat. One can add more complexity such as if the vial breaks or not, but that simply adds additional layers of entanglements. Not only do we see multiple systems being entangled here, we also see another situation of wave function collapse, *i.e.* when we open the box, and find the cat alive, we also know that the measurement of electron's spin is negative.

Treating measurement like an entanglement, the collapse of wave function, as well as the EPR paradox are all open to interpretation, which we will summarize in the next section.

3 Interpretations of Quantum Mechanics [38]

The paradoxes of atom collapse, complementarity of position - momentum, and energy - time, Quantum Zeno's paradox, all had an aesthetically pleasing resolution. But while the causality part of EPR paradox, and measurement being entangled with the system in Schrödinger's cat, the wave function collapse and the paradoxes related to it could also have an acceptable resolution, they remain active areas of interpretation. This nature of quantum mechanics has spawned many interpretations of quantum mechanics. Consider that the interpretations of quantum mechanics is essentially metaphysical in nature, *i.e.* it involves questioning the foundations of physics, or in other words.

Four popular interpretations of quantum mechanics have been discussed here, *viz.*:

1. Copenhagen Interpretation,
2. Spontaneous Collapse,
3. Hidden Variables,
4. Many Worlds.

Note that none of the interpretations preclude any of the others. Certain other interpretations have also been presented here in order to provide a historical context. These interpretations are not new physical laws, of even hypothesis for that matter, but perspectives from which one can view the foundations of quantum mechanics. From where we stand today, it is sufficient to think of these interpretations on equal footing, only ranked by one's own inclinations. Most importantly, this article is in no way to be considered exhaustive or even close to a semi-complete description of the works. This is merely a brief footnote to introduce the general reader to some of the interpretations.

3.1 Copenhagen Interpretation [39]

Oxford dictionary describes logical positivism, that arose in early 20th century Vienna, as "a form of positivism, ..., which considers that the only meaningful philosophical problems are those which can be solved by logical analysis." Copenhagen interpretation was developed by Bohr and Heisenberg in Copenhagen, ergo its name, between 1925-27. This interpretation remains one of the simplest and is widely taught as introductory material to undergraduate physicists all over the world. The Copenhagen interpretation stems from the philosophy of logical positivism, which was gaining acceptance at the time in Europe. Experimental verification of the predictions of quantum mechanics, justified its acceptance, especially for the logical positivists.

The Born's rule of interpreting the wave function [40] was critical to the Copenhagen interpretation. *Born's rule states that the square of the wave function gives us the probability density of measurement*, *i.e.* if one were to measure a quantum system, the chance that one finds a particular outcome is given by the square of the associated wave function. Simply put, the Copenhagen interpretation states that the measurement picks out one of the possibilities within the wave function with a probability obtained using the Born's rule.

This seemingly drawl interpretation opines that it is meaningless to ask the question of an outcome before a measurement. Accordingly, the wave function is a meaningless and fictitious mathematical construct with no physical meaning. Only the probabilities arising from Born's rule has a consequence for measurements. It states that the wave function collapse is not a function of the system, but the doing of an observer. For example, a free electron could have a positive or a negative spin, with equal probabilities, and upon measurement, one finds it in one of the two spin configurations. The fact that the electron's spin measurement has a unique outcome is not the doing of the electron, but of the observer that made a measurement in some instance. One could easily argue, had the measurement been performed (at a different location/time) again, the observer would have measured the opposite value. One could also understand this in terms of (a frequentist's argument where) many observers measuring the spin states of many disconnected electrons, then half of them would measure positive, while the other half would measure negative.

Likewise, the Copenhagen interpretation's view of Schrödinger's cat paradox would be that, the wave function consists of both the possibilities of the cat being alive and dead, and the probability that it is alive or dead upon measurement is 50% in a frequentist's way. Similarly, the Copenhagen interpretation holds fast to the Heisenberg's uncertainty principle and Bohr's complementary, making it straightforward to judge the purview of this interpretation. Also, the issue of causality raised by the EPR paradox is addressed by the Copenhagen interpretation: that once two observers measure the states of an entangled pair, they still have to communicate this information for comparison, which prevents any instantaneous "spooky action at a distance", disallowed by Einstein's special theory of relativity.

3.2 Many Worlds [41]

The Oxford dictionary defines realism as "the attitude or practice of accepting a situation as it is and being prepared to deal with it accordingly." Hugh Everett working with his advisor John A. Wheeler, came up with an interpretation of quantum mechanics as a part of his doctoral dissertation [42], that we today refer to as the many worlds interpretation. The logical positivism used to justify the wave function collapse in the Copenhagen interpretation is entirely replaced by a very realistic approach to the issue in the many worlds interpretation.

Copenhagen interpretation treated the wave function as mathematical jargon, without any real physical meaning. It was only through the Born's rule that physical meaning was associated with the wave function. Instead of relying on hypothetical wave function collapse, many worlds interpretation endows the wave function with physical reality, and makes each outcome represented by the wave function physically possible. The many worlds interpretation posits that, all the possibilities within the wave function are indeed real and do occur, and the universe branches into the many possibilities upon measurement. So this interpretation does not actually require wave function collapse. For example, say we are to measure the spin of an electron; then, after a measurement, there is a 50%-50% chance of it being positive, or negative. The universe splits into two copies where each one of the two possibilities occurred (or could have been measured in this case).

An important physical issue, that was evident early on, was regarding the meaning of associating probabilities to measurements in an Everettian quantum mechanical world, as each of the possibilities definitely occurred in at least one of the copies of the universe post measurement. So seemingly the probabilities are all one, or in other words that all possibilities always occur. But the Born's rule, is very much physical, and is measurably manifest. There have been many ways to reconcile the many worlds interpretation with the Born probabilities. The frequentist approach proposed by DeWitt-Graham [42], Farhi [43] and others posits that if one were to perform the same measurement infinitely many times, then the ratio of the number of different kinds of copies (where each kind of copy corresponding to each of the possibilities), would reflect the ratio of probabilities calculated by the Born's rule. On the other hand, there have been decision theory based methods to address the issue as proposed by Deutsch [44], Wallace [41], and Saunders [45], which posits that the wager (such wagers also arise from a thought experiment called Schrödinger's wager) one would make pre-measurement, as to which possibility is the ultimate outcome post-measurement, would follow the Born probabilities.

