A coupled large eddy simulation-synthetic turbulence method for predicting jet noise

Joshua Daniel Blake

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A coupled large eddy simulation-synthetic turbulence method for predicting jet noise

By

Joshua Daniel Blake

A Dissertation
Submitted to the Faculty of
Mississippi State University
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in the Department of Aerospace Engineering

Mississippi State, Mississippi

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A coupled large eddy simulation-synthetic turbulence method for predicting jet noise

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The noise generated by jet engines represents a significant environmental concern that still needs to be addressed. Accurate and efficient numerical predictions are a key step towards reducing jet noise. The current standard in high-fidelity prediction of jet noise is large eddy simulation (LES), which resolves the large turbulent scales responsible for the low and medium frequency noise and models the smallest turbulent scales that correspond to the high frequency noise. While LES requires significant computational resources to produce an accurate solution, it fails to resolve the noise in the high frequency range, which cannot be simply ignored. To circumvent this, in this dissertation the Coupled LES-Synthetic Turbulent method (CLST) was developed to model the missing frequencies that relate to un-resolved sub-grid scale fluctuations in the flow. The CLST method combines the resolved, large-scale turbulent fluctuations from very large eddy simulations (VLES) with modeled, small-scale fluctuations from a synthetic turbulence model. The noise field is predicted using a formulation of the linearized Euler equations (LEE), where the acoustic waves are generated by source terms from the combined fluctuations of the VLES and the synthetic
fields. This research investigates both a Fourier mode-based stochastic turbulence model and a synthetic eddy-based turbulence model in the CLST framework. The Fourier mode-based method is computationally less expensive than the synthetic eddy method but does not account for sweeping. Sweeping and straining of the synthetic fluctuations by large flow scales from VLES are accounted for in the synthetic eddy method. The two models are tested on a Mach 0.9 jet at a moderately-high Reynolds number and at a low Reynolds number. The CLST method is an efficient and viable alternative to high resolution LES or DNS because it can resolve the high frequency range in the acoustic noise spectrum at a reasonable expense.

Key words: jet noise, LES, synthetic turbulence, SEM, CAA, high frequency
DEDICATION

I dedicate this work to the curious investigators, to those who have come before, to those who now are, and to those who will come after. Keep searching for the new and the interesting. As I have been inspired by the works of others, so may this work inspire you.
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# TABLE OF CONTENTS

DEDICATION .................................................................................. ii

ACKNOWLEDGEMENTS ................................................................. iii

LIST OF TABLES ........................................................................... viii

LIST OF FIGURES .......................................................................... ix

CHAPTER

1. INTRODUCTION ......................................................................... 1
  1.1 Motivation ............................................................................. 1
  1.2 Problem Summary ................................................................. 2
  1.3 Research Goal ...................................................................... 6
  1.4 Contributions ...................................................................... 8

2. REVIEW OF JET NOISE ............................................................ 10
  2.1 Jet Flow Physics and Noise Characteristics ............................. 10
    2.1.1 The Fluid Dynamics of Jets ............................................. 11
    2.1.2 The Sweeping and Straining of Turbulent Eddies ............... 14
    2.1.3 Jet Noise Characteristics ............................................... 17
    2.1.4 Noise Sources in a Subsonic Jet ...................................... 18
  2.2 Methods for CFD-Based Jet Noise Prediction ......................... 23
    2.2.1 RANS-Based Methods .................................................... 24
    2.2.2 LES ............................................................................ 26
    2.2.3 DNS ............................................................................ 29
  2.3 Strategies for Random Turbulence Generation ......................... 30
    2.3.1 Fourier-Based Methods ................................................... 30
    2.3.2 White-Noise Filtering Approaches ................................... 35
    2.3.3 Synthetic Eddy Methods ............................................... 37
    2.3.4 Shortcomings for Noise Prediction ................................... 42
  2.4 High-Frequencies of the Jet Noise Spectra ............................... 44
    2.4.1 The Importance of High Frequencies ............................... 44
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.4.2</td>
<td>Weaknesses of LES-Based Noise Predictions</td>
<td>46</td>
</tr>
<tr>
<td>2.4.3</td>
<td>LES Frequency Resolution Limits</td>
<td>48</td>
</tr>
<tr>
<td>2.4.4</td>
<td>Addressing High-Frequency Cutoff in LES</td>
<td>50</td>
</tr>
<tr>
<td>2.4.5</td>
<td>Models for High-Frequency Noise</td>
<td>52</td>
</tr>
<tr>
<td>2.5</td>
<td>Summary: Literature Gap</td>
<td>58</td>
</tr>
<tr>
<td>3.1</td>
<td>CLST Description</td>
<td>60</td>
</tr>
<tr>
<td>3.2</td>
<td>Assumptions</td>
<td>63</td>
</tr>
<tr>
<td>4.1</td>
<td>Governing Equations</td>
<td>65</td>
</tr>
<tr>
<td>4.1.1</td>
<td>Navier-Stokes Equations</td>
<td>65</td>
</tr>
<tr>
<td>4.1.2</td>
<td>Linearized Euler Equations</td>
<td>68</td>
</tr>
<tr>
<td>4.2</td>
<td>Numerical Methods</td>
<td>71</td>
</tr>
<tr>
<td>4.2.1</td>
<td>Spatial Discretization</td>
<td>71</td>
</tr>
<tr>
<td>4.2.2</td>
<td>Temporal Discretization</td>
<td>72</td>
</tr>
<tr>
<td>4.2.3</td>
<td>Spatial Filtering for Implicit LES</td>
<td>72</td>
</tr>
<tr>
<td>4.2.4</td>
<td>Boundary Conditions and Inlet Forcing</td>
<td>74</td>
</tr>
<tr>
<td>4.3</td>
<td>The VLES-Plus-Stochastic Turbulence Method</td>
<td>76</td>
</tr>
<tr>
<td>4.3.1</td>
<td>The SNGR Method</td>
<td>76</td>
</tr>
<tr>
<td>4.4</td>
<td>Results</td>
<td>79</td>
</tr>
<tr>
<td>4.4.1</td>
<td>Evaluating the SNGR Modifications</td>
<td>79</td>
</tr>
<tr>
<td>4.4.2</td>
<td>Jet at Mach 0.9 and a Reynolds Number of 36,000</td>
<td>83</td>
</tr>
<tr>
<td>4.4.3</td>
<td>Jet at Mach 0.9 and a Reynolds Number of 120,000</td>
<td>88</td>
</tr>
<tr>
<td>4.5</td>
<td>Shortcomings and Issues</td>
<td>97</td>
</tr>
<tr>
<td>4.6</td>
<td>Summary</td>
<td>100</td>
</tr>
<tr>
<td>5.1</td>
<td>Governing Equations</td>
<td>102</td>
</tr>
<tr>
<td>5.1.1</td>
<td>Linearized Euler Equations</td>
<td>102</td>
</tr>
<tr>
<td>5.2</td>
<td>Numerical Methods</td>
<td>105</td>
</tr>
<tr>
<td>5.2.1</td>
<td>Temporal Discretization</td>
<td>105</td>
</tr>
<tr>
<td>5.2.2</td>
<td>Boundary Conditions and Inlet Forcing</td>
<td>106</td>
</tr>
<tr>
<td>5.3</td>
<td>The Coupled LES-Synthetic Turbulence Method</td>
<td>106</td>
</tr>
<tr>
<td>5.3.1</td>
<td>Filtering</td>
<td>107</td>
</tr>
<tr>
<td>5.3.2</td>
<td>Synthetic Eddy Generation</td>
<td>108</td>
</tr>
<tr>
<td>5.3.3</td>
<td>Eddy Size Distribution</td>
<td>111</td>
</tr>
<tr>
<td>5.3.4</td>
<td>Amplitude Calculation from RANS</td>
<td>114</td>
</tr>
<tr>
<td>5.3.5</td>
<td>Convection and Sweeping</td>
<td>115</td>
</tr>
<tr>
<td>Section</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>-------</td>
<td>------</td>
</tr>
<tr>
<td>5.3.6</td>
<td>Recycling of Eddies</td>
<td>119</td>
</tr>
<tr>
<td>5.3.7</td>
<td>Inclusion of Eddies in LEE Source Terms</td>
<td>119</td>
</tr>
<tr>
<td>5.4</td>
<td>Results</td>
<td></td>
</tr>
<tr>
<td>5.4.1</td>
<td>A Comparison of SNGR and SEM</td>
<td>120</td>
</tr>
<tr>
<td>5.4.2</td>
<td>Jet at Mach 0.9 and a Reynolds Number of 100,000</td>
<td>127</td>
</tr>
<tr>
<td>5.4.2.1</td>
<td>RANS Flowfield</td>
<td>127</td>
</tr>
<tr>
<td>5.4.2.2</td>
<td>3D Validation for LEE</td>
<td>130</td>
</tr>
<tr>
<td>5.4.2.3</td>
<td>LES Comparison</td>
<td>137</td>
</tr>
<tr>
<td>5.4.2.4</td>
<td>CLST Noise Spectra</td>
<td>142</td>
</tr>
<tr>
<td>5.4.2.5</td>
<td>CLST Turbulence Investigation</td>
<td>152</td>
</tr>
<tr>
<td>5.4.3</td>
<td>Jet at Mach 0.9 and a Reynolds Number of 3,600</td>
<td>158</td>
</tr>
<tr>
<td>5.4.3.1</td>
<td>RANS Flowfield</td>
<td>159</td>
</tr>
<tr>
<td>5.4.3.2</td>
<td>DNS Comparison</td>
<td>161</td>
</tr>
<tr>
<td>5.4.3.3</td>
<td>CLST Noise Spectra</td>
<td>166</td>
</tr>
<tr>
<td>5.4.3.4</td>
<td>CLST Turbulence Investigation</td>
<td>173</td>
</tr>
<tr>
<td>5.5</td>
<td>Shortcomings and Issues with CLST</td>
<td>176</td>
</tr>
<tr>
<td>5.6</td>
<td>Summary</td>
<td>178</td>
</tr>
<tr>
<td>6</td>
<td>CONCLUSIONS</td>
<td>180</td>
</tr>
<tr>
<td></td>
<td>REFERENCES</td>
<td>185</td>
</tr>
<tr>
<td></td>
<td>APPENDIX</td>
<td></td>
</tr>
<tr>
<td>A.</td>
<td>LINEARIZED EULER EQUATION VALIDATION</td>
<td>196</td>
</tr>
<tr>
<td>A.1</td>
<td>Simple Dipole</td>
<td>197</td>
</tr>
<tr>
<td>A.2</td>
<td>Co-Rotating Vortex System</td>
<td>199</td>
</tr>
<tr>
<td>#</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>----</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>2.1</td>
<td>Maximum Resolvable Frequency in Literature</td>
<td>49</td>
</tr>
<tr>
<td>4.1</td>
<td>Point Locations of Pressure Data Probes VPST Investigation</td>
<td>87</td>
</tr>
<tr>
<td>5.1</td>
<td>Parameters for Inflow Randomization</td>
<td>107</td>
</tr>
<tr>
<td>5.2</td>
<td>Eddy Bin Size Distribution</td>
<td>113</td>
</tr>
<tr>
<td>5.3</td>
<td>Point Locations of Pressure Data Probes for LEE Validation</td>
<td>134</td>
</tr>
<tr>
<td>5.4</td>
<td>Point Locations of Pressure Data Probes for CLST Investigation</td>
<td>146</td>
</tr>
<tr>
<td>5.5</td>
<td>Point Locations of Additional Pressure Data Probes</td>
<td>168</td>
</tr>
</tbody>
</table>
### LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Representation of Resolved and Modeled Scales in the CLST Method</td>
<td>7</td>
</tr>
<tr>
<td>2.1</td>
<td>Schematic of a Turbulent Jet from Colonius and Lele [46]</td>
<td>12</td>
</tr>
<tr>
<td>2.2</td>
<td>Turbulence in A Transitioning Turbulent Jet from Van Dyke (modified) [132]</td>
<td>13</td>
</tr>
<tr>
<td>2.3</td>
<td>Sweeping and Straining of a Small Eddy by a Large Eddy</td>
<td>16</td>
</tr>
<tr>
<td>2.4</td>
<td>Schematic of The Two Noise Sources in A Jet [122]</td>
<td>19</td>
</tr>
<tr>
<td>3.1</td>
<td>Diagram of The CLST Method</td>
<td>61</td>
</tr>
<tr>
<td>4.1</td>
<td>Limited-Domain Cartesian Grid</td>
<td>80</td>
</tr>
<tr>
<td>4.2</td>
<td>Contours of Velocity Magnitude from RANS</td>
<td>81</td>
</tr>
<tr>
<td>4.3</td>
<td>Contours of Velocity Magnitude for Synthetic Turbulent Fluctuations from Iterations of SNGR</td>
<td>82</td>
</tr>
<tr>
<td>4.4</td>
<td>Acoustic Pressure Fluctuations of a Mach 0.9 Jet with an LES Vorticity Isosurface Colored by ( u ) Velocities (From Blake et al. [17])</td>
<td>86</td>
</tr>
<tr>
<td>4.5</td>
<td>SPL Spectrum at Four Observer Points (From Blake et al. [17])</td>
<td>87</td>
</tr>
<tr>
<td>4.6</td>
<td>Velocity Fluctuations of ( v ) (y-Velocity Component) at an YZ-plane Across the Jet (x = 8D(_j))</td>
<td>88</td>
</tr>
<tr>
<td>4.7</td>
<td>Overall View of the Mesh at ( z = 0 ) (From Blake et al. [16])</td>
<td>90</td>
</tr>
<tr>
<td>4.8</td>
<td>RANS Results Along the Jet Centerline</td>
<td>91</td>
</tr>
<tr>
<td>4.9</td>
<td>Acoustic Pressure Fluctuations (Grayscale) of a Mach 0.9 Jet with an LES Vorticity Isosurface Colored by ( u ) Velocities (From Blake et al. [16])</td>
<td>92</td>
</tr>
<tr>
<td>4.10</td>
<td>SPL Spectrum at Four Observer Points (From Blake et al. [16])</td>
<td>94</td>
</tr>
<tr>
<td>Section</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>-----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>4.11</td>
<td>Pressure Histories at Four Observer Points</td>
<td>95</td>
</tr>
<tr>
<td>4.12</td>
<td>A Comparison of Near-Field Vorticity and Pressure Contours for LES and VLES + Stochastic Simulations</td>
<td>96</td>
</tr>
<tr>
<td>5.1</td>
<td>Source Region for Synthetic Eddies</td>
<td>110</td>
</tr>
<tr>
<td>5.2</td>
<td>Example 1-D Gaussian Eddy</td>
<td>112</td>
</tr>
<tr>
<td>5.3</td>
<td>Eddy Size Distribution Shown For A Representative Turbulent Spectrum</td>
<td>114</td>
</tr>
<tr>
<td>5.4</td>
<td>Point-Wise Velocity for a Distorted Synthetic Eddy</td>
<td>117</td>
</tr>
<tr>
<td>5.5</td>
<td>Point-Wise Velocity for a Small Eddy in a Uniform Flow</td>
<td>118</td>
</tr>
<tr>
<td>5.6</td>
<td>Point-Wise Velocity for a Small Eddy in a Uniform Flow with Influence from a Large, Neighboring Eddy</td>
<td>118</td>
</tr>
<tr>
<td>5.7</td>
<td>Contours of Velocity Magnitude Fluctuations for SNGR- and SEM-based Turbulence Methods</td>
<td>124</td>
</tr>
<tr>
<td>5.8</td>
<td>Contours of Y-Velocity Fluctuations for SNGR- and SEM-based Turbulence Methods</td>
<td>126</td>
</tr>
<tr>
<td>5.9</td>
<td>Computational Grid for RANS Simulation</td>
<td>128</td>
</tr>
<tr>
<td>5.10</td>
<td>RANS Results for Re = 100,000</td>
<td>129</td>
</tr>
<tr>
<td>5.11</td>
<td>RANS Results Along the Jet Centerline and Nozzle Lip Line</td>
<td>129</td>
</tr>
<tr>
<td>5.12</td>
<td>RANS Results Across the Jet Shear Layer</td>
<td>130</td>
</tr>
<tr>
<td>5.13</td>
<td>Computational Grid for LES Predictions</td>
<td>131</td>
</tr>
<tr>
<td>5.14</td>
<td>A Comparison of Pressure Fields from NS and LEE Solvers</td>
<td>133</td>
</tr>
<tr>
<td>5.15</td>
<td>Pressure History at LEE Validation Points</td>
<td>135</td>
</tr>
<tr>
<td>5.16</td>
<td>SPL Spectra at LEE Validation Points</td>
<td>136</td>
</tr>
<tr>
<td>5.17</td>
<td>Vorticity and Pressure in the Near-Field for the Jet for Sim. A</td>
<td>138</td>
</tr>
<tr>
<td>5.18</td>
<td>Centerline Mean Velocity In the Jet</td>
<td>140</td>
</tr>
</tbody>
</table>
5.40 SPL Spectra for CLST Along the Line y = 10Dj .......................... 169
5.41 SPL Spectra for CLST At A Distance of 12Dj ........................... 170
5.42 SPL Spectra for CLST Along the Line y = 5Dj .......................... 171
5.43 Contaminated SPL Spectra at Probe P1 (x = 0Dj, y = 5Dj) ............ 172
5.44 Y-Velocity Contours From LES, VLES, and CLST ...................... 174
5.45 Y-Velocity Across Jet for LES, VLES, and CLST ....................... 175
5.46 Velocity Contours From the SEM Method ............................... 175
A.1 Dipole Validation Results: Pressure Isocontours ....................... 198
A.2 Dipole Validation Results: Pressure Along y = 0 ....................... 198
A.3 2D Co-Rotating Vortex Setup from Bogey et al. [30] .................... 200
A.4 Dilatation Contours for a 2D Co-Rotating Vortex ...................... 201
A.5 Dilatation Profiles for a 2D Co-Rotating Vortex ....................... 202
A.6 Sound Calculation For a 2D Co-Rotating Vortex at the Point (x,y) = (25,0) 202
CHAPTER 1

INTRODUCTION

1.1 Motivation

Despite decades of research and technological advances, the sound generated by a commercial aircraft remains a considerable environmental concern. Aircraft noise can disturb communities located near airports, or worse, cause hearing damage to people in the vicinity. Noise reduction is therefore a significant part of NASA’s strategic plan to improve environmental sustainability, with the goal of cutting perceived noise in half by 2035 for transport category aircraft [101]. Lowering noise levels could make a positive economic impact as air traffic is expected to increase [4].

Many systems contribute to the aircraft noise overall (landing gear, flaps and aerodynamic surfaces, etc.), but jet engines are a dominant source, especially at takeoff [135]. Although engines generate noise due to mechanical operation (fan-associated noise), internal flow, and combustion, the fluid jetting from the engine nozzle also generates noise, which is specifically referred to as jet noise. Jets generate a broadband noise spectrum that arises from fluctuations in the jet shear layer and from turbulent interactions in the jet. The noise produced by a jet is certainly influenced by interactions with aircraft structural components (installed jet noise) and the surrounding freestream flow, but the primary field of jet noise investigation focuses on high-velocity fluid jetting into a near-quiescent fluid.
However, jet noise is still not fully understood, and developing a complete theory from first principles does not seem likely in the near future [4, 74, 123]. A significant reason for the lack of understanding is that turbulence plays a major role in jet noise [74, 91, 92, 123], and the inherently chaotic nature of turbulence, with fluid quantities fluctuating at a wide range of length and time scales, makes studying the problem in-depth a challenge. An additional issue is that acoustic fluctuations are orders-of-magnitude smaller than hydrodynamic fluctuations [46] and can be difficult to identify in the region near the jet. Further complicating the issue is that the fluid mechanisms and sources responsible for generating acoustic fluctuations are not clear and are even debated [4, 46, 74, 123]. Identifying and defining these noise sources is an important component of sound prediction [137], but the lack of consensus on jet noise theory is evidence that continued investigation is needed.

Consequently, jet noise is currently a high-interest area of study, with the ultimate goal of understanding and reducing the noise produced by jets. Reducing jet noise, and, therefore, the overall noise level of an aircraft, requires both a proper understanding of the physical jet noise mechanisms and accurate jet noise prediction techniques. Of primary concern are the turbulent noise mechanisms themselves, as well as the propagation of these noise sources to the far-field. Predicting jet noise, especially in the far-field, is the main concern of this work.

1.2 Problem Summary

Numerical simulation of noise generation and propagation problems, often referred to as Computational Aeroacoustics (CAA), is widely-used for jet noise prediction and can be performed with a variety of methods. The two main approaches for simulation are direct (fluid and acoustic
fields are solved simultaneously) and hybrid (acoustic field is solved separately from the fluid field) [137]. Within these two frameworks there exist several methods, including acoustic analogies [62, 91, 92, 138], semi-empirical methods [122, 125], and Computational Fluid Dynamics (CFD). Simulations based on CFD offer high-fidelity numerical simulations that predict both the flowfield and noise field in an accurate and flexible manner, allow for in-depth analysis, and provide an opportunity to directly study the connection between flow structures and acoustics in a manner currently unattainable with experimental investigations [25, 31].

In the case of CFD-methods for jet noise, turbulence modeling is required due to the unsteady and turbulent nature of jet noise. Three increasingly-complex levels of turbulence modeling exist for jet noise simulations: coupled RANS (Reynolds-averaged Navier Stokes) and stochastically-based models [12, 14, 42, 51, 52, 82, 119], LES (Large Eddy Simulation) [4, 23, 25, 46, 74, 137], and DNS (Direct Numerical Simulation) [58]. The simpler approaches, coupled RANS and stochastically-based methods, do not correctly account for all turbulent scales or may require input from experimental data or additional calibration to be generally applicable. The ultimate high-fidelity turbulence model is DNS, but jet noise calculations with this method are prohibitively expensive for even low-to-moderate Reynolds numbers. The standard is currently LES, which introduces modeling of smaller turbulent scales to save computational costs, while still retaining a high level of fidelity by resolving larger turbulent eddies.

However, practical LES noise predictions are still computationally expensive and are a challenge due to the requirement to resolve multiple, disparate spatial and temporal fluid scales in order to capture the turbulent flow dynamics at the medium-to-high Reynolds numbers at which turbulent jets operate [4]. Higher Reynolds number flows require resolution of an even wider range of time
and length scales, which leads to the need for extremely fine computational grid resolution in
the jet shear layer as more realistic jet conditions are simulated. Consequentially, increasing the
grid resolution drives up the computational cost exponentially, although there is no consensus or
standard on what resolution is required [25]. Additionally, propagating the generated acoustic
waves to the far-field, often as far as 100 diameters from the nozzle exit, is inherently costly due to
the sheer amount of grid points needed to resolve such a distance. Furthermore, the scale disparity
between the magnitudes of fluid and acoustic disturbances requires high numerical accuracy in
order to prevent higher frequencies from being damped by dissipation inherent in the numerical
schemes [137], which can also increase the cost.

Along with the high cost, the very nature of the LES formulation prevents the resolution of the
highest-frequencies of the jet noise spectrum. By design, LES filters out the turbulent scales that
are smaller than what a given computational grid can resolve, typically in the inertial or dissipation
range where isotropy can be assumed. Sub-grid stress (SGS) models are used to model the effects
of stress on the filtered sub-grid scales but do not generate pressure or velocity fluctuations that
would normally be present in these smaller scales. Therefore, no acoustics are generated by these
smaller turbulent scales, which typically correspond to the higher frequency range in the spectrum.
The result is that higher-frequency content is missing from the broadband jet noise spectrum when
even well-resolved LES simulations are performed [18, 22, 46].

Simply increasing the grid resolution to resolve additional fluctuations to recover higher fre-
frequencies is impractical for industrial LES use [19]. Colonius and Lele show results of far-field
noise from a Mach 0.9 jet (see Colonius and Lee [46], Figure 8) where the noise spectrum displays
a lack of high frequencies despite increasing grid resolution by an order of magnitude. Although
the sound pressure level (SPL) of these higher frequencies are lower than the peak frequency, the missing spectral content is not inconsequential. Investigations have shown that the moderate-to-high frequencies are needed to correctly predict the far-field sound power spectra [113, 115, 139]. At full-scale jet conditions, this high-frequency noise can correspond to the human ear’s most sensitive frequencies and needs to be resolved [21, 46].

