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Embodied Explorations of Slope

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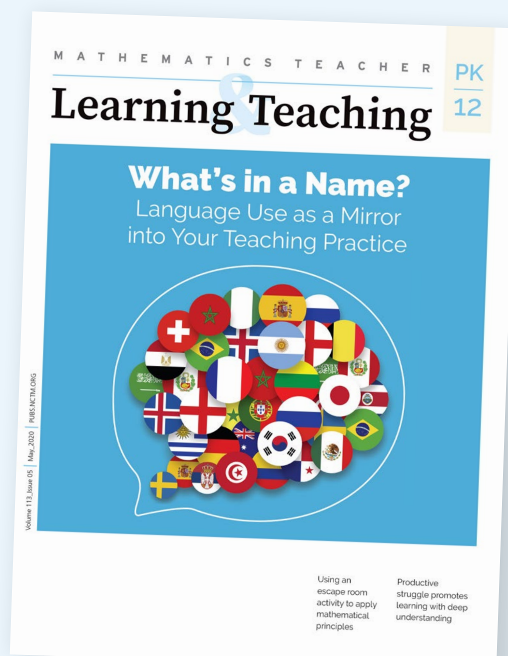

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Embodied Explorations of Slope

Promote student engagement and deep thinking about the meaning of slope.

Rebecca Hudson, Payeton Stevens-Balducci, Liza Bondurant, Lee Dean, and Catherine Putnam

Our bodies help us learn, and our bodies may know something before we can articulate it. Using gestures such as arm motions, manipulatives such as Cuisenaire rods or "C-rods" (Cope, 2015), and simulations such as Desmos explorations are examples of embodiment. Embodied explorations are body-based learning experiences where learners have opportunities to make sense of and convey ideas with their bodies. Embodiment can help us formalize our language about a concept (Abrahamson et al., 2020; Nathan, 2022). In this manuscript, we share how we created and used embodied explorations aimed at promoting student engagement and deep thinking about the meaning of slope.

Although the Common Core State Standards (CCSS) explicitly address slope in eighth grade (e.g., 8.EE, 8.F, 8.G), the proportional reasoning concepts build on students' K–6 work (e.g., 3.OA, 4.OA, 4.MD, 4.NF, 5.OA, 5.NF, 6.RP) and lay the foundation for several high-school content standards (e.g., HS.A, HS.F, HS.G) (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Although many high school students memorize the slope formula in middle school and can procedurally calculate slopes, few have a deep understanding of the concept of slope necessary for advancing their mathematics studies (National Council of Teachers of Mathematics, 2014; Peck, 2020). The specific goal of the embodied explorations was for students to understand that slope is a ratio and that the ratio is defined by the change of the dependent variable (y) divided by the change of the independent variable (x). Developing a conceptual understanding of this ratio for lines will build the foundation for when students learn to use the tangent function to find the angle that the line or curve makes with the independent axis. Through the embodied explorations, students develop an understanding that slope is a measure of the line's (or the curve's) steepness, in which a larger magnitude denotes a steeper ascent/descent from the horizontal. Each of the authors wore teacher and researcher hats during the design, implementation, and writing stages of this participatory research project.

OVERVIEW OF THE LESSON

These embodied explorations were taught at a suburban public high school, in the southern United States, that serves approximately 1,700 ninth-to-twelfth-grade students. We implemented the lesson in an Algebra II class with 15 students that meets for a 100-minute block of time. Students studied slope in their eighth grade and Algebra I classes, but they often needed a refresher. The embodied explorations were designed to help students develop their abilities to explain the meaning of slope, what the formula represents, and real-world applications of slope.

We implemented embodied explorations using materials adapted from the eighth-grade Meet Slope lesson in the Illustrative Mathematics (2019) curriculum, and from the Creating a Measure of Slope lesson by the Mathematics Assessment Project (2014). We enhanced these materials by incorporating two innovative embodied explorations. Our embodied explorations included (1) a physical exploration where students acted the vertical and horizontal changes out by walking up and down steps on a staircase, and (2) a tactile exploration, in which students measured the vertical and horizontal lengths of slope triangles on (centimeter) grid paper using C-rods. Slope triangles are right triangles whose hypotenuses lie on the oblique line. C-rods are a set of colored wooden rods of varying lengths used to teach mathematical concepts and promote hands-on learning (Cope, 2015).