A lesser philosophical problem that immediately arises with the universe making copies of itself upon measurement, is the problem of identity. In the above example, at the point of measurement of the electron's spin, the electron was split into two copies, one which had a positive spin, and another with a negative spin. Clearly these two daughter cases are not identical, so it remains a question as to which one of the two was the case before the measurement. One could sweep the question of identity under the rug claiming it to be an abstract concept, and that rules of identity breaks down at the point of measurement, as done by Parfit [46] and others. But it has also been proposed by Lewis [47] and others that even before the measurement, there are two copies of system though initially their identities match (as being in a superposition of up and down spin states), yet later their identities diverge with one distinct possibility from within the wave function assigned to each case.

One can see that the choice of number of branches, one needs to describe the universe in the many worlds interpretation, is dependent upon the choice of measurements, or choice of mathematical description, or even the meta-interpretation like the arguments by Parfit versus those by Lewis. Some argue that counting the number of branches required to describe the universe is the indeterminacy feature of quantum mechanics. While others like David Wallace embrace this indeterminacy and posit that the number of copies is a mere non-physical way to describe the idea. However, musings on understanding the many worlds interpretation are an active facet of thought.

3.3 Hidden Variables [48, 49, 50]

Newtonian mechanics is deterministic, *i.e. given a set of initial conditions, determinism implies that one can tell precisely what state the system is in at a time in the future.* Quantum mechanics is quite opposite to classical mechanics in that sense. Quantum mechanics, at least, as has been measured so far, is inherently in-deterministic. Quantum mechanics can only tell us what the likelihood of certain outcomes are, but it never tells us precisely what the outcome is. Simply put, hidden variable theories posit that there could be additional variables, that could be unknown to us and even immeasurable (ergo "hidden"), which complete the description of a quantum mechanical system, making it deterministic like classical mechanics.

Hidden variables do not imply simply accessing complementary measurables, like position and momentum, simultaneously with arbitrary precision, as was demonstrated with the help of spin measurement to describe the EPR paradox. Hidden variable theories must respect Heisenberg uncertainty principles, and must also respect the fact that measurement probabilities align with Born's rule of squared wave function. The first of such hidden variable theories was proposed by L. de Broglie [51] as early as in 1927, and was dramatically improved upon by Bohm [52] in 1952.

In de Broglie's version of hidden variable theory, called the pilot wave theory, the (matter-) wave associated with particles followed the Schrödinger formalism, which also respects the Born probability rules, but the particle is accompanied by a "pilot wave" that also depends upon all the other particles with which the particle under consideration maybe interacting with. Whether it be a pilot wave or another hidden variable, such figments gives rise to a deterministic quantum mechanics. For example, the pilot wave in de Broglie's theory would cause the measurement of an entangled pair to always yield opposite measurements of spin, since the pilot wave already depends on properties of all other particles. Hidden variables theories add one or more hidden variables to yield determinism.

Take for example the pilot wave, the fact that the pilot wave is dependent upon all the other particles, is a statement of its non-locality, *i.e.* the EPR paradox like "action at a distance" is already a feature. The non-locality feature flies in the face of Einstein's special theory of relativity, according to which there can be no causally linked action at a distance. Therefore Bell's theorem prompted many, including de Broglie himself, to abandon hidden variables approach of expanding the foundations of quantum mechanics. But Bell didn't imply that hidden variables approach is a non-starter, he merely showed that there is a price to be paid in order to make quantum mechanics deterministic by adding hidden variables to the theory: either the hidden variables incorporate some form of non-locality into them, or we sacrifice the idea that properties of particles are independent of what measurement is performed on them.

Bell did not immediately reject hidden variable theories, as he knew of Bohm's extension of de Broglie's version of hidden variable theorem. In *e.g.* the Young's double slit experiment involving electrons, according to Bohmian mechanics, the (matter-) wave of the electrons are distributed as per Born's rule, but adds a set of "hidden" particles whose positions are always known (ergo the non-locality since the position of each particle is dependent on every other particle's location) and this adds the determinism of finding a particle at a particular location when detected, then the probability of finding the particle also follows the Born's rule owing to the interference from the (matter-) wave. Bohm showed that it is always possible to introduce determinism into the measurement of a particular property (position in the above example) of a quantum mechanical system by augmenting the wave function in a particular way, and conversely, to construct a wave function such that one particular property naturally becomes deterministic. One could then argue that the wave function can be sufficiently augmented (by adding as many hidden variables as it takes) in order to make every property deterministic, thereby violating Bell's inequalities and also Heisenberg's uncertainty principle, but this has been shown to be impossible by Kochen and Specker [53]. In the above case for example, after making position deterministic through the addition of a hidden variable, one could not then add another hidden variable to make momentum deterministic as well. Generally such theories always incorporate non-locality into them, and are called modal approaches.

If non-locality is particularly non-aesthetic, then one could also go about sacrificing the measurement independence assumption, to extend quantum mechanics through hidden variables. In order to imagine sacrificing the measurement independence assumption, one needs to be able to identify an event causally influencing the choice of a measurement. Clearly one can isolate a system, and not perform any measurements on it, ensuring that the no such event happened in the past. However, the act of measuring in the future could constitute such a causal event. Such approaches that involve causation to act backwards in time, are called retro-causal approaches. In retro-causal approaches, one could do away entirely with wave nature, and just consider the particle nature as a basis. Here, a hidden wave carries forward-causal influences on the particle from an initial state, and similarly, backward-causal influences on the particle are carried by the hidden wave from the final state, giving rise to wave-like properties like interference.