The most relevant literature on this topic is a paper by Yao and He [139], who propose the use of synthetic fluctuations to account for the missing LES noise content. They apply the kinematic simulation (KS) method of Fung et al. [61] to model sub-grid scale fluctuations that are not resolved by LES, taking care to match both time and space statistics in order to account for the random sweeping hypothesis [66, 81]. A hybrid CAA method is used for LES where noise is computed by Lighthill’s acoustic analogy. Comparisons with DNS for isotropic turbulence in a box show that a combination of LES and synthetic fluctuations can approximately recover the higher-frequencies content from the missing sound power spectra. However, this method was not applied to more complex flows and cannot be used for turbulent shear flows (i.e. jets) because their implementation of the sweeping hypothesis doesn’t account for the influence of the unsteady resolved fluctuations on the unresolved, modeled fluctuations. Several other authors have investigated the issue of missing LES scales with various methods [9, 10, 11, 18, 19, 21, 113, 115], but none have been applied specifically to jet noise prediction. A more complete literature review will be provided in Chapter 2.

In summary, LES is expensive yet still does not provide the complete spectrum needed for predicting broadband jet noise. Attempting to resolve the missing high frequency noise with increased grid resolution would only lead to more computational cost, and yet, would still never
completely resolve the noise field to the desired level of accuracy. Therefore, new methods are needed to model or approximate the missing high-frequency noise and to do so in a manner that is computationally affordable for LES calculations.

1.3 Research Goal

This primary goal of this work is to develop a model to predict the high-frequency content missing from LES far-field jet noise predictions by coupling LES with synthetic turbulence modeling. This method, hereafter referred to as the Coupled LES-Synthetic Turbulence method (CLST), resolves larger turbulent fluctuations (corresponding to lower-frequency acoustic waves) with VLES (Very Large Eddy Simulation) and generates smaller turbulent fluctuations (corresponding to higher-frequency acoustic waves) with synthetic turbulence modeling. A representation of the process is shown in Figure 1.1. The resulting fluctuations are combined, accounting for sweeping of smaller scales by the larger scales \cite{45, 83, 119, 140}, and are used to generate noise sources, which are then propagated with a Linearized Euler Equation (LEE) solver in a hybrid CAA method. The outcome of this work is to demonstrate that the CLST method can correctly model the missing high-frequency content in the near- and far-field jet noise spectra.

A secondary goal of the CLST method is to reduce overall LES simulation costs while still maintaining a high level of fidelity. A cost reduction is possible due to the use of LEE for propagation instead of the Navier-Stokes (NS) equations used by LES because the LEE are a simplified set of fluid equations and are cheaper to solve than the NS equations on a given grid. Additionally, the increased grid resolution needed to resolve the imposed synthetic fluctuations of the CLST method in the source region will be less expensive when solving LEE instead of NS equations. In
the LEE grid, a coarser resolution can be used away from the jet source region. The use of LEE will allow for either a reduced cost for an equal level of frequency content compared to a typical hybrid LES simulation or an increased frequency content for a similar computational cost. Either way, modeling smaller-scale fluctuations with LEE will be cheaper than simply increasing LES grid resolution, which is the only current alternative option for resolving additional frequency content. The LEE equations also provide the pressure field near the jet, which is desired for qualitative investigations of the CLST method in this work. For future calculations of noise spectra at far-field observation points, a more cost-effective acoustic propagation method, such as Lighthill’s acoustic analogy [91], could be employed.

Predicting a more-complete noise spectrum and reducing simulation costs should lead to equally reliable results in a shorter amount of time, potentially allowing the use of LES jet noise predictions more readily in research and industry. This might enable quicker design cycles or the inclusion...
of wing and pylon installation effects in LES jet noise simulations, as is suggested might soon be possible for business jets [129]. Overall, the CLST method will provide a new framework for LES predictions of turbulence-related noise mechanisms.

The layout of this work is as follows: The next section contains a review of literature on jet noise and simulations methodologies, laying a more concrete argument for the necessity of the CLST method. The rest of this work is divided into two sections describing two different iterations of the CLST method: an SNGR-based method (Stochastic Noise Generation and Radiation) and an SEM-based method (Synthetic Eddy Method). Each section contains a description of the numerical methods and equations and presents results supporting the development of the CLST method. Conclusions about the effectiveness of the CLST concept are drawn in the final section, along with plans for further research to develop a more complete method for CLST.

1.4 Contributions

The contributions of this research are:

- Developing the CLST methodology to supplement the high-frequency noise content that is missing from LES-based noise predictions.

- Coupling synthetic turbulence generation with resolved LES turbulent fluctuations for noise predictions at a reduced-cost.

- Developing and investigating an SNGR-based version of CLST.

- Developing and investigating an SEM-based version of CLST that accounts for sweeping and straining in synthetic turbulence.
• Demonstrating the potential of synthetic turbulence methods to model the missing high-frequency noise content.
CHAPTER 2
REVIEW OF JET NOISE

2.1 Jet Flow Physics and Noise Characteristics

Aeroacoustics is concerned with the study of both the generation and radiation of noise. This includes noise generated by both aerodynamic forces and turbulence in fluid flows, as well as noise generated due to fluids interacting with airframe components. In regards to jet flow, Lighthill [91, 92] theorized in the early 1950’s that turbulence in mixing layers is a significant noise source [74, 123]. By separating noise generation and propagation (via an acoustic analogy), Lighthill’s theory enabled a more thorough investigation of jet noise mechanisms and characteristics.

The continued development of jet aircraft has necessitated the growth of jet noise into a mature field of study, resulting in many experimental and numerical techniques to investigate and predict turbulent jet flows and radiating noise. Since the introduction of Lighthill’s analogy, research continued to support Lighthill’s theory which revealed that turbulence plays a significant role in jet noise despite disagreements about the exact noise-producing mechanisms [74, 123]. This topic remains complex due to interaction between the fluid and acoustic fields. Therefore, two aspects must be considered when discussing the physics of turbulent jet noise: the fluid dynamics of jets and noise sources and the propagation of acoustic waves within the jet flow field. It is therefore useful to understand jet flow phenomena alongside noise source mechanisms in jets.
First, a discussion on the relevant fluid dynamics associated with turbulent jets is warranted, followed by a brief discussion on the convection of turbulent eddies. Then, an overview of jet noise is provided, including influencing parameters and the predominant theories on noise source mechanisms. The focus is on subsonic jet noise and, consequently, supersonic noise phenomena are not discussed.

2.1.1 The Fluid Dynamics of Jets

An isolated jet can simply be described as fluid exiting an orifice or nozzle into a flow that is quiescent or at a different velocity. There are typically four flow regions in a jet: a potential core, a shear/mixing layer, a transitioning region, and a fully-developed jet region [46]. For reference, Figure 2.1 shows a simple diagram of a turbulent jet\(^1\).

The mean flow exiting a nozzle forms a potential core in which the flow (usually laminar) simply moves with the maximum mean centerline flow velocity [7]. The conditions of this core match those exiting the jet (velocity, temperature, pressure, etc.) [46]. The core persists for approximately seven nozzle diameters downstream, depending on the Reynolds number [7, 25]. The boundary layer inside the nozzle, either laminar or turbulent, evolves into annular mixing layer surrounding the potential core in the jet [46]. In a laminar boundary layer, freestream disturbances excite the shear layer, initiating the transition from laminar to turbulent flow. In a turbulent boundary layer, instabilities from inside the nozzle propagate and, along with freestream disturbances, continue to excite the shear layer.

\(^1\)Reprinted from Progress in Aerospace Sciences, Vol 40, Colonius, T., Lele, S., Computational Aeroacoustics: Progress on Nonlinear Problems of Sound Generation, Pages No. 345 - 416, Copyright 2004, with permission from Elsevier.
The mixing layer stimulates much turbulent mixing between the potential core and the quiescent flow, and, consequentially, the mixing layer contains both small and large scale turbulent structures. Strong shear effects in the mixing layer lead to the growth of Kelvin-Helmholtz instabilities at the nozzle exit. The instabilities continue to grow, forming larger coherent structures (eddies) that persist many diameters downstream of the nozzle [7, 46, 75]. These larger turbulent structures eventually break down into smaller eddies through the turbulent cascade process [7].

Additionally, smaller turbulent eddies are formed in the mixing layer due to instabilities from the nozzle lip and to normal stresses in the flow [125]. These eddies are also convected by the mean flow and/or the large turbulent scales and contribute to fine-scale mixing in the shear layer [125]. A
wide range of turbulent eddies exist within the jet. Figure 2.2 shows a transitioning jet, where both small- and large-scale eddies can be observed.

Figure 2.2
Turbulence in A Transitioning Turbulent Jet from Van Dyke (modified) [132]

Further downstream of the nozzle, the mixing layer grows wider and spreads inward due to turbulent mixing [46]. Eventually, mixing layers merge and the potential core collapses downstream as the centerline velocity drops [7, 31]. Higher levels of turbulence intensity and intermittency are found at the end of the potential core, as well as significant turbulent anisotropy [28, 100]. The mixing continues and the flow forms a fully-developed turbulent jet that exhibits self-similarity [46]. This region can be found approximately thirty diameters downstream of the nozzle [107]. Once fully developed, the continual entrainment of mass into the jet at a constant momentum flow rate
leads to the spreading of initial momentum to the point where it is overcome by viscous effects which lead to the dissipation of the jet [7].

Several factors influence the development of a turbulent jet. The initial conditions have a significant impact [7], including: temperature of the jet core, a laminar or turbulent boundary layer, and the exiting jet velocity. Two standard non-dimensional fluid parameters used when discussing jets is the Reynolds number based on the jet diameter and velocity

\[ Re = \frac{U_{jet} \cdot D_{jet}}{\nu} \]  

and the Mach number

\[ M_j = \frac{U_{jet}}{a_\infty}. \]  

At higher Reynolds numbers (above \(1 \times 10^4\) is considered high [107]), the mean velocity profiles, Reynolds stress distributions, and the length of the potential core are not significantly influenced by a change in Reynolds number [46]. However, the jet speed and jet Mach number do have a large impact on the jet, reducing shear-layer spreading and increasing the length of the potential core [46]. Of course, there are more considerations with different types of jets and nozzles, but this research will focus on round subsonic jets. The reader is referred to Ball et al. [7] for a more involved discussion of subsonic turbulent jet physics. Additional compressible flow phenomena such as shocks/expansions or the interaction of those with the noise field must be considered in the case of supersonic jet flow but are outside of the scope of this work.

2.1.2 The Sweeping and Straining of Turbulent Eddies

Turbulence is important to jet noise. Given that this work focuses on turbulence modeling, several relevant turbulent eddy phenomena must be highlighted. Turbulent flow contains many
eddies of various sizes and strengths. An eddy is defined as “a turbulent motion, localized within a region of size, that is at least moderately coherent over this region” [107]. The largest eddies carry most of the energy, and this energy is transferred to smaller and smaller eddies by the energy cascade process, the smallest of which are eventually dissipated by viscous forces. Both large and small eddies are convected downstream by the jet mean flow.

A key observation for turbulence modeling is that smaller eddies exist within or in close proximity to larger eddies [107]. The largest eddies, formed by the most energetic fluctuations, naturally influence the smaller eddies. It is assumed that this influence is significantly more than the amount to which smaller eddies influence the larger eddies. Two phenomena observed in turbulent flows describe aspects of the interaction between large and small eddies: random sweeping and local straining [65, 140].

Random sweeping occurs when the passing of larger energy-containing eddies leads to convection of smaller inertial-scale eddies [45, 83, 119, 140]. It has been hypothesized that sweeping occurs without significant distortion to the small eddies [65, 81] but could lead to increased eddy rotation [45]. Experiments support the presence of random sweeping in turbulent flows [103]. The random sweeping hypothesis, proposed by Kraichnan [81], is primarily used to analyze turbulent velocity space-time correlations [66], and at times, its validity has been under scrutiny [109]. However, the idea that large-scale eddies convect and influence small-scale eddies is evident in turbulent flows.

Local straining is the idea that eddies are stretched and distorted by local fluctuations and accelerations [65, 80]. It is assumed that the large-scale eddies are responsible for straining the weaker, small-scale eddies [45].
The combined effects of sweeping and straining are shown conceptually in Figure 2.3. In the figure, a simplified large eddy and small eddy can be seen at three consecutive points in time as they are convected downstream by the mean flow (direction indicated by blue arrows). The arrows of the eddies indicate their rotation directions. The dashed line indicates the path of the smaller eddy considering only mean-flow convection. The solid line shows the combined effects of sweeping and straining from the large eddy on the small eddy, including both convection and distortion over time.

![Figure 2.3](image)

Sweeping and Straining of a Small Eddy by a Large Eddy

Both sweeping and straining lead to a decoupling of eddies in the turbulent field. Sweeping is widely considered a significant temporal decorrelation mechanism in the flow [45, 61, 103, 113, 140]. Straining also decorrelates turbulent flows, though the process is distinct from sweeping and occurs at a slower rate than sweeping decorrelation [45, 65]. Correlation is the degree to which a turbulent fluctuation at one location and instant in time is coupled to a fluctuation at another location and instant in time [65].
Sweeping and straining are clearly important to turbulent statistics and time correlations. It is therefore reasonable to suggest that the interaction of turbulent eddies, including both sweeping and straining, is important to consider when modeling turbulence, especially given that spatial-temporal correlations are important for jet noise predictions \cite{65}. More importantly, sweeping has a significant impact on the noise field \cite{65, 112, 119}, as is discussed in the following sections.

### 2.1.3 Jet Noise Characteristics

With the introduction of Lighthill’s acoustic analogy theory in 1952 \cite{91} and 1954 \cite{92} came the idea that turbulent mixing layers in jets represent a significant source of noise generation \cite{123}. In a jet, small and large scale turbulent velocity fluctuations give rise to small amplitude acoustic pressure waves. These sound waves propagate through the highly nonuniform jet mean flow to the far-field. The strong gradients in the shear layer and mean flow lead to refraction effects \cite{95, 123}. Due to the resulting refraction, the sound radiating from jets tends to bend outward away from the jet centerline, leading to a cone of silence around the jet core where there is reduced noise intensity \cite{2, 122}. The combination of convection and refraction of the turbulent sources leads to sound radiating mostly downstream, but at an outward angle to the flow \cite{123}. Additionally, it has been observed that the frequency content of the jet noise spectra changes with the angle to the downstream \cite{136}. Scattering from the jet nozzle and refraction from atmospheric noise propagation also exist, but are ignored for this discussion.

The primary concern of jet noise predictions is full-scale aircraft conditions. However, obtaining full-scale measurements in a laboratory setting, where anechoic chambers allow for isolation and study of radiated noise, is often impossible given the size of aircraft geometries. Experiments are
therefore typically performed at model scale [135], typically 5% - 20% of full-scale. To accurately compare between model- and full-scale noise data, spectra are typically displayed as a function of Strouhal number (St), a non-dimensional frequency. In Equation 2.3, \( f \) is frequency, \( D_j \) is the jet diameter, and \( V_j \) is the jet exit velocity.

\[
St = \frac{f \cdot D_j}{V_j}
\]  

(2.3)

Many factors influence both the level and the frequency content of the noise radiating from jets, including: jet exit Mach number, jet Reynolds number, temperature, diameter, distance from the nozzle, angle to the jet nozzle, ambient conditions, and boundary layer state. Jet noise levels, typically measured with Sound Pressure Level (SPL), scale with subsonic jet velocity by \( U_j^8 \), which is referred to as Lighthill’s scaling law [91, 92]. In fact, jet noise can be loud, with typical SPL levels of 120 dB and can contain frequencies in the range most annoying to humans. Despite these influencing factors, jet noise follows generally similar spectral curves regardless of jet velocity and temperature [125]. For more information on jet noise in general, readers are directed to several comprehensive reviews of jet noise [7, 46, 74, 122, 123, 137].

2.1.4 Noise Sources in a Subsonic Jet

Since the 1950’s, many theoretical, experimental and numerical investigations have improved the understanding of noise mechanisms in jet flows [123]. Many sources have been theorized for jets, including quadrupoles, self- and shear-noise, fine-scale turbulence and coherent structures, and instability waves [25]. The current consensus is that there are two noise sources in subsonic jets,
one from large-scale, coherent turbulent structures and the other from fine-scale turbulence [75, 99, 104, 122, 123, 134]. The two sources are represented conceptually in Figure 2.4.

![Figure 2.4](image)

**Figure 2.4**

Schematic of The Two Noise Sources in A Jet [122]

The dominant noise source in subsonic turbulent jets is generated by large-scale turbulent structures [3, 100, 122]. The size of these coherent structures (also considered eddies or vortices) is on the order of $D_j$ or larger [122]. These large structures, usually located at the end of the potential core, generate significant low frequency noise [28, 31, 36, 104, 100, 122]. The associated noise sources are found from $x = 5D_j$ to $x = 10D_j$ downstream of the jet [100, 104]. Higher turbulence intensities at the end of the potential core (around $x = 6D_j$) correlate with the peak noise at small observation angles [100]. Beyond $x = 10D_j$, the turbulence reaches equilibrium and noise.

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generation related to these sources becomes insignificant [100]. The large structures are similar to instability waves [3, 122], and could also be considered wavepackets [3, 72]. In high-speed jets, these large scales lead to the radiation of Mach waves, which are strong, highly-directional pressure waves [122, 123].

A possible mechanism for these low frequency noise sources is the idea that the larger coherent structures are accelerated as they are pulled through the merging shear layers into the jet core and influenced by the intense turbulent mixing found there [28, 100, 136]. The location of these noise sources on the centerline [28] indicates that this mechanism is more likely to explain the low frequency noise. An alternative explanation is that the larger scale turbulent structures rapidly decay at the end of the potential core, leading to supersonic wave components and the radiation of Mach waves, despite the jet being subsonic [122]. The exact mechanism is not fully understood, but the significant contribution to the downstream noise occurs for nearly all Reynolds numbers [25, 28]. The only exception is at very low Reynolds number (around Re = 1,700) where these sources may contribute more to the sideline noise than at higher Re [28].

The low frequency noise generated by these sources radiates primarily in the downstream direction and is most influential at angles of 30° to 50° to the downstream [28, 36, 100, 122] (see Figure 2.4). These low frequency waves correspond to the peak SPL levels associated with jets [48, 100, 104], typically centered around a Strouhal number of St = 0.1-0.3 for subsonic jets [25, 36, 48]. In general, the spectral peak St follows a $u_{jet}^9$ power law [3].

Specifically for a Mach 0.9 jet, the location of noise sources correlating to St = 0.2 are found at x=10Dj and sources correlating to St = 0.4 are found at x = 6Dj. For M = 0.9 jets at higher
Reynolds numbers (greater than $1 \times 10^5$), the peak noise is expected at St = 0.3 for angles around $30^\circ$ measured from the jet axis [28].

A second, less influential noise source is generated from random, fine-scale turbulence in the mixing layer [3, 100, 122, 125]. The scales involved in this noise source are eddies of a size smaller than the jet diameter, down to the Kolmogorov scale [3, 122]. The noise generated by these small-scales is broadband, including high-frequency noise content [25, 36, 122], and it radiates in a nearly uniform direction [25, 122]. Given the highly directional nature of the noise radiated by the larger coherent turbulence scales, the noise generated by fine-scale turbulence is observed mainly at the sideline ($90^\circ$) or larger angles to the jet flow [36, 100, 125]. These noise sources are found closer to the nozzle exit plane, before the end of the potential core, usually within a few diameters downstream of the nozzle [32, 38, 100]. The highest frequency noise sources (St = 2 - 10) are usually found within the first three jet diameters [34, 88, 100]. Tam et al. [122] argue that the fine-scale turbulent noise source locations are actually located aft of the potential core, but this argument does not seem compelling given that Tam reports OASPL rather than specific frequency content.

The fine-scale noise mechanism is assumed to arise from high-intensity turbulent fluctuations in the shear layer that lead to acoustic pressure fluctuations [100]. Tam et al. [122] propose the idea that fine-scale turbulence imposes an “effective turbulence pressure” on the flow field and that noise is generated when the turbulence pressure changes due to fluctuations in velocity and turbulent kinetic energy. Another possible mechanism is due to the distortion of eddies in the shear layer. Bogey and Bailly [26] showed with numerical simulations that this sideline noise is directly
associated with shear-layer turbulence. It is also possible that the interaction of eddies in the shear layer leads to this high-frequency noise [31].

These fine-scale noise sources produce a wide range of frequencies, but at full-scale jet conditions, the higher frequencies can be most annoying to the human ear [21, 46, 100]. For sideline jet noise, the peak broadside (90°) noise tends to occur at Strouhal numbers of St = 1 - 2 [48]. The small-scale noise source inherently depends on Reynolds number [28] and may diminish or be masked at lower Reynolds numbers [3]. Specifically for a Mach 0.9 jet, the peak SPL for a Strouhal number of St = 1 occurs around 4D_j downstream, while for St = 2, the sources were found at x = 3D_j [100].

The noise produced by fine-scale turbulence is at a lower intensity than the peak jet noise SPL levels, but it is still a significant part of the noise spectra, especially for higher frequencies [79, 100]. Even for frequencies up to St = 10, the frequency contributions at the sideline may only be 5-10 dB below the peak [48]. In contrast, for smaller downstream angles, these same frequencies may be 15-20 dB below the peak SPL values [48].

One additional factor influences the noise produced by jets: sweeping. Sweeping is an important noise-generating mechanism that accounts for significant noise propagation and influences the sound spectra [65, 112, 119, 140]. The decorrelation caused by the sweeping intensifies the radiated sound [140]. Because of the sweeping effects on small-scale turbulence, it is assumed that this mechanism primarily affects the sound radiated by fine-scale turbulence.

In summary, the three dominant jet noise phenomena are large-scale turbulent structures, fine-scale turbulence, and sweeping. Although Viswanathan states that there is still no clear, definitive answer regarding the sources of jet noise [134], the two-noise source theory is currently accepted
by most research groups. A more complete understanding of jet noise still remains to be discovered though experiments and accurate computational predictions.

2.2 Methods for CFD-Based Jet Noise Prediction

Although theoretical, analytical, semi-empirical, and numerical methods exist for predicting jet noise (the reader is referred to review articles by Wang et al. [137], Colonius and Lele [46], and Bailly et al. [4] for more information), the focus of this work is CFD-based predictions. Numerical simulation of noise falls into two general categories: direct and hybrid [46, 137].

Direct methods solve for the flowfield and the acoustic sources with the same numerical methods, allowing the acoustics to influence the flowfield as both hydrodynamic and acoustic fluctuations develop. Due to the prohibitive cost of direct methods, far-field extension methods are often used, such as Ffowcs Williams-Hawkings (FWH), to calculate noise at the far-field. In direct methods, acoustic sources are resolved in the flow field.

Hybrid methods decouple the calculation of the flowfield from acoustic propagation, solving first for the flowfield and then obtaining the noise through one-way coupling with a propagation method or acoustic analogy generating sources from the flow field data. In hybrid methods, the sound waves have no influence on the flowfield, but the flow modifies the propagation of the sound waves [137]. Hybrid methods are often less expensive computationally because a simpler method may be used to propagate the sound waves than was used to compute the flow field. The Linearized Euler equations (LEE) and other acoustic analogies are often used in this manner to generate the acoustic sources from flowfield fluctuations.
Sound generation in turbulent shear layers is a non-linear phenomenon, which necessitates solving the full Navier-Stokes equations for jet noise prediction [46]. For CFD-based methods, this involves modeling mean and fluctuating turbulent flow quantities to some degree. Ranging from low fidelity to high fidelity, the options for modeling turbulence are: RANS (Reynolds-Averaged Navier Stokes), LES (Large-Eddy Simulation), and DNS (Direct Numerical Simulation). These three methods are described in brief in the following sections.

2.2.1 RANS-Based Methods

Reynolds-Averaged Navier Stokes (RANS) is a method to model turbulent flow fields by decomposing the Navier-Stokes equations into time-averaged and fluctuating terms, \( u(x,t) = \bar{u}(x,t) + u'(x,t) \). In order to close the resulting system of equations, the fluctuations, along with the resulting fluid stresses, must be modeled [96]. Therefore, in RANS, the fluctuating terms are modeled, allowing for the direct calculation of the time-averaged mean flow field.

The advantage of RANS turbulence models is found in applications with high Reynolds number flows, which are beyond the current capabilities of LES and DNS. At these conditions, the wide range of spatial and temporal scales that comprise turbulent flow can be modeled in a steady simulation with RANS at a low computational cost [89]. In fact, the industry routinely uses steady RANS simulations as inputs to empirical databases for noise prediction [89].