PHYSICALLY CLIMBING STAIRS

Students asked what we would be doing as they entered the room, and Ms. Hudson replied that they would

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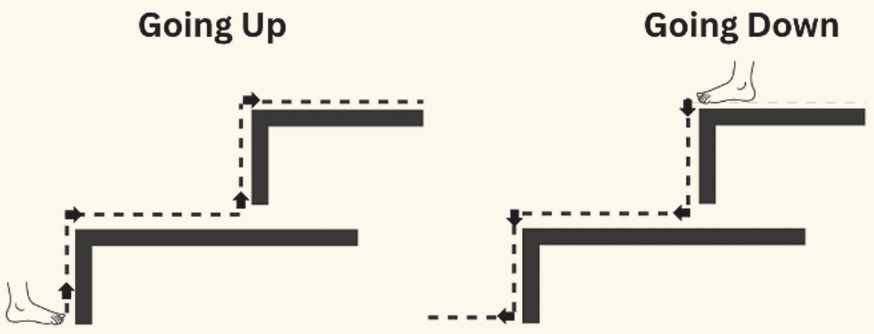
be investigating a few slope explorations. A student exclaimed, "Oh the difference in the x-values over the difference in the y-values." This unprovoked comment demonstrates a common mistake many students make. Rather than correcting the student or giving a lecture, Ms. Hudson replied that we would return to the formula after our explorations. This intentional decision was deliberate because we wanted students to develop a conceptual understanding of the meaning of slope through embodied explorations, rather than memorizing a procedural formula that did not make sense.

We initially engaged students by having them physically climb a staircase in their school and consider the slopes of different stairs in their communities. This portion of the lesson took about 20 minutes. The stairs exploration had several purposes. For one, through the embodied action of physically climbing the steps, students connected the concept of slope to steepness. Secondly, when students moved up and down the steps, we discussed the physical motion of their feet. This was an opportunity to connect gestures to precise vocabulary. Specifically, we discussed how students moved their feet vertically and horizontally while moving up and down the steps. Students are moving up and down the stairs by following the contour or edge of the stairs. Especially when moving downward, students need to start on the edge of the top stair and then follow the contour or edge to allow for the correct connection for the slope (see Figure 1). This connection inspired conversations about positive versus negative slopes. Finally, students realized they could measure the vertical and horizontal dimensions with standard or non-standard units.

We asked students to physically climb the stairs in slow motion and then carefully observe how they moved their bodies as they climbed the stairs (see Figure 2). When students are standing and moving around, they are more engaged and more likely to communicate, with both verbal and non-verbal gestures (Liljedahl, 2020). The whole-class discussion at the stairs resembled the exchange below (however, this is not a transcript).

Teacher: What direction(s) is/are your leg(s) moving?
Student: Up and forward on the way up and down, and forward on the way down.
Teacher: What distance(s) is/are your leg(s) moving?
Student: It seems like the horizontal distance is quite a bit longer than the vertical distance.
Teacher: How much work does it take to climb the stairs?
Student: Not too much work. More work than just walking down the hall, but not as much work as climbing a ladder.
Teacher: How could you quantify the work required to climb the stairs?
Student: The hard part is going up. The farther we have to climb up for every amount we can rest by stepping forward, the more work it takes.
Teacher: How does going up compare to going down the steps?
Student: Going up is like positive work, and going down is like negative work.
Student: The direction that our feet move changes too.

Figure 1 Directions of Stair-Climbing Movement



Start at the edge of the stair and follow the contour and/or edge of the stair to find the appropriate slope.

Teacher: Can someone explain how to properly go down the stairs to find the appropriate slope?

Student: You first have to start at the edge of the stairs. Then, follow the contour of the stairs. Thus, going down vertically and out horizontally.

Teacher: How could you model skipping steps?

Student: We'd have to add all the verticals and compare it to the sum of all the horizontals.

Teacher: If we assume the comparison is a ratio with the vertical in the numerator, this is a solid observation.