While non-locality might indeed be harder to reconcile with special relativity, retro-causal approaches, though counter intuitive, is a commonly used mathematical tool in much of modern quantum field theory, and is usually free of aesthetically uneasy features such as non-locality. Retro-causal approaches particularly, bolstered by experimental verification of the predictions of quantum field theories such as Quantum Electrodynamics (quantum mechanical version of Maxwell's electromagnetism), keep the field of hidden variables an active area of exploration.

3.4 Spontaneous Collapse Theories [54]

Quantum mechanics, as has been recollected many times above, seems to be most manifest in microscopic scales, like with the behavior of sub-atomic particles; while classical mechanics dominates the behavior of macroscopic objects like stars and planets. Most of the interpretations do not concern themselves with the transition from the laws of the microscopic, *i.e.* quantum mechanics, to the laws of the macroscopic, *i.e.* classical mechanics. Copenhagen interpretation and hidden variable interpretations of quantum mechanics seem to be at best agnostic about this transition, while the many worlds interpretation does not require this transition at all.

Spontaneous collapse models, as evident from the name, introduce a collapse term explicitly into the Schrödinger equation. The first of such models was developed by Ghirardi-Remini-Weber [55], referred to as the GRW theory, but this has since been further developed. The solutions of Schrödinger equations yield de Broglie like (matter-) waves, which are spread out over space, but with the introduction of a collapse term, these spread out solutions are instantaneously localized in space at a particular time. The addition of such collapse term has been shown to yield predictions consistent with quantum mechanics [56], *i.e.* generally when one adds additional terms not motivated by first principles the equations no more work, but that is not the case in quantum mechanics upon the addition of the collapse term.

The collapse term is randomly and uniformly distributed over time, but has the same probability density as that obtained from Born's rule. A keen student might note that this might be cyclic: we obtained the wave function by solving the traditional Schrödinger equation and the Born's rule told us that the square of the wave function gives us the probability density, and one might imagine that introduction of the collapse term into Schrödinger equation might change the wave function solutions and thereby affect the probability densities. But there are means by which to constrain the properties of the collapse term (to make it a unitary transformation), such that its only job is to instantaneously collapse the spread out wave function, without affecting the probability distribution.

In this fashion, in a Young's double slit experiment, the electrons can still interfere creating a probability density that resembles the dark and light bands, but the collapse term ensures that the electron is in fact detected locally. One can already see that these collapse theories satisfy Bell's inequalities by violating locality. Similarly, there usually is nothing in collapse interpretations that ensures that, experiments when repeated, yield the same measurements, since collapse could have happened at any time owing to a small uniform chance of collapse over time, however small. This creates an issue for repeatability of experiments, a key tenant of science. So repeatability has be enforced from the outside. An expert would recognize that repeatability over time is what gives us conservation of energy (from Noether's theorem), so spontaneous collapse theories do violate conservation of energy, ever so slightly (immeasurably slightly). In a way, spontaneous collapse interpretations retain the probabilistic nature, which seemed lacking, or at the very least questionable in many worlds interpretations.

More importantly, the spontaneous collapse theories resolve the issue of scales (of quantum mechanics being manifest in only the sub-atomic realm) and applicability of quantum mechanics. They provide a way in which to explain this effect naturally. Individual sub-atomic particles have a very small probability to spontaneously collapse over time, so for the most part, individual particles show the traditional quantum mechanical probabilistic behaviors. But in large objects, made of many trillions of constituent particles, the chance of collapse is proportionally higher, making larger bodies less prone to the probabilistic behaviors. This proportionality can be easily justified in hard solid masses for example, where the constituents are spatially correlated, *i.e.* each constituent (molecule or atom) is spatially fixed with respect to the position of other, and collapse [of the wave function] of any one constituent forces other correlated constituents to also collapse.

Moreover, of all the interpretations, spontaneous collapse theories are the few that tend themselves to experimental check. Though the collapse term only forces the wave function to collapse, and does not change the probability densities predicted by traditional quantum mechanics using the unmodified Schrödinger equation dramatically, addition of these terms do: (i) modify the probabilities ever so slightly around the prediction of the unmodified Schrödinger equation, and (ii) introduce a chance that making the quantum mechanical system larger or subject to longer time periods, would collapse occur with a higher chance (more immediately). So, there have two class of experiments, one testing each of the above hypothesis linked directly to spontaneous collapse theories. Though strong constraints can be places by these experiments, they do not completely rule out these theories categorically.

4 Perception of Quantum Mechanics [57]

Quantum mechanics (an inherently uncertain and probabilistic theory) superseded classical Newtonian mechanics (which was deterministic), and was viewed as an incomplete theory even by physicists due to its indeterminacy, at least to a certain extent. Moreover, the founders of quantum mechanics themselves did not fully fathom the implications of the theory. During its development, even the famous physicist and Nobel laureate, R. P. Feynman opined "I think I can safely say that nobody really understands quantum mechanics" (1965). Since then, tremendous strides have been made to improve the theory over the course of a century, and has lead to a comprehensive quantum field theory, the crowing achievement of 20th century physics: the standard model, which describes all known interactions †. While the predictions and techniques by which quantum mechanical predictions are calculated have become ever more precise and accurate, the underlying foundations are, to say the least, open to interpretations and questions. It is these questions that make the interpretations of the foundations of quantum mechanics ubiquitous in number, and make even a modern day physicist exclaim "Even Physicists Don't Understand Quantum Mechanics!" (S. Carroll, 2020).

Even among formal physicists, their favorite interpretation of quantum mechanics varies wildly. But then again, objective truth (or reality) is not democratic. Copenhagen interpretation dates back to 1920s, whereas many worlds interpretation and the first of hidden variables approaches were formulated in 1950s, while the first of spontaneous collapse theories were formulated in 1980s. Being first itself could be a reason for the prevalence of Copenhagen interpretations in modern quantum mechanics texts. Much of modern texts, center around the applications of quantum mechanics, *eg.* their application to atoms and thereby obtaining the electron configuration. The simplicity of Copenhagen interpretation, and its "as is" approach may also tend itself better for a text book, which wants to focus on the applications of quantum mechanics, by neglecting the foundational aspects. Some have even attributed the prevalence of the Copenhagen interpretation to the orthodoxy in physics academia [58].