However, RANS simulations do not generate turbulent fluctuations but simply model the effects of turbulent fluctuations. Since noise is fundamentally unsteady and generated in part by turbulent fluctuations, RANS models alone cannot predict noise, especially jet noise. Even URANS, which is a time-varying version of RANS, cannot accurately model the turbulent fluctuations needed for jet
noise. Therefore, most RANS-based jet noise methods use the mean flow field provided by RANS as a background flow and either predict noise via a semi-empirical source, physics-based model function, or a reduced-order model or generate artificial turbulent fluctuations based on the RANS flow. An acoustic analogy in the form of a propagation solver or method, such as LEE or a Green’s function, is then used to predict the noise field.

Physics-based approaches generate source terms for acoustic analogies via model functions that represent correlations or other statistical properties of jet noise sources [79]. Two such methods meant to model the fine-scale turbulence present in jet flows are MGBK [78] and Tam and Auriault’s fine-scale model [125]. Both methods are based on modeling turbulent correlations. MGBK uses the assumption that the second order correlation is separable in space and time [79]. Khavaran and Bridges discuss an MGBK model for the two-point correlation function based on Gaussian and exponential functions, with both isotropic and anisotropic forms [79]. Tam and Auriault’s model assumes that fine-scale turbulence comes from normal stresses in the flow and use the analogy that fine-scale turbulence can be represented via the random motion of gas molecules. Despite modeling the noise field quickly, these methods are only applicable to the conditions for which they are tuned. Simulating new nozzle shapes or off-design conditions may lead to erroneous predictions. Of course, there are many other methods, but covering them falls outside the scope of this discussion.

A more flexible RANS-based noise generation is accomplished through stochastic and synthetic turbulence methods. Several methods exist for the synthetic (stochastic) generation of turbulent fluctuations. With these methods, the goal is not to replicate a turbulent field exactly but to generate random velocity fields that match certain statistical characteristics of the flow [119]. These models can be divided into three categories [83]: (1) decomposition of turbulent modes (summation of
Fourier-modes to match an imposed energy spectrum), (2) filtering of white noise (can be filtered to match either a space-time correlation or an imposed energy spectrum) and (3) superposition of turbulent structures. The SNGR (Stochastic Noise Generation and Radiation) method [5, 6, 12] is an example of Fourier-mode methods, while the RPM (Random Particle Method) [50, 54] and FRPM (Fast RPM) [51, 52, 119] are examples of the white-noise based methods. Synthetic Eddy Methods (SEM) [67, 69] are an example of the third type of method. The flexibility of these methods is found in the fact that they generate turbulent fluctuations from a given RANS flow field and are therefore more likely to be applicable to a wider range of conditions than empirical models. A full discussion of these methods is provided in a later section.

2.2.2 LES

There have been major advances in computational aeroacoustics in the past decade and a half [89], in part, due to progress made in simulating jet noise with Large-Eddy Simulations (LES). Large-Eddy Simulations solve a filtered version of the Navier-Stokes equations, accounting for unresolved turbulence by a sub-grid scale model. In other words, LES methods resolve large-scale highly anisotropic turbulent structures and model the smallest turbulence scales, which are statistically isotropic at high enough Reynolds numbers [107]. High-fidelity LES provides an in-depth analysis of fluid flow and noise phenomena and allows for building databases to study sound sources, leading to advances in fundamental understanding of jet noise physics [41, 89]. LES is especially suited to jet noise predictions because the larger turbulent scales resolved by this method are responsible for the majority of peak jet noise in the downstream direction [136].
Even in its infancy, LES jet noise predictions provided researches with insight into noise sources and locations. One of the earliest fully 3D large-eddy simulations of a subsonic jet was Bogey, Bailly and Juvé in 2003 [31] and supported the idea that coherent or large structures in the shear layer are influential to jet noise, especially towards the end of the potential core where the shear layers merge. Several other significant LES contributions include providing support for the two-source theories of jet noise and explaining the spectral changes in noise with varying angles to the downstream direction [27, 136].

A significant effort to improve LES noise predictions was undertaken in the 2000’s and has led to an abundance of available approaches and modeling options [23]. Bodony and Lele [23] review LES method developments and list several prominent research groups. For a more exhaustive discussion on LES and other jet noise prediction methods, the reader is directed to their review article and several others: Colonius and Lele [46], Wang et al. [137], Jordan and Gervais [74], Bailly et al. [4], Ball et al. [7], and Lele and Nichols [89]. For the present discussion, however, a short, non-exhaustive overview of LES options is warranted.

Large-eddy simulations have been performed for a range of Reynolds numbers, Mach numbers (both subsonic and supersonic), and for cold and heated jets providing various insights into jets and jet noise [23]. Large-eddy simulation codes range from high order structured codes [25] to low order finite-volume unstructured approaches [37]. Far-field noise prediction (out to one hundred jet diameters) has been accomplished by a variety of methods [23], including scaling the near-field noise by $1/R$, Kirchhoff surfaces, Ffowcs Williams-Hawkings (FHW) approaches [56], and the Linearized Euler equations (LEE) with source terms [30, 33]. The choice of sub-grid scale (SGS) model to prevent build-up of unresolved small scale energy is also significant [128], with two main
groups focusing on either Smagorinsky-based SGS models that use artificial sub-grid viscosity [46] or implicit LES methods employing higher-order filters [25] for numerical dampening.

Initial LES simulations did not include a nozzle geometry, instead simulating the nozzle exit flow with inflow boundary conditions and only modeling the development of the jet shear layers. However, it quickly became clear that correctly predicting the near-nozzle shear layer was important to noise predictions, and a variety of methods were employed to properly excite the initial shear layer turbulence [23, 26]. Ultimately, a properly simulated internal nozzle flow and boundary layer are required to capture any significant changes in nozzle geometry and design [133], like beveled or chevron nozzles. Including the nozzle geometry in simulations was initially too expensive with LES, but eventually computational power increased enough to simulate the nozzle internal flow in a variety of ways [29, 38, 68, 117, 118].

In addition to including the nozzle internal flow, LES of jets have also investigated multiple jet-streams, installation effects, and unsteady phenomena such as trapped shock waves and crackle [89]. Further advances in computational power and LES methodologies are expected to lead to higher-fidelity simulations with more realistic flight conditions and fluid/noise interactions. A future goal is toward reduced-order modeling, with current research on proper orthogonal decomposition (POD) and wavepackets [89].

It would seem that LES jet noise simulations have reached a “good enough for now” state, or at least a point where databases can be used to develop reduced-order modeling of noise sources [89]. However, there is still some uncertainty regarding LES accuracy when validating and comparing to experimental measurements or at least in the quantifying any expected differences [41]. Furthermore, it is unclear how each of the available LES options affects the accuracy of predicted
results and therefore difficult to ascertain which set of methods is “best.” Although quite popular for jet noise predictions due to a relatively high level of fidelity, LES is not without shortcomings, some of which are discussed in Section 2.4.

2.2.3 DNS

Relatively little progress has been made with direct numerical simulation (DNS) of jets due to the same computational cost issues associated with LES. Direct numerical simulation solves the Navier-Stokes equations without any turbulence modeling for closure, which requires resolving all temporal and spatial scales. Readers are directed to several review articles for more information: Colonius and Lele [46] and Wang et al. [137].

Direct numerical simulations have the potential to resolve all the turbulent scales and generate databases for investigating jet noise mechanisms in depth. In principle, DNS has no high frequency cutoff and could simulate the full jet noise spectra because it does not use sub-grid scale modeling. However, such a simulation would require long simulation run times, large domains, and fine meshes [46], and given current numerical capabilities, simulations at any relevant Reynolds number will still greatly under-resolve small turbulent scales, leading to high frequency cut-off. Pope [107] estimates that the total number of grid points for resolving isotropic turbulence with DNS is approximated by \( N^3 \approx 4.4 Re^{9/4}_L \). For reference, industry-relevant Reynolds numbers are about \( 1 \times 10^6 \) - \( 1 \times 10^7 \) for full-scale jet engines [46]. Such simulations would require approximately \( 5 \times 10^{15} \) grid points by Pope’s estimate. Due to the wide range of temporal and spatial scales present at those Reynolds numbers, DNS is and will be impractical for the foreseeable future. Therefore,
DNS investigations have focused on lower Reynolds numbers. Even at such limited conditions, DNS can provide a helpful in-depth look into jet noise mechanisms [46].

The first DNS of flow-generated sound was for a 2D co-rotating vortex pair [98, 137]. Another early DNS investigation was of a mixing layer [47]. The first applications of DNS to subsonic jet noise for a three-dimensional jet flow flows were performed by Freund for a Mach 0.9 jet at a Reynolds number of 3,600 [58]. This simulation provided an in-depth turbulent jet flow analysis, including investigating two-point velocity correlations and the turbulent kinetic energy budget. The results have also been used for comparisons to LES [22].

A more recent DNS approach investigates simplified methods such as temporally-developing jets that could be used to investigate sound sources in spatially-developing jets [24]. Future progress for jet noise is likely to continue slowly until further advances are made in computing power.

2.3 Strategies for Random Turbulence Generation

Three categories of synthetic, randomized turbulence generation methods are discussed in this section, including Fourier-based methods, white-noise filtering approaches, and synthetic eddy methods. A succinct review of these three approaches, along with additional information, can be found in the work of Lafitte et al. [83]. The following section focuses on the relevance of these method types to the development of the CLST method that is developed in this research.

2.3.1 Fourier-Based Methods

A Fourier-based method employs a superposition of randomized sine waves to generate turbulent fluctuations first introduced by Kraichnan [82] that ideally reproduce certain turbulent statistics. The SNGR (Stochastic Noise Generation and Radiation) method, initially implemented by Bechara
et al. [12] and Bailly et al. [6], was one of the earliest and simplest Fourier-mode methods. The general idea of the SNGR method is to synthesize random turbulent velocities and then use the fluctuations to generate acoustic sources to obtain the acoustic far-field.

Several modifications and improvements have been made to the original SNGR method [83]. Bailly and Juvé, following the approach of Kraichnan [82] and Kareit et al. [76], extended SNGR to three dimensions and introduced time-dependence by convecting the stochastic field via mean velocities from a RANS simulation following the Taylor frozen turbulence hypothesis. Using turbulent kinetic energy and dissipation from a $k-\epsilon$ RANS simulation, the stochastic field simulates locally isotropic turbulence. Bailly and Juvé [5] chose the Linearized Euler equations with source terms as an operator for propagating acoustic perturbations to the far-field. For application to a high-subsonic jet, results were promising.

Di Francescantonio et al. [49] discussed the tendency of the basic SNGR (the method of Bailly and Juvé [5]) to overestimate noise levels. This result is reported by other authors as well [97]. Di Francescantonio et al. attributed this behavior to the lack of a mechanism in SNGR to account for the turbulent energy cascade. Without a mechanism to transfer energy from lower wavenumbers to higher wavenumbers, the authors showed that, as the maximum reconstructed wavenumber increases, the noise levels begin to diverge significantly (up to 50 dB above) from the experimental results. There is no control over the desired time-correlation functions with basic SNGR, and this method reportedly loses spatial correlation over long periods of time due to the convection term [9] and does not properly account for time-correlations, a further consequence of not modeling the energy cascade [49].
Di Francescantonio et al. proposed a modification, DSNG (Damped Stochastic Noise Generation), that attenuates the amplitudes of the random waves to produce a more physical sound radiation behavior. Their results showed a convergence at higher wavenumbers that better matched experimental noise levels at the expense of not matching the values of TKE from the RANS simulation. Of course, it is reasonable that scaling down the TKE at higher wavenumbers would lead to a reduction in SPL for those wavenumbers.

Billson et al. [14] improved Bailly and Juvé’s time-dependent convection term by solving additional advection equations for the stochastic velocities. At each time step, this method generates a new independent random field, which is filtered and added to the convected random field from the previous time step. In this manner, the random field used to generate velocity fluctuations at the current time step is a mixture of the new, filtered random field and a weighted, convected summation of all previous random fields. The influence of previous random fields maintains better control over the time correlation of the stochastic field while saving some computational effort by requiring fewer Fourier modes. Explicitly convecting the stochastic turbulence makes this method more computationally expensive than the method of Bailly and Juvé due to the solution of additional equations. OASPL sound pressure levels predictions for a high-subsonic jet were about 10 dB above measured values and Billson et al. noted that energy was missing from smaller scales in the shear layer. Adjusting a length scale in the SNGR method caused the OASPL to match within 4 dB. Billson et al. [15] further improved their method by introducing modeling for anisotropic turbulence and their results match measured data to within 3 dB; however, discrepancies between measurements at low frequencies were noted.
Further improvements to the SNGR method were made by Lafitte et al. [83, 85] by splitting the stochastic field into large and small scales (similar to Fung et al. [61]), with separate generation methods for each. For the larger scales, Bailly and Juvé’s method [5] was employed, while a modified version of Billson et al.’s method [14] was used for the smaller scales. The main improvement of the method introduced by Lafitte et al. [83] is that it accounts for sweeping, an important decorrelation mechanism in the flow [61, 83].

Sweeping is introduced by convecting the small-scale synthetic velocity field from the previous time step with a combination of the flow bulk velocity and the velocities of the larger turbulent scales. The small-scale turbulent field is then formed by adding a filtered white noise field, which ensures progressive decorrelation of the small scales and hence accounts for sweeping [84]. In this manner, the small-scale turbulent field is a weighted sum of velocity fields from all previous time steps. The small-scale velocity field is combined with the large-scale fluctuations to form the synthetic turbulent field. The method reproduces several spatial and temporal statistical quantities well, including space-time velocity correlations such as two-point, two-time velocity correlation functions. This is appropriate given that their focus was on correctly modeling the synthesized turbulent structures in the shear layer. The authors note that their method only captures turbulence noise and that the only inclusion of anisotropy is due to advection by the mean flow, which also leads to a loss of the TKE of the generated field.

Lafitte et al. [83] also state that previous versions of SNGR overpredicted the far-field acoustic spectrum, and their initial results also followed this trend [85]. They even showed that closely matching statistical quantities for synthetic turbulence did not necessarily lead to more accurate noise predictions. Scaling the source terms [85] and modifying a calibration factor related to the
distribution of energy among the Fourier modes [83] both led to better matches with experimental
data for far-field noise predictions. The latest iteration of this method does not show far-field noise
predictions, and so it is unclear if the method still overpredicts far-field noise.

A downside of the Lafitte et al. variation [83] is the grid resolution needed to obtain a maximum
desired wavenumber. According to the authors, their modification is more expensive than classic
SNGR due to the solution of additional advection equations, but cheaper than solving the Euler
equations. The expense of these equations grows as the grid resolution is increased. Inherently then,
this method is limited in the higher frequency range by the discretization of the numerical grid.

In summary, SNGR is a simple approach to synthetic turbulence generation and one that is still
used in industry [8]. However, several issues with SNGR have been noted in literature. SNGR
commonly overpredicts the far-field noise when compared to experimental jet data [14, 49, 85, 97].
It should be noted that a frequency-domain version of SNGR showed good agreement with far-field
experimental data, possibly because they accounted for variations in the correlation length over the
source domain [8, 44]. Another reported issue is the lack of the turbulent energy cascade [49] and
the loss of spatial and temporal correlation [9, 49]. The absence of sweeping in the basic SNGR
formulation is also significant since it is required for jet noise [83]. Furthermore, the best manner
to properly convect a Fourier-generated stochastic field is debatable, and as Lafitte et al. [83] noted,
Fourier mode methods struggle to account for both convection and time-varying turbulence. A
final issue is that the high frequency content of the far-field depends on the grid resolution of the
acoustic solver.
2.3.2 White-Noise Filtering Approaches

Ewert and Edmunds [54] and Ewert [50, 51, 52] introduced the RPM (Random Particle-Mesh) method, which generates turbulent fluctuations via spatially-filtered convective white-noise. The essential idea of RPM is to generate a white-noise field and then low-pass filter (convolve) the stochastic field to reproduce desired spatial and temporal characteristics [119]. The resulting turbulent velocity fluctuations, with statistics that match the second-order, two-point correlation tensor of homogeneous isotropic turbulence, are used to generate vortex sound sources in a direct CAA framework [51]. Similar to Fourier-based methods, RPM uses the mean flowfield and length scales from RANS data to generate fluctuations that lead to far-field noise. This method has been applied to both airframe interaction noise and jet noise [51]. An alternative description of the RPM method is a velocity field generated by a superposition of randomized vortices that produce noise, where the velocities are represented by random particles [53]. These methods are also referred to as digital filter methods.

Fast RPM (FRPM) is an improved version of the original RPM method [51]. As in the original RPM method, a white-noise field is filtered spatially to realize turbulent fluctuations [51]. The filtering is essentially a convolution operation between the white-noise field and a Gaussian filter kernel. The result is a streamfunction describing turbulent fluctuations with statistics that closely match two-point space-time correlations [51]. The filter has an effect on the desired correlation. Numerically, the white-noise field is represented by a discrete number of particles assigned with frozen random values. The particles are convected on an intermediate mesh with the local mean flow velocity using the assumption of frozen turbulence. The filtering/convolution process is applied to all particles, generating turbulent fluctuations. Using a RANS mean flowfield, parameters are
adjusted so that the fluctuations exhibit the proper turbulent length scales. Together with the mean flow, the stochastic fluctuations are input to an acoustic solver on a CAA grid, generating a sound field.

Modifications were made to the RPM method of its use in jet noise applications. Fundamentally, the temporal fluctuations of turbulence lead to the generation of jet-noise [51]. The RPM and FRPM methods were originally designed using the assumption of frozen turbulence, so a Langevin procedure is used to introduce decorrelation and exponential decay [14, 51] into the random field. In this manner, the proper space and time correlations can be obtained. This time-varying procedure is similar to the method used by Billson et al. [14]. Additionally, fluctuations from RPM are fed into source terms from Tam and Auriault’s fine scale jet noise model [125] to generate noise in the acoustic solver [119]. The RPM method does accurately reproduce Tam’s fine-scale noise curve below the method’s cut-off frequencies. Additional improvements to the method include using a series of one-dimensional recursive filters to save computational time in the convolution operation [119].

A further feature of the RPM method for jet noise is the inclusion of sweeping into a synthetic turbulence model for the first time [119]. Sweeping is captured due to a feedback mechanism in the interplay between the generation of the stochastic and turbulent field that leads to complex mixing of the synthetic turbulent field over time [119]. The hierarchy of recursive filters applied to the white noise field achieves correlations that confirm the presence of the sweeping phenomenon. Furthermore, the importance of including sweeping in jet noise calculations is confirmed by Siefert and Ewert [119]. Additionally, this iteration of RPM shows good agreement with Tam’s spectra for fine scale turbulence.
RPM is a robust approach to synthetic turbulence generation given that it can accurately match second-order, two-point correlations [83, 119]. More recent application of this method shows FRPM used as a backscatter model for a hybrid RANS/LES method called Forced Eddy Simulation [53, 102]. For jet noise predictions, the RPM method accounts for sweeping and can reproduce Tam’s empirical jet spectra for fine-scale noise. However, RPM is complicated to implement and the cost of the convolution process is likely significant [67], even with recursive filters. Additionally, the use of the Langevin equation might lead to discontinuities in the acoustic source term [83]. Furthermore, the use of an intermediate mesh adds extra interpolation and limits the applicability for complex three-dimensional geometries.

2.3.3 Synthetic Eddy Methods

Another class of methods focuses on generating synthetic turbulence by superposing eddy structures. These methods were born from the need to generate realistic inflow turbulence for LES simulations, a common LES issue especially for simulations of engineering importance where saving time is essential. Without a synthetic turbulence method, arriving at realistic fluctuations in LES is a relatively slow process. Developing a realistic turbulent inflow quickly can save computational time. As previously noted, in the context of LES jet simulations, SEM-type methods are often used to generate turbulent fluctuations inside a nozzle.

Jarrin et al. [69] introduced the SEM method as a means of generating spatially decaying homogeneous isotropic turbulence at the inflow to LES by superposing disturbances derived from coherent structures. The main idea is to superimpose many Gaussian eddies to approximate turbulent fluctuations. A Gaussian shape function is chosen for each eddy, and a certain number
of eddies are randomly positioned at the inlet plane to trigger the development of realistic inflow conditions for homogeneous isotropic turbulence. The eddies are convected by the mean flow. The goal of SEM is to produce realistic turbulence over a shorter inflow distance thereby making the simulation less expensive. This method does require input from Reynolds stress tensors via RANS or DNS, but does match one-point statistics well. Overall, SEM is easy to implement and computationally efficient even for complex geometries.

The SEM method has been improved upon with divergence-free formulations [106, 116], which are more physically realistic. One such method is the Divergence-Free SEM (DFSEM) of Sescu and Hixon [116], which was designed for application to CAA [63]. This modification generates a vector potential field with SEM, then obtains turbulent velocities by taking the curl of the random field so that the divergence-free condition is satisfied. This method can model quadrupole sources and is therefore more applicable to jet shear layer turbulence. For a more thorough review of synthetic eddy-based and alternative turbulence methods, see Haywood [63].

Yet another divergence-free SEM method is the Triple-Hill’s vortex method of Haywood [63, 64]. The original Hill’s vortex is ideal for reproducing isotropic turbulence. The Triple-Hill’s vortex uses a superposition of three spherical Hill’s vortices, each with a different amplitude and inclination, to reproduce anisotropic turbulence in a mixing layer. Using a minimal Lagrangian map [111], each eddy can be distorted, which leads to matching higher-order statistics for anisotropic turbulence. Eddies are convected with the frozen turbulence assumption. The method reproduces Reynolds stress tensors in homogeneous isotropic turbulence, channel flows, and turbulent mixing layers. The advantage of this method over traditional SEM is a reduced length for transition to realistic turbulence. The Triple-Hill’s method also matches Reynolds stress profiles well.
The majority of work on SEM-based methods has focused on adding eddies into the Navier-Stokes equations to enhance the development of realistic turbulence. This allows interplay between the NS flow and the convected synthetic field. However, Fukushima and other authors have recently applied SEM methods to generate synthetic turbulence for jet noise predictions [59, 67]. These applications generate turbulent fluctuations with SEM and, together with a RANS mean flow, are fed into the Linearized Euler equations (LEE) to generate a noise field.

Fukushima [59] uses a modified version of SNGR that implements SEM to model turbulent fluctuations in the jet shear layer via a superposition of eddies instead of a superposition of Fourier modes. SEM is more widely-applicable than random Fourier modes for flows other than jets, such as inhomogeneous wall-bounded flows, since Fourier methods may possibly be limited in these cases due to spurious noise that could result from cutting off global Fourier modes at wall boundaries. As with SNGR, a RANS-generated background shear flow is also used in this formulation, and the LEE equations (with source terms) are used to propagate the synthetically-generated noise sources. This method also uses the Building-Cube Method (BCM) to generate a block-structured Cartesian mesh framework. This approach is highly parallelizable. It allows the mesh to be generated quickly for complex configurations, and is compatible with higher-order schemes. Fukushima et al. [60], use this SNGR/SEM method and lay the groundwork for RANS-based full aircraft noise with synthetic turbulence.

In the original SEM method, the method is intended to produce inflow turbulence, so eddies are convected with a constant velocity. Fukushima modifies the original SEM method by generating a new independent random field at each time step and introducing time-variation of the eddies by filtering in a manner similar to the method of Billson et al. [14, 71]. The time filter is a weighting
with the previous random field, which in turn contains history effects from all previous fields [14]. However, this implementation does not appear to require the solution of additional advection equations.

Fukushima compares this modified SEM method to DNS results for a wall-bounded flow, as well as with other synthetic methods: a random Fourier method, the original SEM, and a digitally-filtered white noise method. The digital filter and modified SEM method provided the best comparison to the turbulent kinetic energy spectra. Additionally, the modified SEM method generated a random field in less computational time than both the random Fourier method and the digital filter method, showing that this method is computationally inexpensive. It is unclear which SNGR formulation this is, but it is assumed to be the method of Lafitte et al. [84, 85].

For jet noise, Fukushima’s modified SEM method [59] is compared to experimental results and Lafitte et al.’s SNGR method [84, 85]. The modified SEM method slightly overpredicted noise at lower angles (more downstream), but agreed well for other measurement locations. A grid-based frequency cut-off is noted as a limiting factor for high frequency resolution.