Upon returning to the classroom, the teacher displayed images of stairs in the community (see Table 1). Students worked with a peer to estimate the slopes. The purpose of this discussion was to assess if students identified the vertical change over the horizontal change and to incorporate culturally relevant content (Gutiérrez, 2007). The discussion resembled the following exchange (however, this is not a transcript).

Student: All the slopes are between about one-quarter and three-quarters, and most are about one-half.

Figure 2 Students Climbing Stairs in Slow Motion

Teacher: Why do you think that is?

Student: Probably so they are not too little—or too much—work to climb.

Student: Yeah, like if it was less than that we would need tons of steps and it would waste a lot of space.

Student: I agree. Also, if it was too much, it would be too tiring to climb and it could be dangerous.

Teacher: How were the vertical and horizontal distances measured?

Student: Some were with standard tools, like a measuring tape.

Student: Yeah, but some were measured with non-standard things, like hands or cups.

Teacher: Does it matter whether we measure with standard or non-standard units?

Student: Nope, as long as we measure both the vertical and horizontal distances with the same units.

Student: That's pretty cool, because like if I don't have a tape measure or ruler, I can use almost anything to measure.

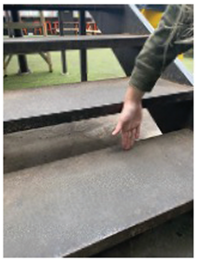
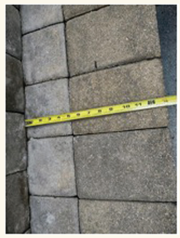
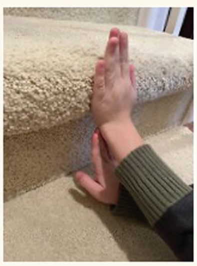
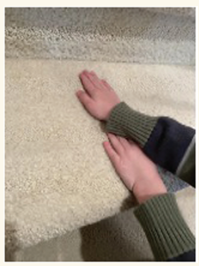
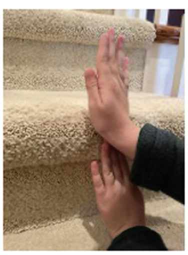
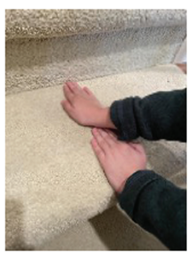
Finally, the teacher asked a student who had described slope at the beginning of class as "the difference in the x-values over the difference in the y-values" how they would describe slope after the stair exploration. The student replied, "It's the steepness, the vertical change over the horizontal change like steeper steps would have a greater slope and require more work to climb." The student's reply provides evidence of an improved understanding of the definition of slope.

TACTILE EXPLORATION TO CONNECT SLOPE TO SIMILAR TRIANGLES

Next, students were given a C-rod key (see Figure 3), a set of C-rods, and a piece of centimeter graph paper with a diagonal line drawn on it (see Figure 4). Notice that the C-rod lengths are measured in centimeters, making C-rods an attractive alternative to centimeter rulers. Students spent about 20 minutes using the C-rods and any two points on the line to determine the slope by creating slope triangles above and below the line (see Figures 4 and 5 and Table 2).

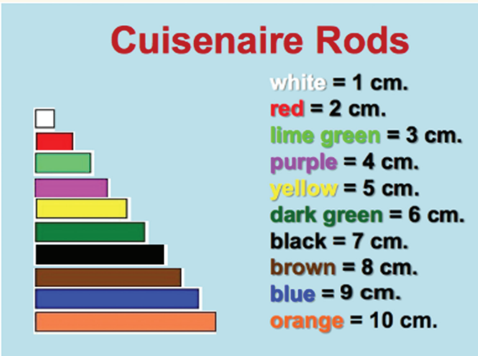
There were three main reasons why we had students measure the dimensions with C-rods. For one, the C-rods provide engaging tactile and visual stimulation (Cope, 2015). Secondly, students often struggle to count the faint blue lines on standard grid paper. The vibrantly colored C-rods and the measurement key helped students accurately measure the dimensions.