As we saw, the interpretations only gave extra depth to our understanding, but the predictions

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[†] Except for neutrino oscillations.

and math are ever so accurate, regardless of the underlying foundations. Deviations from the predictions of quantum mechanics have never been measured thus far [‡]. The tremendous success of these predictions, maybe, has made the quest to understand the foundations take a backseat, so much so that R. P. Feynman is misquoted to have said “shut up and calculate” (quote from D. Mermin, 2004 [59]). This school of thought comes from instrumentalism, which is as defined by Oxford dictionary is “a pragmatic philosophical approach which regards an activity (such as science, law, or education) chiefly as an instrument or tool for some practical purpose, rather than in more absolute or ideal terms.”

An instrumentalist purview of quantum mechanics, itself can also be a considered as an interpretation. Alas, such an interpretation is hard to evaluate and compare with other mathematically rigorous interpretations treated in this work, owing to its stance of disregarding any deeper understanding at all. But, such a view is not indefensible. Karl Popper’s scientific realism, *i.e.* “the view that the universe described by science is real regardless of how it may be interpreted”, may take quantum mechanics’s success as a predictive tool to justify its validity regardless of how it maybe interpreted.

The plethora of interpretations of quantum mechanics, gives the perception that there maybe gaps in our understanding of the foundations. Note that it may very well be the case that the interpretations are exactly what the name entails, *i.e.* that they are simply ways of philosophically reconciling with the probabilistic nature of quantum mechanics, and that they may simply be different perspectives of seeing this very nature of quantum mechanics. In other words, just as Heisenberg’s and Schrödinger’s versions of quantum mechanics were shown to be one and the same, it is possible that certain interpretations (like hidden variables and spontaneous collapse approach) are actually different statements of one and the same feature. It may also be the case that there are many class of comparable interpretations, each class of interpretations stating a particular feature. The ultimate point being that we may never reach, in any realistic way, a point where certain interpretations develop into a coherent underlying mechanism that expands our predictive potential of quantum mechanics. Finally, it is also possible that certain interpretations

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[‡] Except for neutrino oscillations.

(like spontaneous collapse approach) can in fact be experimentally tested and possibly found to be untrue [§].

5 Choosing an Interpretation [61]

Four category of ideas to interpret or augment traditional quantum mechanics were presented in the previous sections. On one side is the Copenhagen interpretation which sees quantum mechanics as is, a probabilistic theory accepting of the impact of observer or measurement; spontaneous collapse interpretation stands at the other end, actively augmenting the very underlying equations governing traditional quantum mechanics, in a very tangibly (and measurably) mathematical way. Then there were the hidden variables and many worlds approaches which viewed the universe through the purview of wave nature, and provided ways to incorporate the particle behavior from happenings outside the tangible, *i.e.* hidden particles and branching into many worlds, respectively, both of which are not measurable.

Some may argue that hidden variables and many worlds approaches are indeed true interpretations as they involve mechanisms outside the tangible, whereas spontaneous collapse theories are extensions of quantum mechanics. Obviously, the scenario might change if tangible tests of these two interpretations are ever realized. Especially the intangible nature of hidden variables and many worlds approaches may lead us to conclude that the measurements one can perform through experiments in the tangible universe, that confirm only the predictions of quantum mechanics but say nothing about the foundations, is under-determining the reality.

The choice of which interpretation to choose therefore lies wholly with personal preferences, at the moment. However, one could evaluate the interpretations along a set of aspects to determine where one falls subjectively. Some of these aspects are: (i) determinism, (ii) ontology of the wave function, (iii) history (of the quantum mechanical system), (iv) (need for) hidden variables, (v) (need of) wave function collapse, (vi) non-locality (in the wave function), (vii) counter factual

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[§] Clearly quantum mechanics is hard and it is relatively new in its public outreach, compared to other classical physics. It has not tended itself yet to become a part of general knowledge of the populace, despite numerous popular books on the topic. This coupled with the inherent features of quantum mechanics has made it possible for some to spin quantum mechanics for their own pseudo-science. This article neglects such avenues as they generally do not add to the constructive discourse.

definiteness, and (viii) the (role of the) Observer, to name a few:

Determinism: Determinism is the concept that the future events are a function of the past. Interpretations such as Copenhagen interpretation submit themselves to the indeterminism seen in measurements, whereas theories, *eg.*, that involve hidden variables are inherently deterministic, but make sure that the measurements in their deterministic world still obey Bell's inequalities and Heisenberg principles.

Ontic wave function: The ontology of the wave function concerns with the factuality of the wave function. The physical implications are not always the same across the interpretations. In interpretations such as the Copenhagen interpretation, the wave function does not really imbue any physical meaning, but only acquires physical meaning through the Born's rule. But in many worlds approach, the wave function is a literal reading of the underlying possibilities. This aspect concerns with the physical meaning imbued by the wave function.

History (of the quantum mechanical system): *This aspect questions, if we rewind the clock do we recover the quantum states. In models like many worlds, the description of the wave function is determined by the choice of what the writer considers complete, so a sense of unique history may at best only be abstract. But in other interpretations discussed here, the history of the system is unique.*

(Need for) Hidden variables: This aspect concerns with the need for additional parameters to discuss the reality, beyond the measurable parameters. Hidden variables approach stands alone here in its need for additional parameters to interpret quantum mechanics. One could also argue that the spontaneous collapse theories also include hidden variables by incorporating a collapse term.

(Need of) Wave function collapse: Wave function collapse, is inherent in almost all models except for in the case of many worlds interpretations where there is no need for a wave function collapse.