The method of Hirai et al. [67] shows promise for the use of SEM methods in jet noise prediction, building on the work of Fukushima [59] with further modifications and by investigating the statistical properties of the synthesized turbulence. The main contribution of Hirai et al. was to introduce a turbulence dissipation mechanism (a time decorrelation process) that modulates the intensity of each eddy in time to better match the temporal decorrelation observed in the experimental data of Fleury et al. [57]. This is needed because neither SEM [69] nor the improved DFSEM [116]) method correctly models the dissipation of turbulent structures in time due to their initial design as inlet turbulence generators. In this formulation, each eddy is convected by a
constant mean flow. This assumption is consistent with the result from Fleury et al. [57] that states that turbulent structures, as a whole, convect with a velocity of $0.6*V_{jet}$. Each time step, the size and strength of each eddy is recalculated based on convection and the background RANS field TKE and $\epsilon$. The newly generated synthetic velocity field is fed into the LEE source terms each time step. The authors show that DFSEM with the added decorrelation process most closely matched spatial and temporal statistics, such as two-point two-time velocity correlations. However, noise was overpredicted by over 25 dB when the statistics were well matched. This result was actually reported for an SNGR-based method by Lafitte et al. [83, 85], where matching statistics for the synthesized velocity field did not lead to more accurate noise prediction. Increasing the time scale of the noise source led to better matching for noise predictions, but the parameters used for this result lack a connection to physical values for the tuned constants. This method may require a large number of eddies (above 50,000) to accurately model turbulence, which would drive up the cost. And it is possible that a more physical temporal decorrelation mechanism may be used, rather than randomly varying the strength of each eddy. More tuning and validation is needed for this method.

Although SEM has seen limited use for jet noise applications, the methods are simple to implement and computationally inexpensive with a reasonable number of eddies. Fukushima [59] indicates that the method is slightly faster than SNGR but with comparable noise prediction abilities. As with other LEE-based methods, SEM high-frequency noise is limited by the numerical grid. Although some differences exist between matching turbulent statistics and predicting the correct noise field, these methods show promise for developing realistic turbulence for jet noise and would benefit from further development.
2.3.4 Shortcomings for Noise Prediction

Synthetic and random turbulence noise models have the advantage over LES and DNS of quickly producing turbulent fields with realistic statistical properties and noise predictions for jet flows. Furthermore, many synthetic methods take advantage of using simpler wave propagation methods, like LEE, which are cheaper to solve than the full Navier-Stokes equations. And while many of these models have their place and find good use, random turbulence methods are not without shortcomings when applied to jet noise.

Turbulent anisotropy has been shown to impact noise levels and directivity in jet noise [15, 73, 75, 110]. However, most synthetic methods assume generate isotropic, homogeneous turbulence and are therefore missing important turbulent physics that are essential to noise generation. While the smallest turbulent scales can be assumed as isotropic, the lack of anisotropy in turbulence will lead to errors in jet noise prediction.

Several synthetic methods do include anisotropy, but this requires the estimation of Reynolds stresses from RANS data and often includes coordinate transformations to match Reynolds stresses in the principle axes. These additional calculations increase the cost and complexity of synthetic methods. For instance, Billson et al. [15] include length-scale anisotropy with SNGR, but still show discrepancies with lower frequencies when applied to jet noise, even missing the peak noise level that occurs at lower frequencies. Since the lower frequencies are the most significant in jet noise, this presents a serious problem for noise prediction. These results suggests that random turbulence methods have difficulty dealing with low frequency content.

Additionally, coherent eddies are not likely well predicted by these methods either since the lower frequency content in jet noise corresponds to these largest turbulent structures. Jet noise
is predominantly generated from these larger structures, and, therefore, properly simulating their behavior is key for accurate noise predictions. In contrast, LES or DNS would produce more realistic large-scale turbulent structures.

A further shortcoming of synthetic methods is that they are limited at low and high frequencies by the grid discretization of the acoustic propagation solver. A consequence is that using these methods to resolve higher frequency content will incur a higher computational cost to resolve the acoustic waves that are generated at these smaller scales.

Locality in the randomized turbulent field is also a difficulty for synthetic turbulence methods. With SNGR, global Fourier modes may be stretched and deformed due to the influence of convection from the background shear layer. Attempting to specify a non-homogeneous length scale with a Fourier-mode method would potentially destroy the properties of the turbulent field [70]. The RPM method uses a series of filters across the entire source term domain, making it difficult to control local quantities. SEM methods do allow locally-specified length scales to influence the synthetic turbulence.

A final shortcoming is that most synthetic models make assumptions in the turbulence generation process and often require some tuning to achieve desirable results. For instance, most of the methods discussed in the previous sections were made only to model noise produced by a turbulent mixing layer. The accuracy of these models for various noise applications and flow conditions typically require tuning of parameters associated with the strength of the random fluctuations. The difficulty of matching both turbulent statistics and far-field noise has already been discussed [67, 85], and properly modeling interaction (such as sweeping) between the synthesized turbulent scales is
challenging. In all, these shortcomings limit the applicability of synthetic turbulence, and logically have lead to the further development of LES- and DNS-based noise predictions.

2.4 High-Frequencies of the Jet Noise Spectra

A closer look at the high frequencies of jet noise is warranted. This section looks at the relevance of high frequencies, discusses shortcomings of using LES for jet noise predictions, and methods and models that were developed in an attempt to address the problem of missing noise in the high frequency jet spectra.

2.4.1 The Importance of High Frequencies

A natural question arises as to the maximum frequencies (or Strouhal number) that are practically relevant to the jet noise spectrum. To review, the peak noise levels in a jet and the majority of far-field jet noise are contained in the range of frequencies from \( St = 0.1 \) to \( St = 1.0 \), at least for subsonic jets \([94, 99]\). Even though higher frequencies (above \( St = 1 \)) are less intense than lower-frequency noise, these frequencies are important.

The primary significance of high frequencies is due to the fact that, in full-scale jet noise, these frequencies lie in the range most annoying to humans \([21, 46, 100, 120]\). Given that the human ear is most sensitive in the 1 - 5 kHz range, jet noise that is generated at those frequencies is most problematic. Thomas et al. \([127]\) show some full-scale experimental spectra, including frequencies from 1 - 5 kHz. On average, at the sideline, frequencies at 1 kHz were about 5 dB below the peak SPL. This same frequency was about 10 dB below the peak at smaller angles to the downstream. These results show that the annoying range of frequencies still contributes a non-trivial amount to the overall SPL levels. Additionally, for investigation of nozzle modifications,
such as chevrons, higher frequencies are vastly more important [127]. Several authors have argued that higher frequencies are ultimately needed for jet noise predictions, including values of St = 8, 10, 20, and up to 30 for full-scale large engine jet mixing noise [23, 118, 129].

To further illustrate this point, a full-scale example is needed. For a large diameter fan high-bypass ratio turbofan with a plume diameter around 2.4 m [118], assume a core jet nozzle with half the diameter (1.2 m) and a sonic exit velocity. Since most LES simulations focus on simulating a single-stream jet, considering only the core flow is a more representative case. For these conditions, a frequency limit of St = 5 corresponds to 1.4 kHz, which barely even reaches to the sensitive range (1-5 kHz). For the sensitive range of frequencies, 1 kHz corresponds to St ≈ 3.5 and 5 kHz corresponds to St ≈ 17.6. From side experimental observations where the higher frequencies are observed, SPL levels can be 5 - 10 dB below the peak levels for higher frequencies in the range from St = 1 to St = 10 [48], confirming that these frequencies are in fact important to the noise.

The upper limit of frequencies required for full-scale noise certification by the FAA is around 10 kHz [55, 135]. In the previous example, a resolvable limit of St = 30 would be around 8.5 kHz. Given the limits for certification, an upper frequency limit of St = 30 is certainly required for any serious industry applications (such as evaluating chevron designs [23, 118]) and cannot be ignored. To simulate the full human-audible range, with a maximum St = 30, would require an estimated 1,000x increase in computational cost compared to resolving St = 3 [129], needing perhaps a grid point count numbering in the tens of trillions, making this sort of simulation unlikely for the foreseeable future. This further indicates the necessity of developing methods to increase the upper frequency noise prediction limits.
2.4.2 Weaknesses of LES-Based Noise Predictions

Jet noise predictions highlight several shortcomings of LES. First and foremost, the main drawback of LES for noise predictions is the extreme computational cost, making this method unlikely to be applied routinely in industry for quite some time. The high cost is due to the fundamental issue that, as the Reynolds number increases to realistic conditions, the range of turbulence scales present in the flow also increases, leading to the need to resolve more temporal and spatial scales with a finer grid spacing. Additionally, the use of highly accurate numerical schemes also adds to the cost. Finally, propagation to the far-field can increase expense, depending on the method used. Saving, managing and analyzing the massive amounts of data generated by LES is a further complication [41].

A second shortcoming is the lack of an agreed-upon standard methodology, leading to a lack of standardization and an abundance of numerical methods. Indeed, validating LES codes at a high-enough resolution for fundamental fluid-acoustic coupling is a challenge, and in response, NASA has been working to produce more in-depth data sets that can measure statistical turbulence quantities, such as space-time correlations through advanced PIV (particle image velocimetry) techniques. One such example data set is from the Small Hot Jet Acoustic Rig (SHJAR) rig, which has developed a database for single-stream round jets at a variety of velocities and temperature ratios for several nozzle designs [40, 41].

Bodony and Lele [23] discuss several additional concerns about LES noise predictions, including the influence of SGS models, of inflow conditions and inflow forcing, of the inclusion of a nozzle geometry, and of propagation methods for far-field noise. The two most relevant of their concerns
are: (1) addressing the limited spectrum of radiated noise and (2) properly simulating the initial shear layer thickness and turbulent behavior.

The issue of limited frequency bandwidth is facilitated by a high-frequency cut-off that has been widely observed [18, 22, 23, 46, 137]. This high-frequency cut-off is especially pronounced in the near-nozzle shear layer of the jet where the smallest turbulent scales are generated [46, 100] and is ultimately related to the spatial resolution of the LES scheme. By design, LES methods only solve for the larger turbulent scales, while in contrast, the effects of the smallest turbulent scales are modeled by a sub-grid scale (SGS) model. In other words, LES simulations lack turbulent fluctuations at scales smaller than the discrete computational grid and numerical scheme can resolve. Without the presence of fluctuations, no acoustic content can be generated for those unresolved, smaller scales. Therefore, a portion of the sound spectra is missing: the higher-frequency noise associated with fine-scale turbulence. Simply put, the spatial resolution for a LES method is not fine enough in the near-nozzle shear layer to resolve the turbulent processes that generate high-frequency noise.

To further compound the issue of missing frequencies, the initial thickness and state of the shear layer play roles in noise resolution issues. If the simulated near-nozzle shear layer is too thick due to the inadequate resolution by the computational grid or has not fully transitioned to a turbulent state, the fine-scale turbulence connected with high-frequency noise cannot properly develop [23, 46]. Consequently, without a fine enough grid resolution, the shear layer cannot develop turbulence properly, which further contributes to the missing high-frequency issue. A thinner shear layer has been shown to promote more rapid turbulence development and to encourage transition, but the exact effect on the noise is unclear since it has been shown to both enhance sideline noise (which
is dominated by higher-frequency content) [26] and increase low-frequency noise [25]. Similarly, recent investigations by Bogey and Marsden have shown that resolving a wide range of fine-scale turbulence in the shear layer leads to the generation of more low-frequency noise, which suggests that fine-scale turbulence is required to properly model the development of low-frequency noise generation mechanisms [34]. A failure to resolve fine-scale turbulence, then, might lead to not only a reduction in high-frequency noise due to unresolved turbulence but also an under-prediction of low-frequency noise.

Since the development of turbulence in the jet directly relates to noise production and ultimately grid resolution, attention must be paid in properly simulating the shear layer. It is clear that correctly simulating the wide range of turbulent scales in the near-nozzle shear layer is essential to proper noise prediction. However, resolving all the required turbulent scales with LES is difficult.

### 2.4.3 LES Frequency Resolution Limits

A discussion on the missing noise warrants a look at the current frequency resolution limits for acoustic spectra predicted by LES. The question arises as to what frequencies should be resolved with LES. The sideline peak noise is observed around \( \text{St} = 1 \) or \( \text{St} = 2 \) [48]. At minimum, these frequencies must be resolved for LES noise predictions.

The maximum practically-achievable Strouhal numbers reported by several selected publications are shown in Table 2.1. Note that the values presented are not necessarily identical due to differing flow conditions. Additionally, other papers may have been overlooked, but the limits presented are useful to observe general trends.
The most noticeable trend is that the maximum resolvable Strouhal number, around 5-6, has remained relatively constant for a decade, and has only approximately doubled in value in the last fifteen years despite significant advances in computational power. However, the grid size has increased several orders of magnitude, most likely due to the inclusion of nozzle geometries, some of which are simple pipes, for better turbulence simulation in the shear layer. This trend in grid count gives some indication of the difficulty in simply increasing grid resolution to simulate higher frequencies, as it is not clear exactly how much resolution is needed [25]. One outlier in this trend, Uzun et al. [130], only extend their simulation domain for a few diameters downstream of the nozzle due to computational cost constraints. In contrast, the other simulations in Table 2.1 include the full jet domain and the far-field to some extent. Uzun et al. are able to predict higher frequencies by capturing fine-scale turbulence in the near-nozzle region, but fail to include the full jet domain and thus miss the peak-SPL low-frequency noise components. Even so, their grid
required 50 million grid points. A more recent study of a pipe nozzle and the entire jet domain required a one-billion node jet simulation and around three billion CPU hours to predict frequencies up to $St = 5$ [25], further demonstrating the extreme cost needed to make any significant increase in resolved frequency bandwidth.

It is reasonable, then, to suggest that a practical limit for present-day LES is to resolve frequencies up to $St = 5$, if not higher [21, 28], even though this range still does not account for all of the high-frequency content in the jet noise spectra.

### 2.4.4 Addressing High-Frequency Cutoff in LES

Bodony and Lele [23] proposed two solutions to address the issue of missing noise content in LES-based noise predictions. The first and most obvious solution to account for the missing high-frequency noise content is to simply increase the grid resolution in the near-nozzle to simulate thinner shear layers and resolve finer scales of turbulence. Of course, this option is costly (exactly how much is unclear [20]) and currently not likely to be viable for routine industrial use any time soon [19, 118, 136].

Colonius and Lele demonstrate the difficulty of resolving additional high-frequency content by showing noise spectra of a Mach 0.9 jet from two LES simulations of varying grid resolutions ($1 \times 10^5$ and $1 \times 10^6$ grid points, see Colonius and Lee [46], Figure 8). The increase in grid resolution brought the predicted high-frequency noise results closer to the curve fit from Tam’s empirical jet spectra [123]. However, the SPL at a moderate Strouhal number of 1 was still significantly under-predicted by 10dB and likely requires a further order of magnitude or more increase in grid resolution to match the empirical curve fit, giving further indication of the cost required to predict
higher frequencies. The overall advantage of simply increasing the grid resolution does not require the development of new models that might limit the applicability of LES. An extension of this option is to include the nozzle geometry in the simulation, since turbulence in the boundary layer of the nozzle leads to fine-scale turbulence in the shear layer [130]. While likely a more physically-accurate alternative, this further increases the computational cost to the point that simulating the entire jet may be impractical.

As computational power improves, so does the upper limits of resolvable frequencies. However, improving computational power alone has not been enough to solve the problem of high cost, especially for industry applications. Bodony and Lele [19] showed in 2003 that the grid resolution required to resolved even moderate fluctuations above Strouhal numbers of 0.5 was impractical for routine industrial LES use. There is still no consensus on the required grid spacing for LES jet noise predictions even for academic studies [25]. Even with increased grid spacing, the full frequency spectrum could not be resolved due to the nature of the LES and sub-grid scale modeling. Only DNS is capable of capturing all scales of turbulent motion and the full range of noise spectra, necessitating the need for developing a missing noise model when using LES.

A second and more reasonable solution for the high-frequency cutoff is to introduce a model to estimate the noise missing from the unresolved sub-grid turbulent scales. This option requires the use of lower-fidelity models in conjunction with LES or the development of new SGS models that account for noise. Either option involves developing new models, which will likely have limited applicability as it is likely they cannot be tuned for every single jet flow condition. However, given current limitations on computational resources, this avenue seems the most promising. Even simpler models have shown some ability to recover the lost acoustic power at higher frequencies [113, 115,
What little progress has been made in this area will be discussed in later sections in detail. For now, it suffices to say that one complete solution has not been presented in the literature.

To summarize, addressing the issue of missing high-frequency spectral content by increasing grid resolution is too costly to employ with LES, and there is a surprising lack of research into modeling the missing small scale turbulence for noise applications. This gap in research suggests the need to develop a new model for the missing noise.

### 2.4.5 Models for High-Frequency Noise

The inability of a pure LES method to resolve the full noise spectrum in a cost-effective manner suggests the need for an alternative method for LES-based noise prediction. Several authors have investigated different approaches for modeling the missing high-frequency noise content.

Seror et al. [115] apply both a priori and a posteriori methods to filtered DNS results of isotropic turbulence in order to evaluate a hybrid LES/Lighthill noise prediction approach. They observed a loss in acoustic intensity in higher frequencies when the contribution of the SGS term was not included in the Lighthill stress tensor. In fact, they demonstrated the need for a parameterization of the subgrid scale part of Lighthill’s tensor in order to properly recover the lost acoustic intensity. These results indicate that the noise from unresolved turbulent scales (or residual SGS stresses) is essential to predict the full acoustic spectra [139]. Significant further development would be needed to apply such a method to transient jet noise simulations.

Rubinstein and Zhou [113] take a preliminary step towards a model for the subgrid contribution to radiated sound. They assume isotropic turbulence and derive a theoretical equation for subgrid-scale sound radiation. Their initial results help confirm the idea that sound from unresolved velocity
fluctuations still contributes significantly to the total acoustic power [113, 115, 139], showing that for larger LES filter sizes, the total power is under-predicted when only considering the sound radiated by resolved velocity fluctuations. While not applied to jet noise, this method is based upon a model for two-point, two-time properties of the subgrid motions (related to the energy spectrum and time correlations) and could therefore be used in an SNGR-type acoustic simulation method for jet noise [5].

Using a framework called Large-Eddy STimulation (LEST) for hybrid RANS/LES modeling, Batten et al. [10] use synthetic turbulence to generate resolved-scale fluctuations for LES when transitioning from RANS to LES regions, such as at the edge of the boundary layer. A variation of Kraichnan’s stochastic method [82, 121] is used to generate the turbulent fluctuations in a manner similar to SNGR. However, this method neglects the sub-grid scale unresolved fluctuations. In an earlier paper, Batten et al. [9] describe using the same method to model sub-grid scale noise sources for a system of non-linear disturbance equations. A combination of resolved fluctuations from LES and sub-grid fluctuations from the statistical model can be used for noise sources with this approach, but the resulting noise field is not thoroughly investigated in their work.

Bodony and Lele [18] propose a method to approximate the missing scale noise by an acoustic analogy approach with source terms based on interactions between the resolved scale and the missing scales. Their method requires an adjoint approach [124] to solve the equations and model the noise of the missing-scales. Starting by linearizing the fluid equations and keeping products of the resolved and missing scales as source terms on the right-hand side, they developed equations for velocity and pressure for the missing scales.
Using a filter to separate DNS results into LES-resolved turbulent scales and LES-unresolved scales, Bodony and Lele determined that the large scales transport compressibility effects while the small scale motions are essentially incompressible [18]. Additionally, although the convection speed changed across the shear layer, they showed that the small scale structures travel at about the same speed as the large scale structures. They also demonstrated that eddies in the shear layer may travel slower or faster than the mean flow in the initial portion of the shear layer, possibly due to the influence of large coherent structures. Further downstream, the eddy convection velocities converge to the mean velocity. They suggest that the effects of the missing scales cannot be computed independently of the resolved turbulent scales, implying that some interaction between the resolved and missing scales is important for a noise model for the missing scales. They ignore several phenomena, such as refraction from the mean flow and scattering due to turbulence.

Bodony and Lele [19] further developed this method into a statistical noise model for the missing noise. The authors derived a set of equations from Goldstein’s general acoustic analogy that describe source terms for the missing noise. However, solving these equations requires the same computational resources as solving for the resolved turbulent flow field. Instead, Bodony and Lele related the space-time correlations of the source terms to the far-field power spectral density of the fluctuating density, obtaining the far-field noise by an adjoint formulation of the generalized acoustic analogy. The adjoint formulation includes scattering and refraction effects.

This method was tested on a DNS database of a turbulent shear flow, where the “resolved” and “missing” scales could be directly obtained via filtering (see Bodony and Lele [18]), simulating LES results. The noise from the statistical method compared favorably to the noise from the DNS database. Applying this method to LES in the future, where the missing scales are not known,
requires estimating the missing scales from the large scales via an approximate deconvolution process that combines low-pass-filtered versions of the resolved velocity field [19]. The interesting idea of estimating missing scales via a deconvolution process might prove ineffective with under-resolved LES or VLES simulations, which are more practical for industry. Although intended for jet noise, no further application of this method was found in the literature.

Independently, Bodony [21] approached the problem of missing sub-grid scale noise in LES by applying a Gabor transform (a short-window Fourier-transform that weighs the signal in time) to separate turbulence scales in a spatial-wavenumber space. With this approach, Bodony derived equations that approximate the behavior of the sub-grid turbulent scales above a cut-off frequency wavenumber. A Lagrangian particle approach [86] was used to discretize the equations, estimate subgrid fluctuations, and reconstruct the Reynolds stress contributions from the modeled subgrid scales [21]. The subgrid stresses, once obtained, can be used with an acoustic analogy to form noise source terms for the subgrid turbulence. The paper develops the method mathematically but presents no results. An advantage of this method is that it directly gives subgrid stresses and fluctuations. However, the use of fast Fourier transforms with the particle-based method likely makes this subgrid noise model expensive [86].

The most relevant method addressing the missing high-frequency noise was introduced by Yao and He [139], who proposed the use of synthetic fluctuations to account for unresolved LES fluctuations. They applied the kinematic simulation (KS) method of Fung et al. [61] to synthesize sub-grid scale fluctuations via a summation of random Fourier modes. The idea of this method, named KS SGS, is that the Kinematic Simulation (KS) method can be used as a sub-grid scale (SGS) model to generate velocity fluctuations for the unresolved scales of turbulence, which in turn,
can be used to recover noise corresponding to the missing high frequency scales in the radiated noise field. The fluctuations are computed on the same grid as the LES simulation.

To ensure some level of physical relevance in the synthesized fluctuations, the random sweeping hypothesis [66, 81] is used to ensure that space-time correlations of the synthesized velocities are consistent with the resolved LES velocity field. The von Karmon energy spectra is used to extrapolate the energy spectra for the unresolved scales based on the resolved LES velocities. This assumption is possible because the universal form of the energy spectra at higher Reynolds numbers allows the use of model spectra.

The KS SGS method was tested on a box of isotropic turbulence, comparing DNS, LES, and LES with KS SGS. Their results show a roll-off of high frequencies in the LES spectra as compared to the DNS spectra. Adding the KS SGS model to LES does increase the higher frequency content after the cut-off. However, with the coarsest grid, the LES cut-off happens at too low of a wavenumber for the added fluctuations to approximate the DNS power spectra. The added content improves the energy drop off due to the LES cut-off and matches the general spectral trends from DNS, but does so at a reduced power level. Even with added high wavenumber content, the DNS levels cannot fully be recovered if sound levels at the lower wavenumbers are not properly simulated. In other words, the benefit of adding synthetic fluctuations for the unresolved scales is limited depending on the underlying grid resolution of the LES simulation. Other results show that the energy spectra with KS SGS is extended to match the DNS results at higher wavenumbers, where the original LES showed a drop-off. Time correlations from LES and KS LES match each other and vary only slightly from DNS results, with LES having slightly slower decay.
The results presented by Yao and He reveal that the concept works: unresolved higher frequency content in LES can be approximately recovered by supplementing the turbulent flowfield with random fluctuations. However, the LES simulation must have a certain resolution to truly take advantage of this added spectral content. The results suggest that a grid spacing ratio of LES to DNS of 1:4 or 1:2 would give the most benefit from a method such as this. An important conclusion of this investigation is that fluctuations were synthesized on the original LES grid, and that higher frequencies were predicted without increasing the grid resolution of the synthesized field. However, it should be noted that the synthetic fluctuations are not actually added into the LES simulation, but their effects on the noise field are incorporated through an equation for the far-field sound power spectra. Another important result is the suggestion that the missing LES scales do have an important contribution to the sound spectra levels.

Although Yao and He demonstrate that the concept of adding fluctuations to LES is promising for resolving the missing higher-frequencies, they do not extend this method to more complex flows like jets. As presented, the KS SGS method cannot be applied to jet noise flows because the current implementation of the sweeping hypothesis cannot account for all of the time correlation mechanisms present in turbulent shear flows. Furthermore, there is no influence of the unsteady resolved fluctuations on the unresolved, modeled fluctuations, and so sweeping in a shear flow cannot be enforced. The authors do mention that modifications could be made for shear flows. The intended application for the KS SGS method is particle dispersion in turbulent media [141], but the random Fourier mode method is similar to the SNGR method for jet noise [5]. A similar methodology could be used for jet noise: this is the intent of the CLST method.
In summary, several approaches have been developed to address the high frequency content that is missing from LES noise predictions. However, no method has been demonstrated satisfactorily for jet noise predictions. The most promising method, that of Yao and He [139], cannot be used for jet noise without modifications. The other methods that were discussed did not see continued development toward jet noise applications, though some might have been promising. Clearly, additional modifications or methods are required to capture the missing noise contributions needed for accurate noise predictions.