Table 1 Stairs in Our Community

Location	Vertical change	Horizontal change	Slope
School with inches as a standard unit of measurement			$\frac{4.5}{17} \approx 0.26$
Local restaurant with teacher's hands as non-standard unit of measurement			$\frac{1}{2} = 0.5$
Local theater with inches as a standard unit of measurement			$\frac{7}{13} \approx 0.54$
Local park with coffee cups as a non-standard unit of measurement			$\frac{1}{2} = 0.5$
Author's home with five-year-old's hands as non-standard unit of measurement			$\frac{1.6}{2.2} \approx 0.73$
Author's home with nine-year-old's hands as non-standard unit of measurement			$\frac{1.4}{1.9} \approx 0.73$

Finally, using the C-rods reinforced the fact that surfaced during the Stairs in Our Community discussion, that we can measure with non-standard units.

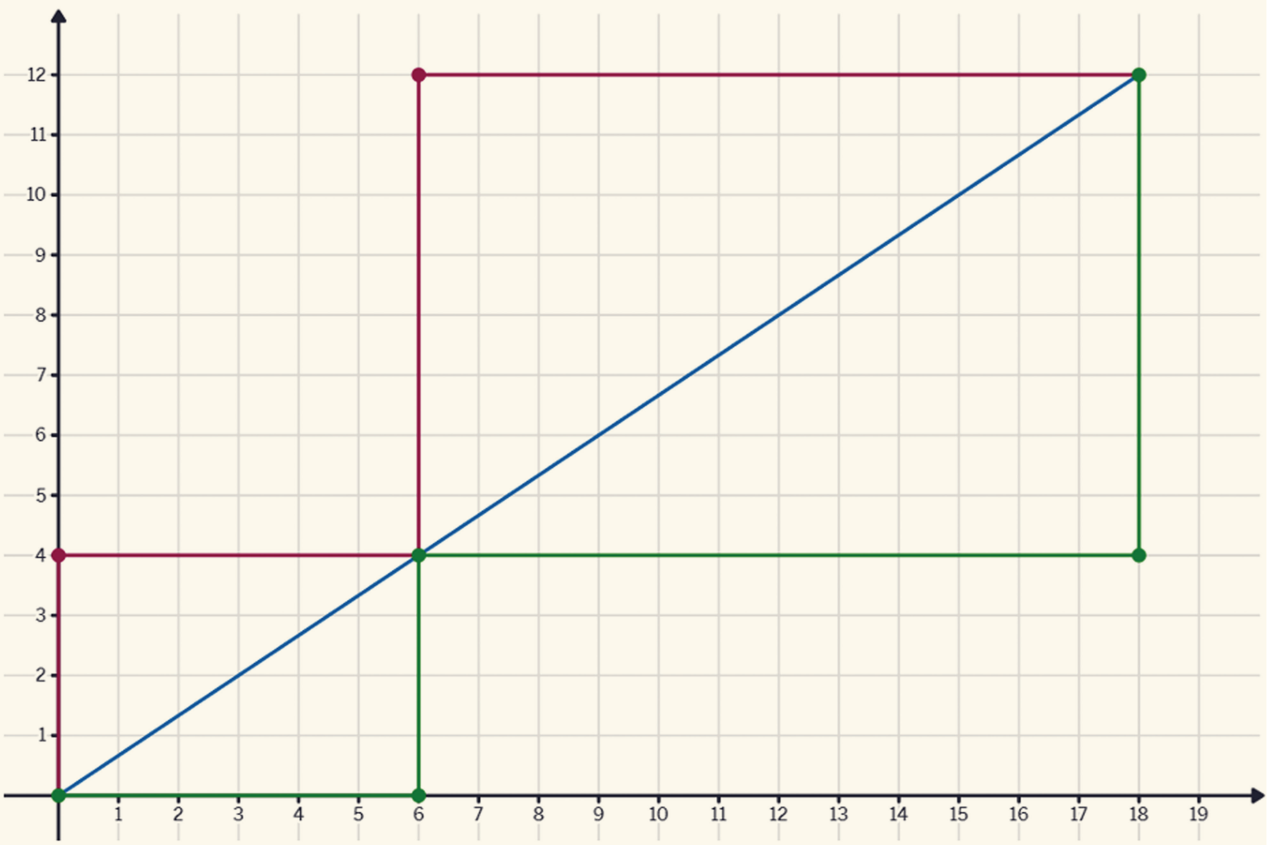
Figure 3 Measurement Key for C-Rods



During this portion of the lesson, the dialogue resembled the following exchange (however, this is not a transcript).