Non-locality (in the wave function): It was seen that in order to satisfy Bell's inequalities or Heisenberg uncertainty relationships, in interpretations involving hidden variables, one had to sacrifice locality (the idea that events are influenced at a distance), which was hard to reconcile with special theory of relativity, or sacrifice the independence in choice of measurement. This aspect refers to the former. Hidden variables approach and spontaneous collapse interpretations sacrifice locality, whereas one might argue that there is no need for hidden variables in the other two so they escape having to make this sacrifice.

Counter factual definiteness (CDF): *Counter factual definiteness speaks to the meaningfulness of definiteness in the measurements that have not been performed. For example, what could we say about an independent particle whose position was measured, destroying its momentum definition, if instead its momentum was measured? If we could say something about the momentum measurement which could have been performed instead of the position measurement which was performed, then the theory can be said to be definite. One could compare this aspect with the determinism that the models were evaluated upon before.*

(Role of the) Observer: The chief method to reconcile with wave function collapse was to attribute it to the observer who made the measurement. This aspect concerns with the need of the observer effect in interpretations. Some interpretations do not need a wave function collapse, like the many worlds interpretation, so the many worlds interpretation has no observer effect. Similarly, deterministic interpretations most definitely no not need an observer effect. However, in the converse case, involving an indeterministic theory, one does not always require an observer effect, *e.g.* the spontaneous collapse theory.

One could base their choice of interpretation based on any of the above aspects. Note that evaluation based on some of these aspects are ill-defined in some interpretations (italicized), *e.g.* asking if the universe has unique history is ill-posed in many worlds interpretations, since the description of the events and its associated possibilities is defined by the evaluator upon the description they choose. So leaving such cases, the remaining features have been summarized in Table 1.

Aspect Interpretation	Deterministic	Ontic WF	Hidden Var.	WF Collapse	Non-locality	Observer
Copenhagen (Bohr-Heisenberg)	✗	✗	✗	✓	✗	✓
Many worlds (Everett)	✓	✓	✗	✗	✗	✗
Hidden var. (de Broglie-Bohm)	✓	✓	✓	✓	✓	✗
Spont. collapse (GRW)	✗	✓	✗	✓	✓	✗

Table 1: Aspects according to which interpretations can be evaluated upon. Coloring of the columns has been discussed in the text.

Occam's razer is a philosophical hypothesis that posits that one must use the simplest technique, or fewest possible adjustments, to explain as many observations as possible. In simple words it means "keeping things simple" is often (not necessarily) the right way to explain things. Arranging interpretations according to a table like Table 1, as cherry picked as the aspects are, is a way to analyze the interpretations upon their merit. One could imagine evaluating the interpretations along various such aspects, and making a table far encompassing than the one shown here. Such a table makes a good case for generating a preference list according to Occam's razer [¶]. The best interpretation requires as few issues, such as non-locality, to interpret quantum mechanics; issues which are hard to reconcile with the body of physics empirically verified to be objectively true today. Evaluation based on such aspects are marked in red in Table 1 (so a tick mark in one of these aspects marked red is a negative remark). The best interpretation also provides sufficient basis for the elements already inherent to description of quantum mechanics, such as the wave function that arises directly from the Schrödinger equation. There is precedent within classical physics to expect physical meaning out of the elements of a basic theory. For example, the electromagnetic field was thought to be a mathematical construct, and pre-Maxwellian physicists considered the concept of a field to simply be a fictitious, with only the force calculated using the field as a manifest physical quantity that could be measured. Modern understanding of electromagnetic fields transcends from them being mere mathematical constructs, and they are treated as physical quantities now. Such aspects are marked in green in Table 1 (so a tick mark in one of these aspects marked green is a positive remark). While the aspects in white represent those qualities which are sometimes ill-defined, or simply are agnostic to the test of Occam's razor [‖].

[¶] The author is self-admittedly a many-worlds proponent.

[‖] It is left as an activity to the reader to generate their own preference list!

A concept relevant here is an analog of the argument of the "god of gaps" for science. The "god of gaps" argument for science, is a call for scientific investigation of the gaps in our knowledge, instead of attributing it to a deity [62]. Similar to the "god of gaps" argument for science, one could make a similar argument calling for understanding the foundations of quantum mechanics. Certain interpretations take a very "as is" approach to the aspects that lead to the paradoxes. Particularly, this purview would be antithetical to the instrumentalist's interpretation, of "shut up and calculate". Though Popperian realism provides a certain degree of protection (from fallacies, in that it assures us of the validity of the predictions of quantum mechanics regardless of the interpretations) to the instrumentalist's interpretation, the instrumentalist's interpretation maybe akin to the "god of the gaps" defense of a deity. If one adopts an instrumentalist's interpretation of quantum mechanics, then one does not need to grapple with the paradoxes or the aspects in Table 1, and our curiosity in thus put out. Therefore, one could make an argument to a call for investigating the foundations of quantum mechanics, to keep our curiosity alive and thereby make progress in our understanding of the foundations quantum mechanics.

6 Conclusion

Paradoxes within quantum mechanics have been clear since the early days of quantum mechanics infancy. In this article, four classes of interpretations were introduced, *viz.* the Copenhagen interpretation, Everett's many worlds interpretation, hidden variables approach (de Broglie-Bohm primarily), and spontaneous collapse interpretation (particularly the GRW theory). These interpretations differ in what they have to say about some key aspects with which quantum mechanics can be evaluated upon, *viz.* determinism, ontology and collapse of wave functions, need for hidden variables, (non-) locality, definiteness, and the role of the observer. But these are not the only class of interpretations. Moreover, the aspects of evaluating the interpretations presented in this article are an incomplete list.

There are also hybrid theories like the consistent histories interpretation [63], which combines aspects of spontaneous collapse interpretations ad hidden variables to attain a consistent history unlike the many worlds interpretation. Some theories have modified aspects, of the above interpretations, such as the transactional interpretation [64] which uses the retro-causal aspect from hidden variables interpretation. This article has not explicitly described certain other interpretations like quantum information interpretation [72], relational interpretation [66], quantum logic [67], conscience-collapse [68], stochastic quantum mechanics [69], quantum-Darwinism [70], ensemble interpretation [71], and quantum Bayesian interpretations [72, 73].