2.5 Summary: Literature Gap

The problem of missing high frequency content in LES simulations due to unresolved small scale turbulent fluctuations has been widely acknowledged in the literature [18, 22, 23, 46, 137]. Several models addressing this issue have been developed, and were discussed in previous sections [10, 18, 19, 21, 113, 115, 139]. These methods were either not suitable for shear flows or not developed to the point of application to a full jet case. In other words, no method was found that adequately supplements the missing high-frequency noise content for LES-based jet noise predictions. Therefore, this problem remains an open research issue.

To reiterate: LES noise predictions are expensive, but the highest frequencies cannot be directly resolved for higher Reynolds number jet flows (the problems of interest) because this would further increase the computational cost. Yet, the higher frequencies (above St = 5) are still desired for more complete noise predictions scenarios.

Synthetic and stochastic turbulent methods have shown the ability to predict portions of the jet noise spectra at a low computational cost [5, 14, 51, 59, 67, 83]. These methods alone are
insufficient for the level of detail desired for noise prediction and are often not appropriate for modeling large, anisotropic turbulent scales and low-frequency noise. However, these models are often appropriate for simulating isotropic turbulence, corresponding, in general, to high-frequency noise.

Based upon the gap in existing methods, the proposed CLST (Coupled LES-Synthetic Turbulence) method couples a synthetic turbulence method with a Very Large Eddy Simulation (VLES) method in order to predict jet noise in a hybrid CAA framework. A majority of the noise spectra and large-scale turbulent fluctuations will be resolved by VLES, while synthetic turbulence will supplement the unresolved, small-scale turbulence at a minimal increase in computational cost. Convection of the synthetic eddies will include sweeping and straining effects to better model turbulent physics. Noise corresponding to the synthesized turbulent scales will supplement the missing high-frequency spectra, modeling fine-scale turbulent noise sources.

The desired goal is to decrease the computational cost associated with resolving higher frequencies. In this work, frequencies up to $\text{St} = 5$ will be modeled, which is approximately 1.5 kHz for a full-scale jet. This limit is reasonable given practical computational limitations [21, 28].

The CLST (Coupled LES-Synthetic Turbulence) method is described and demonstrated for jet noise prediction in the remainder of this dissertation.
CHAPTER 3
INTRODUCTION TO THE CLST METHOD

This section presents an overview of CLST, a method for obtaining a more complete frequency spectrum by coupling LES-resolved fluctuations with synthetic fluctuations.

3.1 CLST Description

An accepted method for far-field jet noise prediction is to resolve the near-jet flowfield with LES and to propagate acoustic fluctuations to the far-field using LEE equations with source terms or acoustic analogies based on integral methods. The use of LES or VLES turbulence modeling ensures the resolution of larger-scale turbulent fluctuations that correspond to lower frequency noise in the far-field. Figure 3.1 shows how the proposed CLST method modifies the VLES/LEE-based noise prediction scheme by adding the generation of synthetic fluctuations to model higher frequency content.

The main idea of CLST as shown in Figure 3.1 is that resolved and modeled velocity fluctuations are generated by VLES and a synthetic turbulence method, respectively. These fluctuations are input into source terms for the LEE (Linearized Euler Equations), which generate acoustic sources that produce pressure fluctuations. These pressure fluctuations generated by the coupled velocity fluctuations from VLES and synthetic turbulence are propagated to the far-field, thus obtaining
far-field noise predictions. The proposed CLST modeling approach involves the following general steps:

- Compute an axisymmetric RANS solution for a jet flow using a $k-\varepsilon$ or $k-\omega$ turbulence model.
- With the RANS solution as an initialization, run the VLES solver until statistically-stationary results are obtained.
- Initialize synthetic turbulence from the mean flowfield.
- Continue the VLES simulation and interpolate VLES fluctuations onto the LEE grid.
- Convect synthetic turbulence with VLES flowfield, accounting for sweeping and straining of the synthetic velocities.

Figure 3.1
Diagram of The CLST Method
• Compute velocity field produced by the summation of each synthetic mode or eddy (SNGR or SEM).

• Combine VLES fluctuations with synthetic fluctuations computed at each time step.

• Generate noise sources from LEE source terms and propagate noise with LEE to obtain far-field acoustic pressure data.

As shown in Figure 3.1, either LES or VLES (Very Large Eddy Simulations) can be used with the CLST method depending on the desired computational cost. For instance, given the high computational cost of LES, it is unlikely to see industrial application soon. However, if the CLST method were used with VLES or even URANS (Unsteady RANS) instead of LES, then it might be possible to save significant computational cost due to the lower-fidelity nature of VLES while achieving LES-level frequency resolution. Alternatively, applying the CLST method to LES, while still expensive, could possibly provide DNS-level frequency resolution at a fraction of the cost of full-resolution DNS.

The novel aspect of CLST is the coupling of synthetic (or randomized) fluctuations to model higher acoustic frequencies. In Figure 3.1, the method chosen to synthesize small-scale turbulent fluctuations is SEM, the Synthetic Eddy Method. Alternative methods, such as the SNGR (Stochastic Noise Generation and Radiation) method can be employed. In this work, an SNGR method was first tested to prove the effectiveness of adding synthetic turbulence fluctuations to the noise field. This was seen as a first step because SNGR is fast and quick to implement. However, SEM can produce a synthetic turbulence field with localized variations, which is a more accurate representation of turbulence. This allows SEM to model sweeping and straining of the synthetic
eddies in a more straightforward manner. The computational cost, however, is increased but is still much lower than running full DNS or high-resolution LES.

Ultimately, SEM was chosen as the synthetic turbulence method for CLST. In this work, CLST will refer to the overarching framework, as well as the specific implementation built on the SEM method. For clarity, the SNGR-based method will be referred to as the VPST (VLES plus Stochastic Turbulence) method. The specific implementation of each method will be explained in Chapters 4 and 5, respectively. Additionally, results are presented and evaluated for each implementation of CLST.

3.2 Assumptions

Two assumptions underpin the CLST method and aid in limiting the complexity of this problem. The first assumption is that the two-source theory of jet noise (see Section 2.1.4) holds and that, consequently, high frequency noise content is generated by fine-scale turbulence in the shear layer. In other words, larger scales of turbulence correspond to lower-frequency noise and smaller turbulent scales correspond to higher-frequency noise. The second assumption is that a wide range of turbulent scales exists in the jet shear layer, and, consequently, there is a separation between large and small turbulent scales. Furthermore, it is assumed that the smaller scales are isotropic (per Kolmogorov’s hypothesis) and statistically similar [107] and can therefore be modeled by a stochastic or synthetic turbulence method.

Additionally, since the hybrid CAA formulation of the CLST method only allows coupling from the LES solver to the LEE solver, the small scales can not influence the larger scales, and it is assumed that backscatter [90, 114] should not be explicitly accounted for. However, the influence
of the larger scales on the smaller scales, known as sweeping, is assumed to be significant for the radiated noise [119, 112]. Propagation of acoustic waves is assumed to be a linear, inviscid phenomenon even if generated by nonlinear phenomena.
CHAPTER 4

SNGR-BASED CLST METHOD

The SNGR-based CLST method is presented in the following section. The governing equations and numerical discretization methods are discussed, as well as specifics of the synthetic generation method, the Stochastic Noise Generation and Radiation Method (SNGR). To differentiate between the SEM-version of the CLST method, in this section, results from the the SNGR-based method will be referred to as the VPST (VLES plus Stochastic Turbulence) method.

4.1 Governing Equations

The Navier-Stokes (NS) equations are used for LES, and the Linearized Euler equations (LEE) are used for acoustic propagation.

4.1.1 Navier-Stokes Equations

The governing equations of fluid motion are the Navier-Stokes equations. The 3D compressible CFD solver used for this research employs a numerical discretization of these equations in conser-
ative form cast in curvilinear coordinates. A generalized curvilinear coordinate transformation in
the three-dimensional form

\[
\tau = \tau(t) \\
\xi = \xi(x, y, z, t) \\
\eta = \eta(x, y, z, t) \\
\zeta = \zeta(x, y, z, t)
\]

is considered, where \(\xi, \eta,\) and \(\zeta\) are the spatial coordinates in computational space, and \(x, y,\) and \(z\)
are the spatial coordinates in physical space. This transformation allows for a seamless mapping of
the solution from the computational to the physical space and vice-versa so that computations may
be performed on a uniform computational grid to obtain an accurate representation of complex wall
and flow geometries while maintaining a consistent grid spacing. All spatial coordinates are scaled
by reference length (e.g., jet nozzle diameter), \(L,\)

\[
(x, y, z) = \left(\frac{x^*, y^*, z^*}{L}\right),
\]

the velocity is scaled by the freestream speed of sound, \(a_\infty,\)

\[
(u, v, w) = \left(\frac{u^*, v^*, w^*}{a_\infty}\right),
\]

the pressure by \(\rho_\infty a_\infty^2,\) and temperature by the freestream temperature, \(T_\infty.\) Reynolds number
based on a given reference length (\(L\)) and freestream velocity, Mach number and Prandtl number
are defined as

\[
Re = \frac{\rho_\infty V_\infty L}{\mu_\infty}, \quad M = \frac{V_\infty}{a_\infty}, \quad Pr = \frac{\mu_\infty C_p}{k_\infty}
\]
where \( \mu_\infty \) and \( k_\infty \) are freestream dynamic viscosity and thermal conductivity, respectively, and \( C_p \) is the specific heat at constant pressure. In conservative form, the Navier-Stokes equations are written as

\[
\dot{Q} + F_\xi + G_\eta + H_\zeta = S. \tag{4.5}
\]

where the vector of conservative variables is given by

\[
Q = \frac{1}{J} \{ \rho, \rho u_i, E \}^T, i = 1, 2, 3 \tag{4.6}
\]

and where \( \rho \) is the density of the fluid, \( u_i = (u, v, w) \) is the velocity vector in physical space, and \( E \) is the total energy. The flux vectors, \( F, G \) and \( H \), are given by

\[
F = \frac{1}{J} \left\{ \rho U, \rho u_i U + \xi_x (p + \tau_1), EU + p\bar{U} + \xi_x \Theta_i \right\}^T \tag{4.7}
\]
\[
G = \frac{1}{J} \left\{ \rho V, \rho u_i V + \eta_x (p + \tau_2), EV + p\bar{V} + \eta_x \Theta_i \right\}^T \tag{4.8}
\]
\[
H = \frac{1}{J} \left\{ \rho W, \rho u_i W + \zeta_x (p + \tau_3), EW + p\bar{W} + \zeta_x \Theta_i \right\}^T \tag{4.9}
\]

where the contravariant velocity components are given by

\[
U = \xi_x u_i, \quad V = \eta_x u_i, \quad W = \zeta_x u_i. \tag{4.10}
\]

With the Einstein summation convention applied over \( i = 1, 2, 3 \), the shear stress tensor and the heat flux are given by

\[
\tau_{ij} = \frac{\mu}{Re} \left[ \left( \frac{\partial \xi_k}{\partial x_j} \frac{\partial u_i}{\partial \xi_k} + \frac{\partial \xi_k}{\partial x_i} \frac{\partial u_j}{\partial \xi_k} \right) - \frac{2}{3} \delta_{ij} \frac{\partial \xi_l}{\partial \xi_k} \frac{\partial u_k}{\partial \xi_l} \right] \tag{4.11}
\]

and

\[
\Theta_i = u_j \tau_{ij} + \frac{\mu}{(\gamma - 1)M_a^2 Re Pr} \frac{\partial \xi_l}{\partial \xi_i} \frac{\partial T}{\partial \xi_l} \tag{4.12}
\]
respectively, and $S$ is the source vector term. The pressure $p$, the temperature $T$, and the density of the fluid are related by the equation of state, $p = \rho T / \gamma M^2_\infty$, when non-chemically-reacting flows are considered. Another symbol is $\gamma$, representing the ratio between the specific heats. The Jacobian of the curvilinear transformation from the physical space to computational space is denoted by $J$.

The derivatives $\xi_x, \xi_y, \xi_z, \eta_x, \eta_y, \eta_z, \zeta_x, \zeta_y, \zeta_z$ represent the grid metrics. The dynamic viscosity and thermal conductivity $k$ (obtained from the Prandtl number) are linked to the temperature using the Sutherland’s equations in dimensionless form,

$$\mu = T^{3/2} \frac{1 + C_1/T_\infty}{T + C_1/T_\infty}$$

where for air at sea level, $C_1 = 110.4K$, $C_2 = 194K$, and $T_\infty$ is a reference temperature.

The equations are solved in conservative form as

$$Q_t + \xi_t Q_\xi + \xi_x F_\xi + \xi_y G_\xi + \xi_z H_\xi$$

$$+ \eta_t Q_\eta + \eta_x F_\eta + \eta_y G_\eta + \eta_z H_\eta$$

$$+ \zeta_t Q_\zeta + \zeta_x F_\zeta + \zeta_y G_\zeta + \zeta_z H_\zeta = S$$

with the grid metrics for the curvilinear transformation given by

$$\frac{\partial}{\partial x} = \xi_x \frac{\partial}{\partial \xi} + \eta_x \frac{\partial}{\partial \eta} + \zeta_x \frac{\partial}{\partial \zeta}$$

$$\frac{\partial}{\partial y} = \xi_y \frac{\partial}{\partial \xi} + \eta_y \frac{\partial}{\partial \eta} + \zeta_y \frac{\partial}{\partial \zeta}$$

$$\frac{\partial}{\partial z} = \xi_z \frac{\partial}{\partial \xi} + \eta_z \frac{\partial}{\partial \eta} + \zeta_z \frac{\partial}{\partial \zeta}.$$
waves. The LEE govern acoustic, entropy, and vorticity wave propagation \[13\]. With source terms, the LEE equations behave as an acoustic analogy.

The governing equations (expanded to 3D from Bogey et al. \[30\]) are written in weak conservation vector form as:

\[
U_t + F_x + G_y + H_z + M = S
\]  \hspace{1cm} (4.20)

where \(U\), the flow variable vector comprised of fluctuating \((u')\) and mean \((\bar{u})\) primitive variables (i.e. \(u_{inst} = \bar{u} + u'\)), is given as

\[
U = \{ \rho' \bar{u}' \bar{p}' \bar{v}' \bar{w}' \}^T,
\]  \hspace{1cm} (4.21)

and where \(\rho\) is density, \(u, v,\) and \(w\) are velocities in the \(x, y,\) and \(z\) directions, respectively, and \(p\) = pressure. The flux vectors \(F, G,\) and \(H\) and vector \(M\) are given as

\[
F = \left\{ \begin{array}{c}
\rho' \bar{u}' + \bar{\rho}u' \\
\bar{\rho}u' + p' \\
\bar{\rho}v' \\
\bar{\rho}w' \\
\bar{u}p' + \gamma \bar{\rho}u'
\end{array} \right\}, \quad
G = \left\{ \begin{array}{c}
\rho' \bar{v}' + \bar{\rho}v' \\
\bar{\rho}v' + p' \\
\bar{\rho}w' \\
\bar{v}p' + \gamma \bar{\rho}v'
\end{array} \right\}, \quad
H = \left\{ \begin{array}{c}
\rho' \bar{w}' + \bar{\rho}w' \\
\bar{\rho}w' + p' \\
\bar{w}p' + \gamma \bar{\rho}w'
\end{array} \right\},
\]  \hspace{1cm} (4.22)

\[
M = \left\{ \begin{array}{c}
0 \\
(\bar{\rho}u' + \rho' \bar{u}) \frac{\partial}{\partial x} + (\bar{\rho}v' + \rho' \bar{v}) \frac{\partial}{\partial y} + (\bar{\rho}w' + \rho' \bar{w}) \frac{\partial}{\partial z} \\
(\bar{\rho}u' + \rho' \bar{u}) \frac{\partial}{\partial x} + (\bar{\rho}v' + \rho' \bar{v}) \frac{\partial}{\partial y} + (\bar{\rho}w' + \rho' \bar{w}) \frac{\partial}{\partial z} \\
(\bar{\rho}u' + \rho' \bar{u}) \frac{\partial}{\partial x} + (\bar{\rho}v' + \rho' \bar{v}) \frac{\partial}{\partial y} + (\bar{\rho}w' + \rho' \bar{w}) \frac{\partial}{\partial z} \\
(\gamma - 1)p' \nabla \cdot \bar{u} - (\gamma - 1)u' \cdot \nabla \bar{p}
\end{array} \right\},
\]  \hspace{1cm} (4.23)
with \( M \) being responsible for mean shear terms (i.e. \( \partial \bar{u} / \partial y \)) and refraction effects [5, 30]. This term was observed to lead to the growth of instability waves in the solution and therefore was set to zero by Bogey et al. [30] under the assumption that the mean flow variations are not significant (a weakly non-uniform mean flow). Colonius and Lele [46] also note that the “unbounded homogenous solution” has posed a problem with LEE. Additionally, growing instabilities were observed in the present simulations so this term was removed from the equations. For comparison, the full LEE equations in strong conservation form are given in Lafitte et al. [85]. The term \( M \) is also noted to be zero in a uniform mean flow [5]. These equations are referred to as the inhomogeneous Linearized Euler Equations (ILEE) [25].

The source term from Bogey et al. [30], based on the Lighthill tensor [67], is given as

\[
S = \beta \begin{pmatrix}
0 \\
S_1 - \overline{S}_1 \\
S_2 - \overline{S}_2 \\
S_3 - \overline{S}_3 \\
0
\end{pmatrix}, \text{ where } S_i = -\frac{\partial \rho u'_i u'_j}{\partial x_j} \text{ and } \overline{S}_i = -\frac{\bar{\rho} u'_i u'_j}{\partial x_j}
\]

(4.24)

where the fluctuating variables come from LES or synthetic turbulence. The parameter \( \beta \) adjusts the amplitude of the sources, which was added in the manner of Lafitte et al. [85]. For this formulation of the LEE, \( \beta \) was chosen as 5.2 so that the amplitude of the LEE fluctuations would match the amplitude of the LES fluctuations. The term \( \rho \) in Equation 4.24 can be the instantaneous density, \( \rho = \bar{\rho} + \rho' \), but since the term \( \rho' u' v' \) is negligible, \( \bar{\rho} \) is simply used. For the initial implementation, the subtraction of the mean source terms has been neglected. However, Bogey et al. [30] observed that subtracting the mean has a non-trivial effect.
4.2 Numerical Methods

This section discusses the numerical algorithms of the CFD solver, including spatial and temporal discretization schemes, filtering for Implicit LES (ILES) turbulence model, boundary conditions, and inlet forcing. Both LES and LEE employ the same spatial and temporal discretizations, so the following description refers to both.

4.2.1 Spatial Discretization

The spatial derivatives are discretized using the seven-point, fourth-order dispersion-relation-preserving schemes of Tam and Webb [126]. Dispersion-relation-preserving (DRP) schemes are designed to ensure that the finite difference schemes calculate the correct wave speeds and propagation characteristics (nondispersive, nondissipative, isotropic) for the main wave modes (acoustic, vorticity, and entropy), trying to achieve the same dispersion relations as the original partial differential equations. These schemes do so by relating spatial and temporal derivatives to the dispersion relations of the governing equations [126]. Rather than compact schemes, DRP schemes are simply optimized, explicit finite-difference schemes. DRP schemes are commonly used for the spatial discretization in LES jet noise simulations [23].

Specifically, the first derivative at the \( l \)th node is approximated using \( M \) values of \( f \) to the right and \( N \) values of \( f \) to left of the node

\[
\left( \frac{\partial f}{\partial x} \right)_l \approx \frac{1}{\Delta x} \sum_{j=-N}^{M} a_j f_{l+j}.
\]

(4.25)

By taking the Fourier transform of the above equation, the coefficients \( a_j \) are found by minimizing the integrated error of the difference between the wavenumber of the finite difference scheme and the wavenumber of the Fourier transform of the finite difference scheme. For the DRP scheme,
the weights of the centered stencils are \( a_1 = -a_{-1} = 0.77088238 \), \( a_2 = -a_{-2} = -0.16670590 \), \( a_3 = -a_{-3} = 0.02084314 \), and \( a_4 = -a_{-4} = 0 \). To damp out the unwanted high wavenumber waves from the solution, high-order spatial filters, as developed by Kennedy and Carpenter [77], are used.

### 4.2.2 Temporal Discretization

For temporal discretization, an explicit third-order Total Variation Diminishing (TVD) Runge-Kutta scheme is employed. This method for time marching is also commonly used in LES jet noise simulations [23]. The numerical algorithm for the Runge-Kutta method [93] used for the LES simulations is written in the form

\[
\begin{align*}
Q^{(0)} &= Q^n \\
Q^{(1)} &= Q^{(0)} + \Delta t L(u^{(0)}) \\
Q^{(2)} &= \frac{3}{4} Q^{(0)} + \frac{1}{4} Q^{(1)} + \frac{1}{4} \Delta t L(Q^{(1)}) \\
Q^{n+1} &= \frac{1}{3} Q^{(0)} + \frac{2}{3} Q^{(1)} + \frac{2}{3} \Delta t L(Q^{(2)}),
\end{align*}
\]  

where \( L(Q) \) is the residual.

### 4.2.3 Spatial Filtering for Implicit LES

Unresolved length and time scales in the LES solution can produce spurious high-frequency content in the flowfield. High-order low-pass spatial filters can be used to damp these non-physical waves in the solution. Explicit filters require less computational effort and are conceptually simpler than implicit filters [77].
Essentially, the filters are designed to attenuate higher wavenumbers relative to the grid resolution but to leave lower wavenumbers untouched, ensuring that only spurious high-frequency signals are damped in the solution. To be clear, this is a separate issue than the missing noise content from LES that is the main topic of this dissertation and these high frequencies, which are not physical, must be damped to prevent non-physical flow phenomena and noise content.

The high-order spatial filtering that provides damping for unwanted waves serves a second purpose as an implicit subgrid scale (SGS) turbulence model for the LES code. Typical SGS models impose an artificial subgrid viscosity to damp the solution so as to limit the buildup of energy from unresolved turbulent scales. Commonly used for LES jet noise by several researchers [23, 25], the implicit filtering approach employs low-pass filters on the flowfield to dissipate or damp wave numbers above a specified threshold [23]. The argument for not explicitly including an SGS model (such as Smagorinsky) is that the main contribution of such a model is to provide the proper dissipation for resolved turbulent scales and stresses and that an arguably cheaper yet identical result can be achieved by numerical dissipation [23].

The explicit filters of Kennedy and Carpenter [77] are implemented in the present implicit LES code as a filter matrix multiplying the vector to be filtered. For an example, the tenth-order explicit central-difference operator [77] for the $(2n)$th-order derivative of a function $f$ is given as

$$
    f_i^{(2n)} = \frac{\gamma f_i}{(\Delta x)^{2n}} + a \frac{f_{i+1} - f_{i-1}}{(\Delta x)^{2n}} + b \frac{f_{i+2} - f_{i-2}}{(\Delta x)^{2n}} + c \frac{f_{i+3} - f_{i-3}}{(\Delta x)^{2n}} + d \frac{f_{i+4} - f_{i-4}}{(\Delta x)^{2n}} + e \frac{f_{i+5} - f_{i-5}}{(\Delta x)^{2n}}.
$$

(4.27)

The filter is applied to the flowfield $u$ through the equation

$$
    \hat{u} = (1 + \alpha_D D) u
$$

(4.28)
where \( \hat{u} \) is the filtered vector, \( \alpha_D = (-1)^{n+1}2^{-2n} \) for a \((2n)\)th-order filter, and \( D \) is a filter matrix containing coefficients for the operator.

The filter function is designed to have a value of zero at a wavenumber of \( \pi \) (relative to the grid) and a value of one at a wavenumber of zero, with the order of filter determining how quickly the filter drop-off occurs. Additionally, the filter function is guaranteed to be completely dissipative because the eigenvalues of \( D \) are always negative.