Teacher: What is the relationship between the triangles you drew?
Student: Their hypotenuses are all along the same line.
Student: They all have the same ratio of the length of the vertical leg to the length of the horizontal leg.
Student: I agree, the triangles are similar.
Teacher: Excellent! What do we know about similar triangles?
Student: Their side lengths are proportional.
Teacher: Awesome! How are you measuring the sides?
Student: We used trial and error to see which C-rods matched the length of the side. The color told us the length.
Teacher: What triangle are you using to determine the slope of the line?
Student: It doesn't matter which triangle we use, because they are all along the same line.

Figure 4 Similar Triangles Along the Same Line on Centimeter Graph Paper



Student: I agree. They both have the same slope. It does not matter whether you make a big triangle, a little triangle, a triangle above the line, or a triangle below the line.

Teacher: How are you using the triangle to determine the slope of the line?

Student: The slope is the vertical change divided by the horizontal change.

Teacher: Does this remind you of anything?

Student: Yes . . . climbing the stairs!

Teacher: Does it matter which two points you use?

Student: Nope, because the ratio of vertical change over horizontal change will be the same.

Teacher: Why do two different slope triangles along the same line have the same slope?

Student: Because we are building right triangles with their hypotenuses along the same oblique line. The steepness of right triangles will be the same for any right triangles we build along the same oblique line.

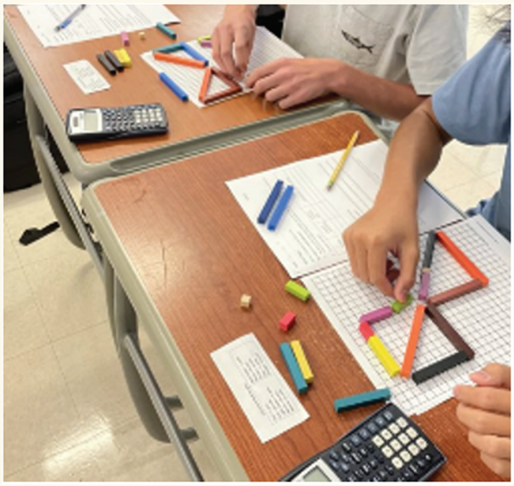
Teacher: What if the oblique line was going down from left to right?

Student: We would have to take the sign of the vertical change into account.

Teacher: Do y'all agree? Can someone please add on?

Student: If the slope triangle was below the oblique line with a negative slope, we would have a negative in the vertical direction because we are moving down, over a positive in the horizontal direction because we are moving right. If the slope triangle was above the oblique line with a negative slope, we would have a positive in the vertical direction because we are moving up, over a negative in the horizontal direction because we are moving left. Either way, the slope would be negative.

Figure 5 Students Using C-rods to Measure Dimensions of Similar Triangles Above and Below a Line



SYNTHESIZING KEY CONCEPTS

We synthesized the lesson by asking students to draw lines with the following slopes: 3, ½, −3, −½, 0, and undefined. We circulated while students worked and engaged in a discussion. We were impressed by how students made use of embodiment in their explanations (e.g., holding their forearms vertically). This is evidence that the use of embodiment helps them to retain and recall these concepts. The discussion resembled the following dialogue (however, this is not a transcript).

Teacher: How do the slopes compare?

Student: Two are positive [holding forearm slanted upward] and two are negative [holding forearm slanted downward]. Then, there's zero [holding the forearm horizontally] and undefined [holding the forearm vertically].

Teacher: Okay, I see you making some arm gestures. Can you explain what your gestures mean? Which lines are increasing from left to right?

Table 2 Ratios of the Vertical and Horizontal Sides of Similar Slope Triangles

Points	Length of vertical leg	Length of horizontal leg	Length of vertical leg/ length of horizontal leg
A & C	4	6	2/3
C & F	8	12	2/3

Student: The ones with positive slopes are increasing. The ones with negative slopes are decreasing.

Student: Agree, and zero is a horizontal line because the line has no steepness.