While the Copenhagen interpretation is one of the oldest and arguably the most wide spread, hidden variables and spontaneous collapse approaches have niche support. More recently, the many worlds interpretation is experiencing something of a renaissance with recent works from Vaidman [74], Sebens and Carroll [75]. **While each interpretation has a particular take on the paradoxes (especially the ones involving the effect of the observer, measurement, and wave function collapse), and some interpretations even tend themselves to possible experimental check, no particular interpretation can be eliminated beyond reasonable doubt at the moment; with the possibility that many of them may actually be comparable, stating the same underlying truth in multiple ways. An appeal to Occam's razor maybe made to justify a preference of interpretations, and finally a call for investigating the gaps in our understanding could lead to new realizations regarding the foundation of quantum mechanics.**

References

* *Secondary sources.*

[1] R. Descartes, “The Meditations, and selections from the Principles of René Descartes”, Chicago: Open Court, 1903 (1641). URL: archive.org/details/117693832.

[2] I. Newton, “Opticks”, Oxford University Press (1730). URL: archive.org/details/opticksoratreat00newtgoog.

[3] R. Hooke, “Micrographia”, London: Printed for James Allestry, printer to the Royal Society, and sold in shop at the Rose and Crown in Duck-Lane (1667). URL: archive.org/details/micrographiaorso1670hook.

[4] C. Huygens, “Treatise On Light”, Macmillan and Company Ltd. (1690). URL: archive.org/details/treatiseonlight031310mbp.

[5] T. Young, “II. The Bakerian Lecture. On the theory of light and colours”, Phil. Trans. R. Soc. **92**, (1802): 12-48. DOI: 10.1098/rstl.1802.0004.

[6] J. C. Maxwell,“VIII. A dynamical theory of the electromagnetic field”, Phil. Trans. R. Soc. **155**, (1865): 459-512. DOI: 10.1098/rstl.1865.0008.

[7] A. H. Compton, “A Quantum Theory of the Scattering of X-rays by Light Elements”, Phys. Rev. **21**, 483 (1923). DOI: 10.1103/PhysRev.21.483.

[8] H. Hertz, “Ueber einen Einfluss des ultravioletten Lichtes auf die electrische Entladung”, Ann. Phys. **267**, 8 (1887): 983-1000. DOI: 10.1002/andp.18872670827.

[9] M. Faraday, “VIII. Experimental researches in electricity.-thirteenth series”, Phil. Trans. R. Soc. **128**, (1838): 125-168. DOI: 10.1098/rstl.1838.0009.

[10] E. Goldstein, “ ”Vorläufige Mittheilungen über elektrische Entladungen in verdünnten Gasen (*Preliminary communications on electric discharges in rarefied gases*)”, Monatsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin (*The*

Royal Prussian Academy of Sciences of Berlin), (1877): 279-295. URL: books.google.com/books?id=7-caAAAAYAAJ.

[11] J. J. Thomson, "XL. Cathode Rays", Philosophical Magazine and Journal of Science **44**, 269 (1897): 293-316. DOI: 10.1080/14786449708621070.

[12] L. de Broglie, "XXXV. A Tentative Theory of Light Quanta", Philisophical Magazine Letters **86**, 7 (1924): 411-423. DOI: 10.1080/14786442408634378.

[13] C. Jönsson, "Electron Diffraction at Multiple Slits", Am. J. Phys. **42**, 4 (1974). DOI: 10.1119/1.1987592.

[14] A. Einstein, "Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt (*On a Heuristic Point of View about the Creation and Conversion of Light*)", Ann. Phys. **17**, 6 (1905): 132-148. DOI: 10.1002/andp.19053220607.

[15] M. Planck, "Uber das Gesetz der Energieverteilung im Normalspektrum", Ann. Phys. **309**, 3, 553–563 (1901). DOI: 10.1002/andp.19013090310.

[16] N. Bohr, "I. On the constitution of atoms and molecules", Philosophical Magazine and Journal of Science **26**, 151 (1913): 1-25. DOI: 10.1080/14786441308634955.

[17] I. Newton, "Philosophiae Naturalis Principia Mathematica", New-York: Published by Daniel Adee and translated from Latin to English by A. Motte, 1730, (1687). URL: archive.org/details/newtonspmathema00newtrich.

[18] E. Schrödinger, "Quantisierung als Eigenwertproblem" (*Quantization as an Eigenvalue Problem*)", Ann. Phys. **384**, 361 (1926). DOI: 10.1002/andp.19263840404.

[19] W. Heisenberg, "Über quantentheoretishe Umdeutung kinematisher und mechanischer Beziehungen (*About quantum theoretical reinterpretation of kinematic and mechanical relationships.*)", Zeitschrift für Physik **33**, (1925): 879-893. DOI: 10.1007/BF01328377.

[20] * Y. Aharonov and D. Rohrlich, "Quantum Paradoxes: Quantum Theory for the Perplexed", WileyVCH, Weinheim (2005). ISBN: 978-3-527-40391-2. URL: wiley.com/en-us/Quantum+Paradoxes%3A+Quantum+Theory+for+the+Perplexed-p-9783527403912. D. Rohrlich, "Errors and Paradoxes in Quantum Mechanics", Compendium of Quantum Physics, eds. D. Greenberger, K. Hentschel, F. Weinert, Springer Berlin-Heidelberg, Berlin-Heidelberg, Germany (2009): 211-220. DOI: 10.1007/978-3-540-70626-7_68.

[21] F. Bacon, "Novum organum", Oxford: The Clarendon press, 1889, ed. T. Fowler (1620). URL: archive.org/details/baconsnovumorgan00bacoiala.

[22] E. Rutherford, "LXXIX. The Scattering of α and β Particles by Matter and the Structure of the Atom", The Philosophical Magazine **6**, 21 (1911). DOI: 10.1080/14786440508637080.

[23] R. A. Millikan, "On the Elementary Electrical Charge and the Avogadro Constant", Phys. Rev. **2**, 109 (1913). DOI: 10.1103/PhysRev.2.109.