In \( D \), the interior scheme is order \( 2n \), but skewed stencils (order \( n \)) are used at the boundaries. Filters of up to order twenty are presented in Kennedy and Carpenter [77]. For the tenth-order scheme, the coefficients are \( \gamma = 252, a = -210, b = 120, c = -45, d = 10 \), and \( e = -1 \). Filtering the Navier-Stokes variables by a tenth-order filter [77] provides the best performance in both suppressing spurious content and providing dissipation for subgrid stresses in the present LES code.

### 4.2.4 Boundary Conditions and Inlet Forcing

The state of the initial boundary layer due to turbulent development in the nozzle has been shown as significant for LES jet simulations [133]. However, simulating the development of flow in the nozzle is both computationally costly and complex when using LES. Rather than resolve the interior flow of a nozzle, the jet is modeled as a hyperbolic tangent velocity profile where the “exit” of the jet nozzle corresponds to the simulation’s inlet plane. It is acknowledged that neglecting a nozzle geometry will have an influence on the development of the shear layer, jet flow field, and thus on the radiated jet noise [36, 133]. However, in the context of developing the CLST method, the use of an inflow boundary condition is seen as an initial step. If the method predicts adequate
results without a nozzle geometry, then including the nozzle geometry can be seen as a future step for improving the method. Furthermore, the use of synthetic turbulence to encourage nozzle flow development is also a possibility [68].

Without a nozzle geometry, imposing fluctuations at the inlet is required to promote the natural transition of the shear layer from quasi-laminar to turbulent [23] and to prevent an artificial reduction in noise levels [25, 26]. In light of this observation, divergence-free fluctuations are forced at the inlet in a ring around the edge of the jet “nozzle” at the inlet plane. For higher Reynolds number jet cases, disturbances are imposed in a manner similar to Bogey and Bailly [26], who employ a summation of azimuthal vortex ring modes to excite the axial and radial velocities as

$$
\begin{align*}
\left\{ \begin{array}{c}
    u \\ v 
\end{array} \right\} &= \left\{ \begin{array}{c}
    u \\ v 
\end{array} \right\} + \alpha u_j \sum_{i=n}^{m} \epsilon_i \cos(i\phi + \phi_i) \left\{ \begin{array}{c}
    u^{ring} \\ v^{ring} 
\end{array} \right\} \\
\end{align*}
$$

(4.29)

where the amplitude, $-1 \leq \epsilon_i \leq 1$, and the phases, $0 \leq \phi_i \leq 2\pi$, are randomly generated. The velocities of the resulting unit vortex ring are then expressed as

$$
\begin{align*}
\left\{ \begin{array}{c}
    u^{ring} \\ v^{ring} 
\end{array} \right\} &= \frac{2r_0}{r\Delta y} \exp \left[ -\ln(2) \frac{\Delta(x, r)^2}{\Delta y^2} \right] \left\{ \begin{array}{c}
    r - r_0 \\ x_0 - x 
\end{array} \right\} \\
\end{align*}
$$

(4.30)

where $\Delta(x, r)^2 = (x - x_0)^2 + (r - r_0)^2$, and the x location is at the origin, where the x-axis extends from the nozzle in the downstream direction.

The number of modes used in the present study is 15 with an amplitude of 0.06. At higher Reynolds number, numerical instabilities are likely to introduce additional fluctuations, so less excitation is needed than at lower Reynolds numbers.

Boundary conditions for the remainder of the jet simulation domain are far-field boundaries that employ extrapolation. A combination of grid stretching and sponge layers are used at the far-field.
boundaries to dissipate and filter out fluctuations before they generate spurious acoustic reflections at the boundaries.

4.3 The VLES-Plus-Stochastic Turbulence Method

This section presents the overview of the SGNR-based CLST method, referred to as the VPST (VLES-Plus-Stochastic) turbulence method. Using a hybrid CAA method, the jet flow is simulated with the Navier-Stokes (NS) equations in a finite difference multi-block, structured LES solver. Velocity fluctuations produced by the NS LES solver are passed to the Linearized Euler equations (LEE) to generate acoustic sources. The LEE solver then simulates the resulting noise field. Velocities generated by the SNGR (Stochastic Noise Generation and Radiation) method are added to the source terms in the LEE to generate the missing high frequency noise. To eliminate potential errors or increased cost due to interpolation between the solutions, the LES, VLES (Very Large Eddy Simulations), SNGR, and LEE flowfields are calculated on the same computational grid.

4.3.1 The SNGR Method

The VPST method uses the Stochastic Noise Generation and Radiation (SNGR) method to generate small scale isotropic turbulent fluctuations. This section describes the baseline SNGR method in more detail, as well as modifications that were made for the SNGR method.

A modified SNGR method [5] was employed, which is simple to implement and computationally inexpensive. The method generates stochastic velocity fluctuations \((u''_{st}, v''_{st}, w''_{st})\) in a specified source region via a summation of randomized Fourier modes over a discretized wave number range. A linear discretization of \(k_n\) was chosen in the manner of Billson et al. [14]. The method and equations are given below.
At each discrete wave number, $k_n$, a randomly-oriented wave vector, $k_n$ is chosen on a sphere of radius $k_n$ using the randomized angles $\theta_n$ and $\varphi_n$. On this wave vector $k_n$, the $n^{th}$ Fourier mode is chosen with an amplitude ($\tilde{u}_n$), a randomized phase ($\psi_n$), and a direction vector ($\sigma_n$). The condition $k_n \cdot \sigma_n = 0$ is enforced as an incompressibility assumption for all $n$. When summed, the $N$ modes provide turbulent velocity fluctuations that approximate a specified energy spectrum. The following equation is the summation of the $N$ Fourier modes that produces the three fluctuating velocities ($u'_{st}$, $v'_{st}$, $w'_{st}$)

$$u_t(x, t) = 2 \sum_{n=1}^{N} \tilde{u}_n \cos(k_n(x - tu_c) + \psi_n + \omega_n t)\sigma_n$$

(4.31)

where $u_c$ is the local convection velocity (from RANS) and $\omega_n$ is the randomly-generated angular frequency of the mode. The inclusion of a convection term $tu_c$ allows for time-dependent convection of the stochastic field via a mean flow, and setting all $\omega_n$ to zero provides simple convection [5].

The amplitude ($\tilde{u}_n$) of the $n^{th}$ mode is calculated so that the summation of modes models the von Kármán-Pao energy spectrum for isotropic turbulence [14], where RANS values of turbulent kinetic energy ($k_{RANS}$), dissipation ($\epsilon_{RANS}$), and viscosity ($\nu_{RANS}$) are used to properly scale the energy spectrum for a given problem where

$$E(k_n) = \frac{u'^2}{k_e} \frac{(k_n/k_e)^4}{\left[1 + (k_n/k_e)^2\right]^{17/6}} \exp[-2(k_n/k_\eta)^2], \quad \tilde{u}_n(k_n) = \sqrt{E(k_n) \cdot \Delta k_n}$$

(4.32)

where $\Delta k_n$ is the distance between the adjacent discretized wave numbers and

$$k_n = \frac{\epsilon_{RANS}^{1/4}}{\nu_{RANS}^{3/4}}, \quad u' = \sqrt{\frac{2}{3} k_{RANS}}, \quad k_e = \frac{0.725 \epsilon_{RANS}}{u'^3}, \quad \alpha \simeq 1.453.$$  

(4.33)
The SNGR method uses turbulent kinetic energy and dissipation rate to synthesize the fine-scale turbulence that is not resolved by LES. The parameter $\omega_n$ is typically generated from a Gaussian probability function, but was simply set to zero for this implementation. Sweeping is not explicitly accounted for in this formulation [83].

In the current implementation, the randomization of variables is done once prior to the time loop, and the calculation of stochastic velocities is performed inside the time loop to account for convection of the stochastic field. Rather than using a specific source region, velocities are calculated over the entire domain to prevent wave cut-off. However, the stochastic velocities are essentially zero in regions where turbulent kinetic energy and dissipation rate are zero, while the extra computational cost is negligible. More details of the randomization and calculations can be found in both Bailly and Juvé [5] and Billson et al. [14].

A key modification was made to the SNGR method that focused on convection of the synthetic turbulence. The implementation of SNGR by Bailly and Juvé [5] convects the stochastic fluctuations by the RANS velocities in the $tu_c$ term. However, in an attempt to account for sweeping, the instantaneous LES velocities (mean + fluctuations) are used to convect the stochastic field in the current formulation.

Additionally, it was observed (see following sub-sections) that the original time-convection term, $tu_c$, continually stretched and elongated the stochastically-generated eddies as they propagated downstream, leading to unphysical results. Therefore, the time term was modified to become a local convection time equal to the period associated with the corresponding mode in the stochastic model. In other words, a certain eddy is convected for a time length equal to the period, $T = 2\pi/(k \times u_c)$,
where $k$ is the wave number vector, and $\mathbf{u}_c$ is the convection velocity vector. These modifications, together with the SNGR method in Equation 4.31, give the SNGR method for the VPST method.

### 4.4 Results

This section presents results for the SNGR-based VPST method as applied to jet noise predictions. The VPST method was evaluated for two moderate-Reynolds number jets. This section first discusses modifications to the basic SNGR method, and then describes the simulation setups, results, and method shortcomings. Selected results from Blake et al. [16, 17] are presented for LES of a Mach 0.9 jet.

#### 4.4.1 Evaluating the SNGR Modifications

Several modifications were made to the SNGR method due to observed shortcomings in generating stochastic turbulence. One particular shortcoming of the VPST implementation is the convection provided by the current SNGR formulation, or lack thereof, for the imposed stochastic fluctuations. Ideally, imposed fluctuations would interact with and be convected by the mean flowfield and larger turbulent scales (known as sweeping). The initial implementation of SNGR in VPST does not properly account for convection or sweeping. What remains to be done is to implement a more appropriate convection method for SNGR that allows better control over the stochastic fluctuations and also accounts for sweeping. This section demonstrates the issues with convection in the stochastic generation method by investigating Mach 0.9 jet (Re = 100,000) simulations on coarsely-discretized grids. These simulation results are qualitative only and are not meant to be well-resolved.
The grid for this study, shown in Figure 4.1, was 32 blocks with 2.6 million grid points, with a spacing of 0.055Dj. The domain extended from 0Dj to 16Dj in x, and from -2.5Dj to 2.5Dj in y and z. The stochastic velocity fluctuations are added to a RANS mean flow, shown in Figure 4.2.

As previously discussed, the initial implementation by Bailly and Juvé [5] (see Equation 4.31) supposedly accounts for convection by the term x – tu_c. However, the actual effect of this term is to continually stretch the eddies as they propagate downstream in the jet. Figure 4.3 (a) demonstrates
this organized stretching, as “C” shapes are visible in the contours of velocity magnitude fluctuations from the SNGR method of Bailly and Juvé [5]. The SNGR method used 150 modes and a range of wavenumbers from $4/D_j$ to $20/D_j$ to produce these results. A more realistic imposed fluctuating field would likely show a less organized structure, and therefore, these results are undesirable for time-varying synthetic turbulence.

For comparison, results\(^1\) from the SNGR method of Lafitte et al. [83] are shown in Figure 4.3 (b). In this method, the imposed turbulent fluctuations are split into large and small scale structures. The large scale is modeled with the method of Bailly and Juvé [5], and the smaller scales are modeled by the method of Billson et al. [14], who account for convection explicitly with an additional equation. This explicit time convection models sweeping and ensures that the flowfield is properly decorrelated. Evidence of stretching and semi-organized turbulent streaks are visible in Figure 4.3,

\(^1\)Reprinted with permission from the authors.
and the presence of randomness and some larger structures in the flowfield break up some of the “C-shaped” patterns.

(a) SNGR Bailly and Juvé [5]

(b) Explicit Time Convection from Lafitte et al. [83]

(c) SNGR with Modifications

Figure 4.3

Contours of Velocity Magnitude for Synthetic Turbulent Fluctuations from Iterations of SNGR
In an attempt to fix this stretching from the basic SNGR method of Bally and Juvé but without adding an explicit time convection like Lafitte et al. [83], a characteristic periodic time re-setting was implemented (eddies are convected for a time length equal to the period, \( T = \frac{2\pi}{(k \times u_c)} \)). Figure 4.3 (c) shows velocity contours at the same time step but with using this periodic time-convection fix. The stretching is not observed in the development of the imposed fluctuations, and the imposed velocity field appears remarkably different. However, it can be noted that what actually occurs is a re-imposing of a given eddy at its initial position after its periodic time has passed. This leads to a build-up of velocity fluctuations in the LEE, which is not pictured due to the difficulties of obtaining an uncontaminated LEE solution on this limited-domain grid. This build-up or concentration of velocities is not desired because the fluctuations no longer become truly randomized.

Therefore, it is likely that convection and sweeping are not handled correctly by the modified SNGR method of Bailly and Juvé that is currently implemented in the VPST method. Since sweeping is an important decorrelation mechanism in jet flows, the lack of such a mechanism may lead to the fluctuations forming orderly “C-shaped” flow structures. These indicate that further work is needed on the convection of the stochastic fluctuations for VPST. However, the current SNGR method with modifications can still give an indication as to the effectiveness of VPST for modeling high-frequency jet noise, as the following sections demonstrate.

### 4.4.2 Jet at Mach 0.9 and a Reynolds Number of 36,000

The VPST method was applied to a Mach 0.9 jet at a Reynolds number (based on the jet diameter) of 36,000 as an initial test case. A multi-block structured grid was employed. The grid
resolution was increased in the area near the nozzle exit and in the shear layers, but was decreased (grid stretching) to the far-field spacing in order to damp the outgoing waves that would eventually interact with the far-field boundary and reflect back to the domain to contaminate the solution. The same grid was used for LES and LEE for simplicity.

In the following results, stochastic fluctuations are simply added to the LES fluctuations and propagated with LEE to see what effect the stochastic velocities might produce on the radiated noise spectra without increasing grid resolution. After a statistically-steady LES result was obtained, the LEE solver was used to solve for the propagation of acoustic waves to the far-field. In order to test the stochastic method, two LEE simulations were run. The two simulations, LES and LES + stochastic are compared in the following results.

For acoustic analysis, the fluctuating pressure ($p'$) from the LEE equations was investigated. For a qualitative comparison, contours were plotted to capture the fluctuating acoustic pressure (shown in Figure 4.4 in black and white, with levels an amplitude $\pm$ 460 Pa), and a vorticity iso-surface is included, colored by $u$ velocity to approximate the jet. The vorticity comes from LES velocities only. Figure 4.4 (a) shows only the LES fluctuations propagated by the LEE, with the image window extending from the nozzle to approximately 20$D_j$-25$D_j$ downstream. Acoustic waves are observed propagating both upstream and downstream, although the upstream propagating waves are not observed further downstream. In Figure 4.4 (b), the stochastic fluctuations are added alongside the LES fluctuations and propagated by the LEE. The result is a noticeable increase in waves radiating from the jet around 10 to 15$D_j$ downstream. Additional fluctuations are seen propagating further into the near-field compared to Figure 4.4 (a). As is expected, fluctuations that appear to have
larger wavelength (lower frequency) propagate mainly downstream, while the apparent smaller wavelength (higher frequency fluctuations) propagate in all directions.

For a simple comparison between the current VPST method and the original SNGR method, Figure 4.4 (c) shows the results of LEE propagating only the stochastic fluctuations, with a vorticity isosurface from RANS mean flow velocities to approximate the jet. For the RANS + stochastic case, a pressure contour level of ± 920 Pa is used. This implementation clearly lacks the resolution of the LES simulation. Additionally, the development of turbulence upstream of the jet is not observed.

For a quantitative analysis of the VPST method, SPL (sound pressure level) data are presented from 4 pressure probes at locations shown in Table 4.1. Figure 4.5 (a)-(d) shows the SPL spectra obtained from the pressure time-histories with the LES in red and the LES + stochastic in black. In all four cases, the stochastic method shows an increased SPL prediction for some frequencies in the range St = 1-3. In Figure 4.5 (a) and (b), the two methods show similar results, but the inclusion of the stochastic fluctuations leads to additional higher frequencies. However for Figure 4.5 (c) and (d), the LEE + LES fluctuations alone resolve higher frequencies over the LEE + LES + stochastic method. In Figure 4.5 (d), the stochastic method shows a near 10 db boost in the lower frequencies over the LES only method. The reasons for the differences observed between the four data points are unclear, but the VPST method was able to increase the spectral content for two of the four points.

For additional quantitative analysis, Figure 4.6 shows a comparison of the LES and LES + stochastic y-velocity components, v, plotted against y, taken at a YZ- plane across the jet at a location of 8Dj downstream. From Figure 4.6, it is clear that the LES + stochastic results show higher frequency fluctuations compared to the LES results.
The added fluctuations shows limited promise that the higher frequency content can be supplemented with the VPST method, possibly without further increasing the numerical grid resolution. The modeled SPL range is currently limited by the LEE grid, which was taken as the LES grid for simplicity. The question arises whether the LES for this case is well-enough resolved or whether adding fluctuations truly increased the higher-frequency fluctuations. It is likely that a finer-resolution LEE grid is needed to demonstrate whether adding fluctuations to LES would be
Table 4.1

Point Locations of Pressure Data Probes VPST Investigation

<table>
<thead>
<tr>
<th>Point #</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>x/D_j</td>
<td>1.0</td>
<td>3.0</td>
<td>6.0</td>
<td>10.0</td>
</tr>
<tr>
<td>y/D_j</td>
<td>2.9</td>
<td>3.2</td>
<td>3.5</td>
<td>3.9</td>
</tr>
<tr>
<td>z/D_j</td>
<td>-2.8</td>
<td>-3.1</td>
<td>-3.4</td>
<td>-3.8</td>
</tr>
</tbody>
</table>

Figure 4.5

SPL Spectrum at Four Observer Points (From Blake et al. [17])
truly effective. Ultimately, the exact effect of the added fluctuations is not clear with the current results because there was no attempt to evaluate the “missing” scales from the LES simulation.

4.4.3 Jet at Mach 0.9 and a Reynolds Number of 120,000

The previous results demonstrate that VPST has some limited potential for increasing the higher-frequency content but do not provide a quantitative comparison as to whether the added content produces the desired noise spectrum. Overall, the results were inconclusive because the effect of the missing scales was unknown or simply not measured. A more thorough investigation is needed.

Conceptually, adding smaller-scale fluctuations to well-resolved LES results should yield little effect due to the fact that the grids for both LES and LEE limit the resolved wavenumbers. If the same grids are used for LEE and LES, then it is expected that they both share the same high
wavenumber limit. Conversely, using a finer LEE grid than LES grid should allow for higher frequencies to be modeled by LEE if those fluctuations are added into the LES solution. However, using a finer LEE grid would make it difficult to determine the influence of any added fluctuations. To solve this issue, the current test case was designed to demonstrate, through the use of filtering, the exact effect on the noise spectra that is produced by adding stochastic fluctuations.

The current test case can be thought of as simulating an artificially coarser LES grid spacing through the use of low-pass filtering, which equates to using a finer grid for LEE but allows for an equal comparison between resolved and added acoustic fluctuations. In actuality, both LES and LEE grids are the same, which simplifies the problem by removing the need for interpolation from LES to LEE and ensures a better comparison between the acoustic fields.

In order to test the VPST method and the effectiveness of simple stochastic turbulence model at extending the higher frequency range of LES simulations, three LES simulations were performed: \textit{LES}, \textit{VLES (filtered)}, and \textit{VLES + stochastic}. The first simulation, \textit{LES}, was moderately-well resolved basic LES and after a statistically stationary LES flow field was obtained, the LEE solver was used to generate acoustic waves from the LES fluctuations only and propagate those waves. The second simulation, \textit{VLES (filtered)}, was obtained as the result of spatially filtering the \textit{LES} flowfield to simulate an under-resolved LES simulation. Then stochastic fluctuations were added to the \textit{VLES (filtered)} results to identify the degree to which the stochastic fluctuations influenced the radiated noise spectrum. The modified SNGR method was used to generate stochastic fluctuations, employing 150 Fourier modes across a non-dimensional wavenumber range of 6 to 30.

The three LES simulations were performed for a Mach 0.9 jet at Reynolds number (based on the jet diameter) of 120,000 because several prominent LES noise simulations use similar jet
conditions [22, 23, 25, 130]. For the selected conditions, the diameter of the jet was set to 0.004 m to ensure proper scaling. The multi-block structured mesh consisted of 6.8 million nodes in 182 blocks. Figure 4.7 shows an image of the mesh in the sectional plane $z = 0$. The grid resolution was increased in the area near the nozzle exit and in the shear layers, but was decreased (grid stretching) to the far-field spacing in order to damp the outgoing waves that would eventually interact with the far-field boundary and reflect back to the domain to contaminate the solution. For simplicity, the same grid was used for LES and LEE. Additionally, the grid extends backwards behind the nozzle “inlet” to prevent noise from being reflected near the nozzle or generated at the nozzle edges artificially.

![Figure 4.7](image)

**Figure 4.7**

Overall View of the Mesh at $z = 0$ (From Blake et al. [16])

Initialization of the LES flowfield was provided by a RANS simulation obtained with ANSYS Fluent (version 17.2) using a $k - \epsilon$ closure model. Figure 4.8 shows the mean velocity and the
turbulent kinetic energy along the centerline from the RANS solution (the results are scaled by the freestream velocity).

\begin{figure}[h]
\centering
\begin{subfigure}[b]{0.4\textwidth}
    \centering
    \includegraphics[width=\textwidth]{centerline_U.png}
    \caption{Centerline Velocity}
\end{subfigure}\hfill
\begin{subfigure}[b]{0.4\textwidth}
    \centering
    \includegraphics[width=\textwidth]{centerline_TKE.png}
    \caption{Centerline TKE}
\end{subfigure}
\caption{RANS Results Along the Jet Centerline}
\end{figure}

Results from Blake et al. [16] are presented in Figures 4.9 and 4.10. An LES solution was first obtained (Figure 4.9 (a)), then filtered with a 6th-order filter to obtain a representative VLES (Very Large Eddy Simulation) result by removing some of the resolved higher-frequency content (Figure 4.9 (b)). Stochastic fluctuations were then added to the VLES results (Figure 4.9 (c)). Acoustic pressures (greyscale contours) shown in Figure 4.9 are taken from the LEE solver, with isosurfaces of vorticity (colored by u-velocity) taken from the NS solver. Comparing the LES and the VLES + stochastic results in Figure 4.9 (a) and (c) shows that the stochastic method generates additional waves qualitatively similar to those observed in the LES results.
Acoustic Pressure Fluctuations (Grayscale) of a Mach 0.9 Jet with an LES Vorticity Isosurface Colored by $u$ Velocities (From Blake et al. [16])

For a more quantitative investigation of the additional higher-frequency content, pressures were recorded at four observation points listed in Table 4.1. These points have been non-dimensionalized by the jet diameter. The SPL spectra at each of these observation points were obtained and plotted vs. Strouhal number (shown in Figure 4.10). Figures 4.10 (a) and (b) (corresponding to observation points 1 and 2, respectively) both show a large decrease in higher frequencies in the VLES results as compared to the LES results, as is expected due to the imposed filtering. However, when the
stochastic turbulence is added to the VLES results, the SPL curves of the VLES + stochastic case include additional higher-frequency content similar to the pure LES results. Figures 4.10 (c) and (d) (corresponding to observation points 3 and 4, respectively) show a similar trend with the stochastic turbulence adding higher-frequency content, but the effect is reduced. It should be noted that observation points 3 and 4 are located further downstream and correspond to smaller angles with respect to the jet axis. Therefore, the reduced effect of adding higher-frequency content at observation points 3 and 4 is expected since the lower frequencies tend to dominate further downstream, while the higher-frequency waves are more influential at higher angles. It is unclear if the relatively straight nature of the SPL content in the range of a Strouhal number from 5 to 7 is actually resolved and modeled by the grid or if this is an artifact of the Fourier transform of the pressure and exists due to a lack of resolution. Overall, the spectra at all four points confirm that the stochastic turbulence modeling adds additional higher-frequency content.

The large amplitudes of the stochastic fluctuations observed in Figure 4.10 (c) might indicate the need to re-calibrate $\beta$ or implement a scaling parameter to adjust the level of stochastic fluctuations when they are combined with the LES fluctuations. Additionally, the range of $k_n$ needs to be investigated based on the turbulent kinetic energy spectrum. For the present simulations, subtracting the mean of the fluctuations from the LEE source terms has been ignored, as has selecting a non-zero $\omega_n$ in the stochastic terms. However, both of these have a non-trivial effect on the SNGR method and should be investigated further [5, 14, 30].

Figure 4.11 shows the pressure time-histories for the same four points. The VLES time-histories appear smoother, indicating a lack of higher frequency fluctuations. This further shows the effects of the filtering. Adding in stochastic fluctuations leads to a similar pressure history to the unfiltered
LES, further confirming the effectiveness of adding back in the stochastic fluctuations. These combined results show that the VPST method has promise and could be used to supplement under-resolved LES high-frequency content.