Student: Yup, and vertical lines have an undefined slope because the horizontal change is 0, and we cannot divide a number by 0.

Teacher: Okay, so which line is steeper, the one with a slope of three or the one with a slope of one-half?

Student: The one with a slope of three is more steep, for sure, because three is greater than one-half.

Teacher: Okay, so one-half is greater than negative three. Is a line with a slope of one-half steeper than a line with a slope of negative three?

Student: Nope, the one with a slope of negative three is steeper. I should have said the line whose slope has a greater absolute value will be steeper.

Teacher: Okay, so did you make the slope triangles above or below the line?

Student: It doesn't matter.

Teacher: Can you say more about why?

Student: Because the vertical and horizontal lengths are the same above and below the line.

Teacher: Okay, what if these grids were on the coordinate plane, where moving down vertically and left horizontally from the origin are considered negative directions and moving up vertically and right horizontally from the origin are considered positive directions?

Student: We could still find the slope above or below the line.

Student: Agree, for positive slopes. From left to right, we would be measuring up vertically and right horizontally. This is a positive direction over a positive direction, and a positive divided by a positive results in a positive slope. From right to left, we would be measuring down vertically and left horizontally. This is a negative direction over a negative direction, and a negative divided by a negative is a positive.

Teacher: What do others think? What about negative slopes? Could we measure those above or below the line too?

Student: Yes, it works for negative slopes too. From left to right, we would be measuring down vertically and right horizontally. This is a negative direction over a positive direction and a negative divided by a positive is a negative. From right to left, we would be measuring up vertically and left horizontally. This is a positive direction over a negative direction and a positive divided by a negative is a negative.

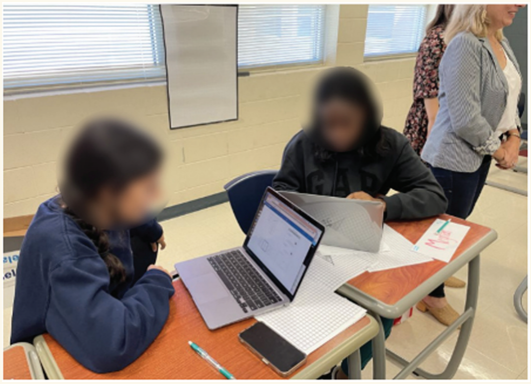
OPTIONAL VOCABULARY PRACTICE GAME

Ms. Hudson prepared a Desmos Polygraph game for students who completed the explorations before the end of the block. This game took about 15 minutes to complete. For the game, Desmos randomly pairs students. One student selects 1 out of 16 cards. Each card has an image of a line graphed on it. Next, the other student asks yes-or-no questions to determine which card their partner selected. The Desmos Polygraph allows students to actively engage in practicing the slope vocabulary and terminology that was explored with the stairs and C-rods. As an additional challenge for students who completed one round, Ms. Hudson asked students to see who could determine their partner's card with the least number of questions, which required students to be precise with their vocabulary. Students expressed that they thoroughly enjoyed the game, and their engagement was evident, as many turned their laptops in a competitive stance (see Figure 6).

FINDING SLOPE AND GRAPHING LINES

We evaluated student mastery of the lesson objectives by first asking students to reflect on their level of agreement with the following two self-assessment prompts: (1) I can draw a line on a grid with a given slope, and (2) I can find the slope of a line on a grid. Finally, students were asked to find the slope of two lines and graph two lines with given slopes. We purposefully included positive, negative, whole numbers, and fractional values for the slopes in this formative assessment.

Figure 6 Students Competing During the Slope Polygraph Game



The students' responses indicated a conceptual understanding of slope. All of their self-assessments indicated that they were confident drawing a line on a grid with a given slope and finding the slope of a line on a grid. Moreover, all students correctly found the slope of the lines and graphed lines with positive, negative, whole number, and fractional valued slopes. Following participation in these embodied explorations, students demonstrated the ability to articulate the meaning and definition of slope, comprehend representations of slope, and recognize applications of slope in real-world scenarios.

CONCLUSION

Supporting a conceptual understanding of slope is important. We hope our implementations aimed at engaging students in thinking deeply about the meaning and definition of slope inspires the reader to try embodied explorations in their classrooms.

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