[24] N. Bohr, "Discussion with Einstein on epistemological problems in atomic physics", Philosopher-Scientist, ed. Paul A. Schilpp, Tudor Publishing Company, New York (1951): 201-41. URL: philpapers.org/rec/BOHDWE.

[25] Scully et. al., "Quantum optical tests of complementarity", Nature **351**, (1991): 111–116. DOI: 10.1038/351111a0.

[26] W. Heisenberg,"Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik (*About the descriptive content of quantum theoretical kinematics and mechanics*)", Zeitschrift für Physik **43**, 3-4 (1927): 172-198. DOI: 10.1007/BF01397280.

[27] L. Landau and R. Peierls, "Erweiterung des Unbestimmtheitsprinzips für die relativistische Quantentheorie (*Extension of the indeterminacy principle for the relativistic quantum theory*)", Zeitschrift für Physik **69** (1931): 56-59. DOI: 10.1007/BF01391513.

[28] * G. S. Kirk, J. E. Raven, M. Schofield, "The Presocratic Philosophers: A Critical History with a Selection of Texts" 2nd ed., Cambridge University Press. ISBN 0-521-27455-9. URL: books.google.com/books?id=kFpd86J8PLsC.

[29] Aristotle, "Physics", Trans. by R. P. Hardie and R. K. Gaye, The Internet Classics Archive. II 9 (350 BC). URL: classics.mit.edu/Aristotle/physics.2.ii.html#646.

[30] B. Misra and E. C. G. Sudarshan, "The Zeno's paradox in quantum theory", J. Math. Phys. **18**, 756 (1977). DOI: 10.1063/1.523304.

[31] D. Leibfried, R. Blatt, C. Monroe, D. Wineland, "Quantum dynamics of single trapped ions", Rev. Mod. Phys. **75**, 281 (2003). DOI: 10.1103/RevModPhys.75.281.

[32] C. D. Anderson, "The Positive Electron", Phys. Rev. **43**, 491 (1933). DOI: 10.1103/PhysRev.43.491.

[33] A. Einstein, B. Podolsky, N. Rosen, "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete", Phys. Rev. **47**, 777 (1935). DOI: 10.1103/PhysRev.47.777.

[34] A. Einstein, "Zur Elektrodynamik bewegter Körper (*On the electrodynamics of moving bodies*)", Ann. Phys. **322**, 891 (1905). DOI: 10.1103/PhysRev.47.777.

[35] J. S. Bell, "On the Einstein Podolsky Rosen paradox", Physics Physique Fizika **1**, 195 (1964). DOI: 10.1103/PhysicsPhysiqueFizika.1.195.

[36] S. J. Freedman and J. F. Clauser, "Experimental Test of Local Hidden-Variable Theories", Phys. Rev. Lett. **28**, 938 (1972). DOI: 10.1103/PhysRevLett.28.938.

[37] Y. Aharonov and D. Z. Albert, "Is the usual notion of time evolution adequate for quantum-mechanical systems? II. Relativistic considerations", Phys. Rev. D **29**, 228 (1984). DOI: 10.1103/PhysRevD.29.228.

[38] * J. A. Wheeler and W. H. Zurek, “Quantum Theory and Measurement”, Princeton University Press (1983). URL: press.princeton.edu/books/hardcover/9780691641027/quantum-theory-and-measurement.

[39] * H. Wimmel, “Quantum Physics & Observed Reality: A Critical Interpretation of Quantum Mechanics”, World Scientific (1992). DOI: 10.1142/1691.

[40] M. Born, “Zur Quantenmechanik der Stoßvorgänge”, Zeitschrift für Physik **37**, (1926): 863-867. DOI: 10.1007/BF01397477.

[41] * S. Saunders, J. Barrett, A. Kent, D. Wallace, “Many Worlds?: Everett, Quantum Theory, & Reality”. Oxford University Press, (2010). DOI: 10.1093/acprof:oso/9780199560561.001.0001.

[42] H. Everett et al., Eds. B. S. DeWitt and G. R. Neill, “The Many-Worlds Interpretation of Quantum Mechanics”, Princeton Series in Physics, Princeton, NJ, Princeton University Press (1973). URL: press.princeton.edu/books/hardcover/9780691645926/the-many-worlds-interpretation-of-quantum-mechanics.

[43] E. Farhi, J. Goldstone, S. Gutmann, “How probability arises in quantum mechanics”, Annals of Physics **192**, 2 (1989): 368-382. DOI: 10.1016/0003-4916(89)90141-3.

[44] D. Deutsch, “Quantum theory of probability and decisions”, Proc. R. Soc. Lond. A **455**, (1999): 3129-3137. DOI: 10.1098/rspa.1999.0443.

[45] S. Saunders, “Derivation of the Born’s rule from operational assumptions”, Proc. R. Soc. Lond. A. **460**, (2004): 1771-1788. DOI: 10.1098/rspa.2003.1230.

[46] D. Parfit, “Personal Identity”, The Philosophical Review **80**, 1 (1971): 3-27. DOI: 10.2307/2184309.

[47] D. K. Lewis, “On the Plurality of Worlds”, Wiley-Blackwell Publishers, (1986). URL: www.wiley.com/en-us/On+the+Plurality+of+Worlds-p-9780631224266.

[48] J. S. Bell, “Speakable and unspeakable in quantum mechanics: Collected papers on quantum philosophy”, Cambridge University Press, (2004). DOI: 10.1017/CBO9780511815676.

[49] * N. D. Mermin, “Quantum mysteries for anyone”, The Journal of Philosophy **78**, (1981): 397-408. DOI: 10.2307/2026482.

[50] * H. Price, “Time’s arrow & Archimedes’ point: New directions for the physics of time”, Oxford University Press, (1997). DOI: 10.1093/acprof:oso/9780195117981.001.0001.