Bogey [25] performed LES on a jet at similar conditions (Mach 0.9, Re 100,000), studying the effect of the boundary layer on the jet flow. A qualitative comparison was made for the present LES simulations. Figure 4.12 shows results from the present simulation (both LES and VLES + Stochastic) in a style similar to results published by Bogey. The contours displayed in Figure 4.12
Figure 4.11
Pressure Histories at Four Observer Points

show vorticity in the jet and pressure fluctuations outside the shear layer. The acoustic pressure field is obtained from the LEE solver.

The developing turbulent shear layer of the LES results, as shown by the vorticity contours in Figure 4.12 (a), appears more similar to Bogey’s case with an initially laminar boundary layer (Figure 21 (a) in Bogey’s paper), which might indicate that the turbulent forcing at the inlet may be too weak and needs to be investigated further. Another possibility is that the grid is not fine enough in the shear layer to resolve fluctuations, so grid resolution should also be investigated further. At
the edges of the domain, dissipation is observed in the pressure field for both Figure 4.12 (a) and (b), indicating a lack of grid resolution for the acoustic waves at those locations.

The larger scale fluctuations (likely lower-frequency waves) seen radiating outward from the jet indicates that the VPST method models the lower-frequency behavior correctly. Another promising result is that the pressure field of the VLES + Stochastic case (Figure 4.12 (b)), which includes the influence of added stochastic fluctuations, contains both large and small scale structures. A lack of symmetry is observed in the pressure field in Figure 4.12 (b), which possibly is a further indication that convection of the stochastic velocities needs to be reconsidered. Despite some significant differences likely due to grid resolution, the results of the present simulation are acceptable for an initial investigation.

In summary, a Mach 0.9 jet at a Reynolds number of 120,000 was simulated with the VPST method. The use of filtering helped demonstrate the effectiveness of adding stochastic fluctuations and resolving finer-scale fluctuations with a finer LEE grid. These results also showed promise that
the VPST method could model the effects of missing high-frequency content due to under-resolved LES simulations. More work is needed in the areas of grid resolution, inflow forcing, imposed fluctuations, and the radiated noise field. A shortcoming of this approach was that it gave no indication of whether the VPST method actually saves computational resources over traditional LES methods.

Overall, the VPST method requires further development and testing.

4.5 Shortcomings and Issues

The results reveal several shortcomings with the current implementation of the VPST method that should be addressed in future development. The issues that remain to be addressed fall into two general categories: SNGR-related concerns and simulating the correct flow and noise physics.

- Convection and Sweeping: Although successful for initial results, the basic SNGR method of Bailly and Juvé [5] is not properly suited to time-accurate simulations due to the convection term leading to a distorted, non-physical flowfield over long periods of time. A modification was proposed to better account for convection since initial implementations of SNGR with a time-accurate method led to significant distortion and stretching of the convected eddies. However, this modification simply resets the eddies after they have been convected for a time length equal to their period and is a temporary fix to the issue. Given preliminary results, it seems that SNGR does not properly account for convection or sweeping of the smaller synthetic turbulent eddies even with several modifications.

- SNGR Wavenumber Range: Another complication of SNGR is deciding on the wavenumber range of stochastic fluctuations to impose. With SNGR, it is easy to select a new limit for
given conditions, but developing a universal method for this could be complex. Currently, the wavenumber range is selected manually. This selection should be automated for future flexibility, perhaps based on grid spacing or a desired frequency content range. The wavenumber range issue also plays into convection for sweeping and deciding where the large/small scale transition occurs. This, in turn, will influence the choice of grid resolution needed for the LES simulation.

- SNGR Isotropic Turbulence Assumption: The current formulation of SNGR models isotropic turbulence. It is recognized that the assumption of isotropy may only be valid in the smallest turbulent scales and may not be appropriate as SNGR models larger turbulent scales in jet flows. An anisotropic formulation of SNGR [15] may be investigated.

- SNGR Truncation of Fourier Modes: SNGR uses a summation of global Fourier modes that are assumed to run through the entire domain. Applying the method in a localized sense results in truncating Fourier modes at the edge of the source region, which may cause unwanted spurious fluctuations. With the current formulation, the only parameter that accounts for the local strength of the fluctuations generated by the Fourier modes is the turbulent kinetic energy from the RANS simulation. As the TKE dissipates away from the jet, so do the imposed fluctuations. It is unclear therefore whether any spurious fluctuations generated by the truncation of Fourier modes is a significant issue. Truncation destroys the divergence-free condition can could lead to the generation of extra noise. Even so, the SNGR method is currently applied throughout the whole computational domain to prevent truncation. Additionally, the global nature of the Fourier mode method does not allow for local control of turbulence, which could be an issue if anisotropic turbulence is considered.
• Mean Flowfield: Additionally, the SNGR method relies on a steady-state RANS flowfield obtained prior to the LES simulation. As the LES flowfield becomes fully developed, this solution will likely diverge somewhat from the initial RANS flowfield. It may be more advantageous to base the generation of stochastic fluctuations off of the mean LES field rather than the RANS field if possible, although issues with dissipation of turbulent quantities may be encountered due to sub-grid scale modeling.

• LES Grid Resolution: The grid resolution needs to be increased, especially in the shear layer and far-field, to better predict the LES flow physics and the noise physics predicted by LEE. The exact increase in resolution will require further investigation, but comparisons to flowfield and far-field noise results will help in answering this question. Even though the point of the VPST method is to limit the required grid resolution for LES simulations, a certain level of accuracy is required. On this point, no investigation has been made into whether the VPST method saves computational costs.

• LEE Grid Resolution: If the VPST method is tested again with the filtering approach, better control over the filtered wave numbers or frequencies is required to better adjust the imposed stochastic fluctuations. Alternatively, the use of a finer LEE grid when compared to the LES grid will likely aid in modeling additional higher-frequency content from the stochastic fluctuations. This will require interpolation between the LES and LEE solutions.

• Turbulent Inflow Forcing: For accurate flow physics, the turbulent inflow forcing for the jet shear layer requires further investigation due to the nature of the jet development as observed in the preliminary jet results. The current forcing may be too weak or the method may be ineffective. Stronger inflow turbulence will likely lead to a more-developed turbulent jet.
• Far-field Propagation: Another shortcoming of the preliminary results is that the sampled data points are in the near-field of the jet but outside the turbulent shear layer. For a true comparison to simulation or experimental data, the grid should be extended to the far-field and data must be sampled at a distance of 45 to 75 $D_j$ and a variety of angles to the downstream jet. It may also be possible to use LEE to propagate acoustics to some near-field distance at which point a Kirchhoff surface may be used to project the pressure fluctuations to the desired far-field locations.

Additional comparisons and investigations are needed to investigate both the LES turbulent flow field and the far-field noise predicted by the VPST method. Demonstrating effective modeling of higher-frequency content for a variety of jet conditions will require significant development and testing.

4.6 Summary

In summary, an initial version of the CLST method (VPST) was implemented for two Mach 0.9 jets at moderate-to-high Reynolds numbers. Investigations into the SNGR method revealed the need for a different way to account for convection of the imposed stochastic fluctuations. The VPST method used to simulate a jet at a Reynolds number of 36,000 showed slight promise for modeling additional higher-frequency content, but the results were inconclusive overall because the effect of the missing scales was unknown. When applied to a jet at a Reynolds number of 120,000 with the intent of evaluating the added fluctuations, the VPST method displayed much higher potential in modeling unresolved high-frequency content.
Initial comparisons to similar simulations from Bogey [25] revealed similar flowfield structures and pressure signatures, but the difference in grid resolution limits the comparison. Increasing grid resolution will likely increase the similarity between the simulations. The comparison also revealed that more work is needed to correctly simulate the developing turbulent jet and the radiating noise field with the CLST method.
CHAPTER 5
SEM-BASED CLST METHOD

The SEM-based CLST method is presented in the following section. The governing equations and numerical discretization methods are discussed, as well as specifics of the method for generating synthetic eddies.

5.1 Governing Equations

The Navier-Stokes (NS) equations are used for LES and the Linearized Euler equations (LEE) are used for acoustic propagation. The equations for the Navier-Stokes equations are the same as those in Section 4.1.1.

5.1.1 Linearized Euler Equations

An alternative, non-conservative form of the LEE are used for the CLST method cast in terms of the primitive variables. The same source terms from Bogey et al. [30] are used. Similar to the previous LEE formulation, the mean-flow derivatives are neglected. The governing Linearized Euler equations (from Fukushima et al. and Hirai et al. [60, 67]) are written in vector form as:

\[
\frac{\partial Q'}{\partial t} + \bar{A}_j \frac{\partial Q'}{\partial x_j} + A'_j \frac{\partial \bar{Q}}{\partial x_j} = S, \text{ for } j = 1 \ldots 3 
\]  

(5.1)

where \( \bar{Q} \) is the mean flow and \( Q' \) is the fluctuations:
\[ \mathbf{Q} = \begin{bmatrix} \bar{\rho} \\ \bar{u}_1 \\ \bar{u}_2 \\ \bar{u}_3 \\ \bar{p} \end{bmatrix}, \quad \mathbf{Q}' = \begin{bmatrix} \rho' \\ u'_1 \\ u'_2 \\ u'_3 \\ p' \end{bmatrix} \]  

(5.2)

\[ \bar{A}_j = \begin{bmatrix} \bar{u}_j & \delta_{1j} \bar{\rho} & \delta_{2j} \bar{\rho} & \delta_{3j} \bar{\rho} & 0 \\ 0 & \bar{u}_j & 0 & 0 & \frac{\delta_{1j}}{\bar{\rho}} \\ 0 & 0 & \bar{u}_j & 0 & \frac{\delta_{2j}}{\bar{\rho}} \\ 0 & 0 & 0 & \bar{u}_j & \frac{\delta_{3j}}{\bar{\rho}} \\ 0 & \gamma \delta_{1j} \bar{\rho} & \gamma \delta_{2j} \bar{\rho} & \gamma \delta_{3j} \bar{\rho} & \bar{u}_j \end{bmatrix} \]  

(5.3)

\[ \mathbf{A}'_j = \begin{bmatrix} u'_j & \delta_{1j} \rho' & \delta_{2j} \rho' & \delta_{3j} \rho' & 0 \\ 0 & u'_j & 0 & 0 & -\frac{\delta_{1j} \rho'}{\bar{\rho}^2} \\ 0 & 0 & u'_j & 0 & -\frac{\delta_{2j} \rho'}{\bar{\rho}^2} \\ 0 & 0 & 0 & u'_j & -\frac{\delta_{3j} \rho'}{\bar{\rho}^2} \\ 0 & \gamma \delta_{1j} \rho' & \gamma \delta_{2j} \rho' & \gamma \delta_{3j} \rho' & u'_j \end{bmatrix} \]  

(5.4)

This formulation is simpler to implement since there is no need to convert between mean and fluctuating quantities. The terms \( \frac{\partial \mathbf{Q}}{\partial x_j} \) represent the mean shear terms and mean flow gradients, which are set to zero to damp instabilities resulting from unbounded homogeneous solutions to the original LEE [30, 46]. Bogey et al. [30] refer to these equations as the Inhomogeneous LEE (ILEE).
The source terms, described below, reduce to the sound sources from Goldstein’s acoustic analogy for the third-order wave equation and are at least appropriate for simple jet flows [46].

The source terms are the same as Bogey et al. [30]:

$$
S = \beta \begin{bmatrix}
0 \\
S_1 - \overline{S}_1 \\
S_2 - \overline{S}_2 \\
S_3 - \overline{S}_3 \\
0
\end{bmatrix} \tag{5.5}
$$

where

$$
S_i = -\frac{\partial \bar{\rho} u_i u'_j}{\partial x_j}, \quad \overline{S}_i = -\frac{\partial \bar{\rho} u_i u'_j}{\partial x_j} \tag{5.6}
$$

The source term $S_1$ is expanded below as an example:

$$
S_1 = -\frac{\partial \bar{\rho} u'_i}{\partial x} + -\frac{\partial \bar{\rho} u'_j}{\partial y} + -\frac{\partial \bar{\rho} u'_k}{\partial z} \tag{5.7}
$$

where the fluctuating variables come from LES or synthetic turbulence and $\beta$ is a parameter to adjust the intensity of the sources [85]. The mean density ($\bar{\rho}$) is used rather than the instantaneous density ($\rho = \bar{\rho} + \rho'$) because the multiplication of the three fluctuating quantities $\rho' u' v'$ is negligible. The bar signifies a time-averaged quantity. The time-average of the source terms is not zero, necessitating the subtraction of the mean of the source term [30]. The governing equations are non-dimensionalized by the mean flow density $\bar{\rho}$, speed of sound $a_{\infty}$, and reference length $D$ [60, 67].
Re-arranging the LEE equations \((n = 1 \ldots 5)\) so that the time-derivative is on the LHS, with \((x_j\) for \(j = 1 \ldots 3\)) with \((m = 1 \ldots 5)\), gives

\[
\frac{\partial Q_n'}{\partial t} = -(\hat{A}_j)_{nm} \frac{\partial Q_m'}{\partial x_j} + \beta S_n \tag{5.8}
\]

With the curvilinear transformations (see Equation 4.1), the LEE equations \((n = 1 \ldots 5)\) with the Cartesian \(x, y, z\) coordinates \((x_j\) for \(j = 1 \ldots 3\)), \((m = 1 \ldots 5)\), and curvilinear \(\xi, \eta,\) and \(\zeta\) coordinates \((c_k\) for \(k = 1 \ldots 3)\) become

\[
\frac{\partial Q_n'}{\partial t} = -(\hat{A}_j)_{nm} \left[ \frac{\partial c_k}{\partial x_j} \frac{\partial Q_m'}{\partial c_k} \right] + \beta S_n \tag{5.9}
\]

with the source term given by

\[
S_n = -\frac{\partial c_k}{\partial x_j} \frac{\partial \tilde{p}u_n'u_j'}{\partial c_k} + \frac{\partial c_k}{\partial x_j} \frac{\partial \tilde{p}u_n'u_j'}{\partial c_k} \tag{5.10}
\]

A validation of this LEE formulation is presented in Appendix A for two test cases in two-dimensions.

5.2 Numerical Methods

This section discusses the numerical algorithms of the present CFD solver, including spatial and temporal discretization schemes, filtering for Implicit LES (ILES) turbulence model, boundary conditions, and inlet forcing. The spatial equations are the same as those in Section 4.2.1 and the equations for spatial filtering are the same as those in Section 4.2.3.

5.2.1 Temporal Discretization

The time integration is performed using a second order Adams-Bashforth method (Butcher [43]) written in the form
\[ Q^{n+1} = Q^n + k \left[ \sum_{\nu=0}^{K} \beta_{\nu} L(Q^{n-\nu}) \right] \]  

(5.11)

where the constants \( \beta_{\nu} \) are chosen to give either the maximum order of accuracy (Butcher [43]) or the lowest dispersion and dissipation, and \( L(Q) \) is the residual. This temporal discretization method is faster than the previous RK-TVD method.

5.2.2 Boundary Conditions and Inlet Forcing

The disturbances for the higher Reynolds number cases are found in Section 4.2.4. Additionally, for lower Reynolds number cases investigated with CLST, the disturbances of Freund [58] were used. The disturbances are approximately solenoidal, and are specified by randomly modulating a thickness parameter, \( b(\theta, t) \), and prevent auto-excitation of the jet via spurious numerical modes [58]. The parameter is given as

\[ b(\theta, t) = 12.5 + 2 \sum_{m=0}^{2} \sum_{n=0}^{1} A_{mn} \cos \left( \frac{St_{mn}U_j t + \phi_{mn}}{D_j} \right) \cos(m\theta + \psi_{mn}) \]  

(5.12)

where \( mn \) are subscripts that are increased or decreased by a 1-in 20 chance in a random-walk fashion at each timestep. Table 5.1 shows the \( \Delta \) per timestep and the minimum and maximum of each parameter. The nozzle inlet velocities are specified by

\[ u = \frac{U_j}{2} \left[ 1 - \tanh \left[ b(\theta, t) \left( \frac{r}{r_j} - \frac{r_j}{r} \right) \right] \right] \]  

(5.13)

5.3 The Coupled LES-Synthetic Turbulence Method

This section outlines the novel CLST method for predicting jet noise, including the description of a synthetic eddy method that incorporates sweeping. The CLST method combines Very Large
Table 5.1

Parameters for Inflow Randomization

<table>
<thead>
<tr>
<th></th>
<th>min</th>
<th>max</th>
<th>Δ per timestep</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.01</td>
<td>0.14</td>
<td>0.0001</td>
</tr>
<tr>
<td>St</td>
<td>0.1</td>
<td>0.7</td>
<td>0.00085</td>
</tr>
<tr>
<td>φ</td>
<td>0.0</td>
<td>2π</td>
<td>0.00085</td>
</tr>
<tr>
<td>ψ</td>
<td>0.0</td>
<td>2π</td>
<td>0.00085</td>
</tr>
</tbody>
</table>

Eddy Simulations (VLES) and a modified Synthetic Eddy Method (SEM). Using a hybrid CAA method, the jet flow is simulated with the Navier-Stokes (NS) equations in a finite difference multi-block, structured LES solver. Velocity fluctuations produced by the NS LES solver are passed to the Linearized Euler equations (LEE) to generate acoustic sources. The LEE solver then simulates the resulting noise field. Velocity components generated by the synthetic eddy method (SEM) are added to the source terms in the LEE to generate the missing high frequency noise. To eliminate potential errors or increased cost due to interpolation between the solutions, the LES, VLES (Very Large Eddy Simulations), SEM, and LEE flowfields are calculated on the same computational grid.

5.3.1 Filtering

In order to evaluate the effectiveness of synthetic velocities supplementing the jet noise spectra, the “missing” LES frequency content must first be measured or estimated. The simplest and most efficient approach is to artificially generate an under-resolved VLES flowfield by removing resolved fluctuations from a LES simulation in a controlled and quantifiable manner. The effects of added synthetic velocities can then be directly investigated.
Applying a low-pass filter to the NS velocities produces an approximate VLES flowfield by damping the turbulent structures smaller than a cut-off wavelength specified by the filter. Since the effects of the filter are known, the removed fluctuations are quantifiable and can be considered the “missing” noise produced by VLES on an effectively “coarse” grid resolution. A comparison of the acoustic spectra produced by the LES (unfiltered) and VLES (filtered) results reveals the missing noise. Filtering thus provides a method to measure the “unresolved” noise.

In the CLST method, the fourth-order filter from Kennedy and Carpenter [77] is applied to remove the small scales from the flow. The stencil is widened by a point in each direction to increase the effectiveness of the filter. The filter cut-off frequency is estimated using $\xi_c = k \cdot \Delta x_{min}$, where $\xi_c$ is the cut-off wavelength assuming the cut-off occurs at 80% of the maximum wave amplitude ($\xi_c \approx 1.47$ from Figure 6 in Kennedy and Carpenter [77] is used), $k$ is the maximum resolvable wavenumber of $2\pi$, and $\Delta x$ is the minimum global grid spacing.

Additionally, filtering enables the use of the same computational grid for LES, VLES, SEM, and the acoustic flowfield without the need for costly, high-quality interpolation between the solutions. Thus a comparison can be made between coarsely- and finely-resolved flowfields.

### 5.3.2 Synthetic Eddy Generation

In CLST, small-scale turbulent velocities are synthesized by a superposition of randomized Gaussian eddies generated by a Synthetic Eddy Method (SEM). An initial RANS simulation with the same flow field parameters as the LES simulation provides turbulent quantities to calculate the eddy amplitude, while the eddy size, orientation, initial position, and “lifetime” are calculated from
random values. These randomized values are set on eddy initialization and do not change until an eddy is re-initialized after getting recycled.

For eddy placement, a source region (see the solid lines in Figure 5.1) is formed around the jet shear layer, centered on $D_j/2$, where $D_j$ is the jet diameter. The source region is comprised of a cone, with an outer angle, $\alpha_s$ and an inner angle, $\beta_s$. The cone is shaped to mimic the initial shear layer and TKE contours from the LES-only flow field. For this work, $\alpha_s = 10.0^\circ$ and $\beta_s = 6.3^\circ$, $x_{\min} = 1.3$, and $x_{\max} = 11.5$, all scaled by $D_j$.

Eddies are randomly placed in the three-dimensional source region by first choosing a random $x_0$ value from $x_{\min}$ to $x_{\max}$. Given an $x$, the projection of the source region on a y-z plane is known and $y_0$ and $z_0$ are chosen from inside the source region. A future improvement would be to automatically size the source region based on underlying TKE values.

Synthetic eddies are generated using the following Gaussian shape function:

$$f(x, y, z, t) = \exp \left[ -\frac{x_p + y_p + z_p}{\sigma^2} - \frac{(t - t_0 - 0.5t_L)^2}{\sigma_t^2} \right]$$

(5.14)

where

$$x_p = (x - x_0 - u_c t)^2$$

(5.15)

$$y_p = (y - y_0 - v_c t)^2$$

(5.16)

$$z_p = (z - z_0 - w_c t)^2$$

(5.17)

and where the initial eddy center is located at $x_0 = (x_0, y_0, z_0)$ at the time $t_0$, the eddy size is given by $\sigma$, the eddy life time is given by $t_L$, the time ramp width is given by $\sigma_t$, the eddy convection velocity is given by $u_c = (u_c, v_c, w_c)$, and the current eddy position and time are given by $x$ and
For $N$ eddies, the following equations randomize the $n^{th}$ eddy’s size and orientation for the synthetic velocity fluctuations $\mathbf{u}^n = (u^n, v^n, w^n)$:

\[
\begin{align*}
    u^n(x, t) &= A (cz_p - by_p) \cdot f(x, t) \\
    v^n(x, t) &= A (bx_p - az_p) \cdot f(x, t) \\
    w^n(x, t) &= A (ay_p - cx_p) \cdot f(x, t)
\end{align*}
\]  

(5.18) \hspace{1cm} (5.19) \hspace{1cm} (5.20)

where, $A$ is the eddy amplitude and $a$, $b$, and $c$ are randomized values between -2 and 2. The orientation randomization of the velocity components in Equations 5.18-5.20 ensures the divergence free condition for the velocity field.
The fluctuations generated by all eddies are summed in a point-wise manner to form the synthetic velocity field. At a point \( x = (x, y, z) \) and time \( t \), the synthetic velocity fluctuations, \( u_s \), generated by all \( N \) eddies is given by:

\[
    u_s(x, t) = \sum_{n=1}^{N} g^n(x, t)
\]

where

\[
    g^n(x, t) = \begin{cases} 
        u^n(x, t) & \text{if } (x^n - x^n_0 - u^n t) \leq \beta_c \sigma^n. \\
        0 & \text{otherwise.}
    \end{cases}
\]

Even though the eddy function extends to infinity, the value approaches zero within several eddy diameters. To minimize the computational cost incurred by performing calculating for thousands of eddies across the entire computational domain, the calculation is omitted for any point further than a given distance from the eddy, forming a calculation “window.” The convection velocity is included in the distance calculation so that the calculation “window” follows the eddy downstream. Jarrin et. al [70] employ a similar cut-off in their SEM formulation. In this work, \( \beta_c = 2.0 \) was observed to give good performance while not cutting off eddies stretched by sweeping. Figure 5.2 shows that for a one-dimensional eddy \( exp\left(\frac{(x-x_0)^2}{\sigma^2}\right) \) centered at \( x = 0 \), this cut-off distance is reasonable.

### 5.3.3 Eddy Size Distribution

In realistic turbulent jet flows, smaller eddies are more numerous than larger eddies. To mimic this behavior in the synthetic velocity field, the eddies are assigned to a “generation” or bin based on size. Three bins are chosen and distributed in the percentages shown in Table 5.2. From Figure 5.2,
it should be noted that eddy size ($\sigma$) and the actual eddy diameter are related by diameter $\approx 2\sigma$. For the rest of this discussion, eddy “size” will refer to $\sigma$, not the actual eddy diameter.

Given that the grid spacing is small near the jet nozzle and larger downstream, an average minimum grid spacing, $\Delta x_{avg}$, is calculated across the grid blocks in the eddy source region. The smallest eddy size is taken from this minimum average global grid spacing, $\sigma_{min} = 2.0\Delta x_{avg}$, which is assumed to be the minimum eddy size that the grid can support. The largest eddy size, $\sigma_{max}$, is taken from the wavelength corresponding to the filter cut-off, $\lambda_c = \frac{2\pi\Delta x_{avg}PPW}{\xi_c}$. The factor $PPW = \frac{5}{2} = 2.5$ is a ratio of the minimum number of points required to resolve the smallest wavelength by the currently-employed DRP scheme (5) to the points per wavelength required in
Kennedy and Carpenter (2) [77]. From Kennedy and Carpenter [77], a value of $\xi_c \approx 1.47$ is used.