[51] L. de Broglie, “La mécanique ondulatoire et la structure atomique de la matière et du rayonnement *(Wave mechanics and the atomic structure of matter and radiation)*”, J. Phys. Radium **8** , (1927): 225-241. DOI: 10.1051/jphysrad:0192700805022500.

[52] D. Bohm, “A Suggested Interpretation of the Quantum Theory in Terms of ”Hidden” Variables”, Phys. Rev. **85**, 166 (1952). DOI: 10.1103/PhysRev.85.166.

[53] S. Kochen, E. P. Specker, ”The problem of hidden variables in quantum mechanics”, Journal of Mathematics and Mechanics, **17**,1 (1967): 59-87. DOI: 10.1512/iumj.1968.17.17004.

[54] * D. Z. Albert, “Quantum Mechanics and Experience”, Harvard University Press, (1995). URL: www.hup.harvard.edu/catalog.php?isbn=9780674741133.

[55] G. C. Ghirardi, A. Rimini, T. Weber, “Unified dynamics for microscopic and macroscopic systems”, Physical Review D **34**, 2 (1986): 470-491. DOI: 10.1103/PhysRevD.34.470.

[56] A. Bassi, ”Collapse models: analysis of the free particle dynamics”, Journal of Physics A: Mathematical and General **38**, 14 (2005). DOI: 10.1088/0305-4470/38/14/008.
“Do dynamical reduction models imply that arithmetic does not apply to ordinary macroscopic objects?”, The British Journal for the Philosophy of Science **50**, 1 (1999): 49-64. DOI: 10.1093/bjps/50.1.49.
“Discussion. More about dynamical reduction and the enumeration principle”, The British Journal for the Philosophy of Science **50**, (1999): 719-734. DOI: 10.1093/bjps/50.4.719.

[57] * S. Carroll, “Something Deeply Hidden: Quantum Worlds and The Emergence of Spacetime”, Penguin Random House Publisher (2020). URL: www.penguinrandomhouse.com/books/566988.

[58] * J. T. Cushing, “Quantum mechanics: Historical contingency and the Copenhagen hegemony”, University of Chicago Press (1994). URL: press.uchicago.edu/ucp/books/book/chicago/Q/bo3646498.

[59] N. D. Mermin“Could Feynman Have Said This?”, Physics Today **57**, 5 (2004): 10. DOI: 10.1063/1.1768652.

[60] K. R Popper, “Conjectures and Refutations: The Growth of Scientific Knowledge, London: Routledge 2003 (1963), URL: archive.org/details/conjecturesrefut0000popp.

[61] * A. Cabello, “Interpretations of Quantum Theory: A Map of Madness”, Cambridge University Press (2017). DOI: 10.1017/9781316494233.009.

[62] N. dG. Tyson, “The Perimeter of Ignorance”, Natural History Magazine (2005). URL: www.naturalhistorymag.com/universe/211420/the-perimeter-of-ignorance.

[63] R. B. Griffiths, “Consistent histories and the interpretation of quantum mechanics”, Journal of Statistical Physics **36**, (1984): 219-272. DOI: 10.1007/BF01015734.

[64] J. G. Cramer, “Transactional Interpretation of Quantum Mechanics”, Eds. D. Greenberger, K. Hentschel, F. Weinert, Compendium of Quantum Physics, Springer, Berlin (2009). DOI: 10.1007/978-3-540-70626-7_223.
“The transactional interpretation of quantum mechanics”, Rev. Mod. Phys. **58**, 647 (1986). DOI: 10.1103/RevModPhys.58.647.
“An overview of the transactional interpretation of quantum mechanics”, International Journal of Theoretical Physics **27**, (1988): 227-236. DOI: 10.1007/BF00670751.

[65] C. G. Timpson, Quantum Information Theory and the Foundations of Quantum Mechanics", Oxford University Press (2013). DOI: 10.1093/acprof:oso/9780199296460.001.0001.

[66] C. Rovelli, "Relational quantum mechanics", International Journal of Theoretical Physics **35**, (1996): 1637-1678. DOI: 10.1007/BF02302261.

[67] G. Birkhoff and J. Von Neumann, "The Logic of Quantum Mechanics", Annals of Mathematics **37**,4 (1936): 823-843. DOI: 10.2307/1968621.

[68] "Symmetries and Reflections, Scientific Essays", American Journal of Physics **35**, 1169 (1967). DOI: 10.1119/1.1973829.

[69] M. Davidson, "The origin of the algebra of quantum operators in the stochastic formulation of quantum mechanics", Letters in Mathematical Physics **3**, (1979): 367-376. DOI: 10.1007/BF00397209.

[70] A. M. Burke, R. Akis, T. E. Day, Gil Speyer, D. K. Ferry, B. R. Bennett, "Periodic Scarred States in Open Quantum Dots as Evidence of Quantum Darwinism", Phys. Rev. Lett. **104**, 176801 (2010). DOI: 10.1103/PhysRevLett.104.176801.

[71] L. E. Ballentine, "The Statistical Interpretation of Quantum Mechanics", Rev. Mod. Phys. **42**, 358 (1970). DOI: 10.1103/RevModPhys.42.358.

[72] C. G. Timpson, "Quantum Bayesianism: A study", Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics **39**, 3 (2008): 579-609. DOI: 10.1016/j.shpsb.2008.03.006.

[73] N. D. Mermin, "Commentary: Quantum mechanics: Fixing the shifty split", Physics Today **65**, 7, 8 (2012). DOI: 10.1063/PT.3.1618.

[74] L. Vaidman, "Probability in the Many-Worlds Interpretation of Quantum Mechanics", Eds. Y. Ben-Menahem, M. Hemmo, Probability in Physics, The Frontiers Collection, Springer, Berlin (2012). DOI: 10.1007/978-3-642-21329-8_18.

[75] C. T. Sebens, S. M. Carroll, “Self-locating Uncertainty and the Origin of Probability in Everettian Quantum Mechanics”, The British Journal for the Philosophy of Science **69**, 1 (2018) 25-74. DOI: 10.1093/bjps/axw004.