To further control the eddy sizes, the maximum eddy size was adjusted such that $\sigma_{max} = 0.2\lambda_c$. With these parameters, the smallest eddies are discretized by approximately four grid points on average.

The eddy bins are divided into equal ratios for each bin, with $\delta_\sigma = (\sigma_{max}/\sigma_{min})^{1/3}$. Table 5.2 lists the limits for each bin. Figure 5.3 shows the bins, along with with $\Delta_{x_{avg}}$ and $\lambda_c$. It should be noted that the eddy size related to the fourth-order filter cut-off corresponds to the most energetic large eddies, which is ideal for approximating VLES in the CLST framework. The eddy size adjustments described in the previous paragraph results in a rather narrow range for the synthetic eddy diameters on the current computational grid. However, the distribution of eddies ensures that they reside within the range for isentropic turbulence and do not model the most energetic large eddies. The size of the synthetic eddies do not approach the dissipation range, but this could be amended by using a smaller-spaced computational grid.

The eddy size is randomly chosen uniformly over the limits of the bin. Upon re-initialization, the eddies stay in their assigned bin, but the size can change within the limits of the bin. This ensures that the desired size distribution is maintained but allows for randomness for the eddy sizes.

### Table 5.2

<table>
<thead>
<tr>
<th>Eddy Size ($\sigma^n$)</th>
<th>% of $N$ eddies</th>
<th>Minimum Size</th>
<th>Maximum Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>90</td>
<td>$\sigma_{min}$</td>
<td>$\sigma_{min}(\delta_\sigma)$</td>
</tr>
<tr>
<td>Medium</td>
<td>9</td>
<td>$\sigma_{min}(\delta_\sigma)$</td>
<td>$\sigma_{min}(\delta_\sigma)^2$</td>
</tr>
<tr>
<td>Large</td>
<td>1</td>
<td>$\sigma_{min}(\delta_\sigma)^2$</td>
<td>$\sigma_{max}$</td>
</tr>
</tbody>
</table>
5.3.4 Amplitude Calculation from RANS

The amplitude for the \( n^{th} \) eddy is calculated from the von Kármán-Pao energy spectrum for isotropic turbulence following the SNGR method of Bailly and Juvé [5]. For a given eddy size, \( \sigma^n \), the wavenumber associated with the \( n^{th} \) eddy is from \( k^n = \frac{2\pi}{2\sigma^n} \). The energy associated with that wavenumber is given by the von Kármán-Pao energy spectrum as:

\[
E(k^n) = \alpha \frac{u'^2}{k_e} \frac{(k^n/k_e)^4}{1 + (k^n/k_e)^2}^{(17/6)} \exp \left[ -2 \left( \frac{k^n}{k_\eta} \right)^2 \right] \tag{5.23}
\]

Figure 5.3

Eddy Size Distribution Shown For A Representative Turbulent Spectrum
where

\[ \alpha \simeq 1.453, \quad u' = \sqrt{\frac{2}{3} k_{\text{RANS}}}, \quad k_e = 0.747/L_e, \quad \epsilon = \frac{u'^3}{L_e}, \quad k_\eta = \frac{\epsilon^{1/4}}{\nu^{3/4}}. \] (5.24)

In these equations, \( \nu = 1.4146 \times 10^5 \frac{m^2}{s} \) was taken as a constant and \( \epsilon \) was essentially calculated from TKE from RANS (\( k_{\text{RANS}} \)) so that the amplitude of the synthetic eddies is only dependent upon the TKE. The wavelength of maximum energy, \( L_e \approx 0.3 \ast D_j \), is estimated from turbulent jet data in Pokora and McGuirk [105].

Finally, for the \( n^{th} \) eddy,

\[ A^n = \beta_{\text{amp}} \sqrt{E(k^n)}/N \] (5.25)

where \( \beta_{\text{amp}} \) is an amplification factor adjusted to produce fluctuations at the desired magnitude. The factor of \( 1/\sqrt{N} \) is taken from Jarrin et al. [69] and adjusts the amplitude based on the number of eddies.

For an example, at the point \((x,y,z) = 4, 0.6, 0\) in the jet shear layer, the von Kármán-Pao energy spectrum is plotted, along with the grid cut-off and the filter cut-off, in Figure 5.3.

### 5.3.5 Convection and Sweeping

In the simplest SEM approach, eddy properties such as amplitude and size are fixed at initialization of the eddy, and each eddy is convected with a constant velocity or with the mean RANS flow velocity at the eddy center. These methods of convection do not allow for any distortion to the eddy shape that would occur via sweeping.

Explicitly tracking and convecting an eddy (which involves interpolating to find the location and velocity of an eddy’s center) are not straight-forward tasks in a parallelized, multi-block grid approach if the convection velocity is non-constant and taken from the mean flow. Furthermore,
accounting for the distortion of an eddy shape from sweeping by large-scale velocity fluctuations would further complicate these tasks as the eddy center would be difficult to identify and would likely increase the computational time.

To solve these difficulties, the eddy convection velocities in CLST are taken from each discrete point across the eddy. If the background flow is uniform, the eddy convects undistorted. If there is a shear flow, the eddy is sheared. In the presence of additional large-scale fluctuations, the eddy is further distorted to account for sweeping and straining. Figure 5.4 shows the pointwise velocity distribution for an eddy distorted by the presence of a non-uniform background flow. Using the velocity in this pointwise manner saves complexity and computational time by avoiding explicit eddy tracking (and point-searching and interpolation) that would be required to update the velocity at the center of a given eddy as it convects downstream. Furthermore, the most important reason for using pointwise convection is to easily account for the influence of sweeping and straining on the synthetic eddies.

To this point, another important feature of the CLST method is that eddies convected via Eq. 5.14 use the instantaneous filtered VLES velocities rather than the mean flow velocities. The primary reason is to capture the effects of sweeping and straining on the synthetic eddies. In other words, the synthetic eddies are convected by the mean jet flow plus any fluctuations resulting from large-scale eddies. Using the instantaneous velocities to convect the eddies produces sweeping, where the small-scale synthetic eddies are influenced by the movement and rotation of larger-scale eddies. Straining is produced by non-uniformities in the local flow field due to larger eddies. To restate, the filtered VLES velocity components are used in Eq. 5.14 to convect and distort the synthetic eddies. Representative diagrams of pointwise convection and sweeping are given in
Figures 5.5 and 5.6. Figure 5.5 illustrates point-wise convection for a small eddy in the presence of a uniform mean flow. The influence of a larger eddy is added to the uniform flow in Figure 5.6 and leads to a distortion of the eddy shape.

Pointwise convection leads to changes in certain eddy properties as the eddy is convected. As a given eddy is convected downstream, the eddy size remains constant, but the eddy amplitude and convection velocity can vary with spatial position. The initial Gaussian eddy shape can also change due to shear, sweeping, or straining effects.

Each synthetic eddy is small in size compared to spatial changes in the mean flow field. The lifetime of each eddy is also small compared to the overall temporal fluctuations of the jet. Therefore, it is assumed that changes in the eddy properties over the eddy shape are small, as well as changes
in eddy properties over the eddy lifetime since an eddy does not convect very far downstream. Any errors due to violating the divergent-free condition are expected to be minimal to the overall noise produced by the jet.

118
5.3.6 Recycling of Eddies

At the beginning of the simulation, \( N \) eddies are created and inserted into the flow. They are initialized with an eddy lifetime based on the eddy size, such that smaller eddies exist for a shorter time as is expected. As the simulation progresses, the amplitude of each eddy is modulated in time with a Gaussian function (see Equation 5.14) so that the eddy fades in on initialization and fades out as it reaches the end of its lifetime. At the end of its life, the eddy is recycled, meaning that the eddy is re-initialized with a new random size, orientation, lifetime, and position in the jet shear layer. The equations for eddy lifetime, \( t^n_L = 10\sigma^n \), and the time ramp width, \( \sigma_t = t^n_L/4 \), were chosen to match observed time-development behavior of eddy structures in LES. Initially, eddies are inserted over a time range of ten times the lifetime of the maximum eddy size. Staggering the eddy insertion in this manner prevents similarly-sized eddies from syncing in time, which eliminates global pulsing behavior that can occur if thousands of eddies are recycled simultaneously.

5.3.7 Inclusion of Eddies in LEE Source Terms

Once the total contribution of each synthetic eddy to each point in the jet domain is computed, the synthetic velocity fluctuations are added to resolved LES fluctuations and passed to the LEE source terms. The source terms of Bogey et al. [30] are used (the same as Equation 5.5).

However, the fluctuating velocities \( u'_i \) and \( u'_j \) come from the combined LES and synthetic turbulent fluctuations, \( u' = u'_{\text{LES}} + u_s \). The parameter \( \beta \) adjusts the amplitude of the sources, which was added in the manner of Lafitte et al. [85]. In the present simulations, \( \beta \) was chosen as 1.3125 so that the the amplitude of the LEE fluctuations would match the amplitude of the LES fluctuations.
5.4 Results

This section presents results for the SEM-based CLST method as applied to jet noise predictions for two \( M = 0.9 \) jets. First, a moderately-high Reynolds number (\( Re = 100,000 \)) is simulated. This case is used to evaluate both the LES and LEE solvers. Then the CLST method is applied to the same jet case. A second jet case at a lower Reynolds number (\( Re = 3,600 \)) is simulated and compared to DNS data to better evaluate the high frequency resolution of the CLST method.

5.4.1 A Comparison of SNGR and SEM

This section presents the advantages of the SEM-based CLST method over the SNGR-based VPST method. As previously discussed in Section 4.4.1, one particular shortcoming of the SNGR-based method (referred to in preceding sections as VPST) was related to convection, or lack thereof, of the stochastic fluctuations. The main issue is that SNGR employs a superposition of Fourier modes to generate random turbulence. These modes, which conceptually are sine waves, exist globally in the simulation domain without a start or end point. While the superposition of many randomly oriented modes might generate appropriate fluctuations for homogeneous isotropic turbulence at one given instance in time, accounting for the changing flowfield over time is difficult. In fact, the appropriate method to convect the stochastic field generated by SNGR has been a subject of debate and several new SNGR methods [83].

Returning to the analogy of Fourier modes as sine waves, if the mean flow velocity is used to convect each mode in time, the sine waves get stretched, elongated, and distorted along the length of the wave. Given that the wavenumber is specified as constant along the Fourier mode in SNGR, stretching and elongating this mode presents a significant problem: the wavenumber of the mode is
no longer constant along the length. The more the mode is stretched, the more variation will occur over the length of the mode. This results in a local disturbance being applied to a global mode, which is not conceptually appropriate and which likely invalidates the divergence-free condition on which the SNGR method is based. Additionally, the summation of Fourier modes, which was designed to follow the von-Karman spectra, will likely no longer follow the intended distribution since distorting the Fourier modes would make it difficult to ensure that the total TKE generated by the summation of all modes at a given point in the domain holds to the prescribed value.

Most importantly, convecting Fourier modes and the resulting stochastic fluctuations cannot be truly considered convection. Given that a Fourier mode exists over the length of the entire domain, separate spatial regions of the flowfield are inevitably tied together. Localized regions of the flowfield are not allowed to move independent of one another due to being tied together via global Fourier modes. For instance, the presence of a large eddy in the downstream shear layer of a jet may influence the turbulent fluctuations in the initial shear layer if a Fourier mode connecting these two points is significantly distorted. Furthermore, accounting for sweeping where large scale eddies convect and distort the synthetized field is out of the question, given that local velocity contributions from large-scale eddies would further distort the Fourier modes.

In fact, true localized convection of SNGR-generated stochastic velocity fluctuations by the background flow is not possible without accounting for some explicit convection of the velocity field as in the framework of Lafitte et al [83]. The alternative methods for SNGR essentially abandon the idea of convecting turbulent structures generated by SNGR in favor of convecting a time-weighted velocity field generated by the addition of new randomly generated fields [14, 83].
In contrast, SEM methods inherently address locality issues because each eddy is convected by a local background flow. The superposition of individually-convected eddies generates a turbulent field with both coherent structures and randomness expected from a turbulent field. Sweeping is also easily achieved by using instantaneous fluctuations from resolved large-scale fluctuations to distort the eddy shape. These features allow SEM methods to generate more realistic synthetic turbulence, which in turn, results in more realistic noise predictions.

To demonstrate the benefits of using SEM, results were calculated for a subsonic jet (M = 0.9, Re = 100,000) on a limited-domain Cartesian grid (see Figure 4.1). The grid was comprised of 32 blocks with 2.6 million grid points, with a grid spacing of 0.055D. Although the grid domain was not sufficiently sized to obtain usable LES results or capture turbulence-generated noise, it was large enough to simulate the jet shear layer. These simulation results are qualitative only and are not meant to be well-resolved.

Starting with a RANS mean flow (shown in Figure 4.2), synthetic eddies were added using the CLST method described in Section 5.3. A total of 15,000 eddies were inserted into a source region ranging from x = 0.5D to 12D. For comparison to SNGR, results from Section 4.4.1 are presented alongside those from the synthetic eddy method.

Figures 5.7 (a) and (b) show the SNGR- and SEM-generated fluctuations, respectively, convected by the RANS mean velocity field. The evidence of global modes (and their distortion) can be seen in vertical streaks in Figure 5.7 (a), especially downstream of x=8D. These streaks show up repeatedly over time due to the nature of the SNGR convection modification (see Section 4.3) that resets each mode periodically over time. In contrast, the SEM-generated fluctuations in Figure 5.7 (b) show
an organization of turbulent structures and streaks that might be expected in a turbulent flowfield, especially in the early portion of the shear layer.

It should be noted that the local nature of the SEM method allows the source region to be much more confined to the jet shear layer, which is why the source region appears smaller compared to the SNGR-based method. This prevents the placement of eddies outside the shear layer where they are not needed. For the SNGR-based method, the modes only taper off when the amplitude (determined here by RANS TKE) decreases. Additionally, the locality of the SEM method can be seen from the convection and stretching of eddies. Even with simple RANS-based convection, the SEM method appears to generate more realistic turbulence over the SNGR-based method. The SEM method solves the convection issues of SNGR simply by convecting each eddy locally. The superposition of all synthetic eddies leads to convection and sweeping of the entire synthetic turbulence field.

However, the turbulent fluctuations in Figures 5.7 (b) are still not ideal because they lack significant randomness, at least in a qualitative sense. Figures 5.7 (c) shows the same SEM field with synthetic eddies convected by the instantaneous LES field (this is the CLST method), which allows the larger eddies to sweep the synthetic eddies. Figures 5.7 (c) shows increased randomness in the turbulence fluctuations due to distortion of the synthetic eddies by the larger eddies. This is expected because sweeping is a decorrelation mechanism [61, 83, 119].

The differences between SNGR, SEM, and CLST are further emphasized in Figure 5.8, which shows the y-component velocity fluctuations. The streaks in the SNGR-synthesized field due to the Fourier modes are more clearly evident in Figure 5.8 (a) around $x = 10D_j$, $14D_j$, and $15D_j$. Figure 5.8 (b) and (c) shows fluctuations generated by the SEM-based method, without sweeping and with sweeping, respectively. Localized sweeping from the LES velocities is evident.
Figure 5.7

Contours of Velocity Magnitude Fluctuations for SNGR- and SEM-based Turbulence Methods
in Figure 5.8 (c) when compared to the simple convection via RANS in Figure 5.8 (b). The effect of sweeping on the turbulent field in these results is to increase randomness.

The use of an SEM-based turbulence method in CLST should generate more realistic turbulence over an SNGR-based method, which should result in a better prediction of the high frequency noise. The inclusion of sweeping in the synthetic field should also lead to more accurate noise prediction from synthetic turbulence [119]. A trade-off is that the SEM method will be slightly more expensive (15,000 eddies vs. 150 Fourier modes). However, the convection of each synthetic eddy by the local instantaneous LES velocity fluctuations seems most physically appropriate for modeling small-scale turbulence.
Figure 5.8

Contours of Y-Velocity Fluctuations for SNGR- and SEM-based Turbulence Methods
5.4.2 Jet at Mach 0.9 and a Reynolds Number of 100,000

Given that the goal of the CLST method is to account for the effects of fine-scale turbulence, the method was tested at a higher Reynolds number where the separation between large and small turbulent scales is greater. A round, isothermal jet is simulated at $M = 0.9$ and $Re = 100,000$. The jet parameters were chosen to compare with a recent LES study from Bogey and Marsden [34] that presented results of both the jet flow physics and near-field jet noise spectra.

This section contains several results for these jet parameters. The first section describes the RANS simulation used to initialize LES and calculate the synthetic eddy amplitudes. The second section is a validation of the three-dimensional LEE solver, comparing pressure results from NS and LEE. The third section is a comparison of the LES results to results from Bogey and Marsden [34] and Bogey [25] to investigate the jet shear layer and the associated near-field radiated noise. Following is an evaluation of the CLST method, comparing LES, VLES via filtering, and CLST with synthetic eddies added. The final section provides a summary of the results.

5.4.2.1 RANS Flowfield

The LES flowfield was initialized by a RANS simulation obtained using a $k - \epsilon$ closure model, using the same jet parameters, $M = 0.9$ and $Re = 100,000$. The computational grid employed for RANS simulations is shown in Figure 5.9.

Figure 5.10 shows contour plots of velocity magnitude, turbulent kinetic energy (TKE), and turbulent dissipation rate ($\epsilon$). The flow domain and contours are scaled by the jet diameter and by the jet exit velocity, respectively. As expected, the jet velocity shows a potential core, where fluid travels at the jet exit Mach number, surrounded by a shear layer. The jet velocity drops after
the end of the potential core. In general, both TKE and $\epsilon$ increase to a peak before the end of the potential core and then decrease downstream. Higher levels of TKE are found along the inside of the shear layer where turbulent fluctuations are likely strongest. Both TKE and $\epsilon$ show large values just after the inflow, which is a non-physical solution likely resulting from the inlet boundary condition. However, since these results are used only to generate the amplitudes for the synthetic eddies and no eddies are generated in this region, these results are acceptable.

Figure 5.11 shows the development of velocity, TKE, and $\epsilon$ along the center line and “nozzle” lip line. In Figure 5.11 (a), the velocity along the centerline begins to drop 7-8 $D_j$ downstream, indicating the end of the potential core. The increase in TKE and $\epsilon$ along the center line near 7-8 $D_j$ shows the merging of the shear layers, which also suggests the end of the potential core. This agrees well with the LES results from Bogey and Marsden [34], which show that the end of the potential core is approximately 7.5 $D_j$ for these jet parameters.
Figure 5.10

RANS Results for Re = 100,000

Figure 5.11

RANS Results Along the Jet Centerline and Nozzle Lip Line
Figure 5.12 shows development of the velocity, TKE, and $\epsilon$ profiles across the jet for several downstream stations. Figure 5.12 (a) also demonstrates the spreading of the jet downstream and the reduction in axial velocity after the potential core. Both TKE and $\epsilon$ decrease as the jet begins to weaken after $x = 15 D_j$. Together, these plots demonstrate that the RANS solution produced sufficiently accurate results to provide a starting point for LES and to calculate amplitudes for the CLST method.

![Figure 5.12](image)

(a) Velocity  (b) TKE  (c) $\epsilon$

**Figure 5.12**

RANS Results Across the Jet Shear Layer

### 5.4.2.2 3D Validation for LEE

In addition to the two-dimensional validation cases in Appendix A, a simple validation was performed to test the LEE formulation in three dimensions. Pressures and noise spectra from the NS and LEE simulations are compared in the near-field of a jet. The computational grid for both solvers is the same in this work, so it is expected that the spectra should compare well and provide
validation for the noise spectra predicted by LEE. It is assumed that any non-linear effects in the NS flow field (and not present in the LEE) will not significantly affect the noise predictions.

The computational grid for the jet simulations has 148 blocks and 4.88 million grid points. The domain extends from \(x=-15D_j\) to \(x = 60D_j\) in the \(y\)- and \(z\)-directions at the widest point. Slices through the grid at an \(xy\)-plane and a \(yz\)-plane are shown in Figure 5.13. The grid is stretched towards the outer boundaries to prevent spurious waves from reflecting back into the domain. Also a sponge layer is used at the outflow boundary to further dampen reflecting waves and instabilities that may exist in proximity to the outflow boundaries. This grid is used for all other high Reynolds number jet investigations in this dissertation.
Instantaneous contours of non-dimensional pressure \( \frac{p}{\rho \cdot a_\infty^2} \) taken from and xy-plane, \( z = 0D_j \), are shown in Figure 5.14. Figure 5.14 (a) for NS and Figure 5.14 (b) for LEE were obtained at the same instance in time. In both images, large-scale Mach waves are observed, radiating to the downstream at approximately 30° angle measured from the jet axis. Additionally, smaller-scale pressure waves are seen radiating from the shear layer before \( x = 5D_j \). These are the expected essential components of jet noise. The large disturbances along the jet shear layer are due to hydrodynamic fluctuations. Additional small-scale pressure waves are seen in the NS pressure field near the inlet plane, which are generated by the disturbances imposed at the inflow boundary. They appear to quickly die out and likely do not influence the near-field noise spectra. The two contour plots in Figure 5.14 demonstrate that there is excellent qualitative agreement between the NS and LEE pressure fields.

Pressure probes were placed in the near-field at the points listed in Table 5.3. These probes are placed along a line of \( y = 5D_j \) and cover most of the angles at which high-frequency noise contributes to the spectra. Time-histories of the pressure fluctuations from both the NS and LEE solvers are plotted in Figure 5.15, with the pressure displayed in Pascals. The histories are taken over the same time range for both NS and LEE. Excellent agreement is observed between the fluctuations. The only difference is a slight decrease of the maximum amplitude of the LEE pressure fluctuations at Point V6, Figure 5.15 (f).

Noise spectra from NS and LES at probes V1 - V6 are shown in Figure 5.16. The pressure histories were analyzed via FFT and the spectra are plotted from the range \( St = 0.2 \) to \( St = 7 \). All plots display excellent agreement across the frequency spectra, especially in the range \( St = 2 \) to \( St = 5 \). Exceptions are observed, mainly for the lower frequencies, which vary by 1-2dB in
Figure 5.14
A Comparison of Pressure Fields from NS and LEE Solvers

Figure 5.16 (a), Figure 5.16 (b), and Figure 5.16 (f). Above the frequency $St = 5$, the NS and LEE spectra do not agree well, which is most likely an indication that the grid cannot resolve these frequencies. However, there is confidence that the data agree up to $St = 5$. This will be used as a limit for high-frequency content in evaluating the LES and CLST results.

Probe V6 (Figure 5.16 (f)) shows the greatest differences between NS and LEE, with LEE under-predicting frequencies below $St = 2$ by 3-5 dB. Probe V6 is at 45°, an angle at which the spectra is influenced more by the larger-scale turbulence. The LEE formulation eliminates instability waves, and this may influence the mechanisms that lead to the generation of the lowest-frequency noise. Alternatively, it is possible that some non-linear phenomena are captured in the NS solver and not in LEE, which could explain the under-prediction of the lower-frequencies. Yet another possibility
could be reflections of the lowest frequencies (below St = 0.1) from the outflow boundaries, like was observed in Bogey and Marsden [34].

The similarity observed between pressure contours, pressure histories, and noise spectra demonstrate good agreement between the NS and LEE solvers to give confidence that the LEE solver is validated for jet noise predictions. Additionally, this gives confidence that noise from pure LES can be obtained, in the near-field, from either NS or LEE for this jet case.
Figure 5.15
Pressure History at LEE Validation Points
Figure 5.16

SPL Spectra at LEE Validation Points
5.4.2.3 LES Comparison

Data from Bogey and Marsden [34] and Bogey [25] are used to evaluate the accuracy of the present LES simulations for the Re = 100,000, Mach 0.9 jet. Bogey and Marsden [34] investigate the effect of different grid resolution parameters on the jet flow and noise spectra. Their results are quite detailed, using computational grids with up to one billion grid points and modeling the jet noise out to 75D_j from the nozzle. Additionally, Bogey and Marsden simulate the jet flow with a pipe nozzle geometry. Bogey [25] presents additional results from identical simulations.

In comparison, the grid of the present simulation (referred to as Sim. A) does not extend to the far-field with adequate resolution to resolve pressure fluctuations past 15D_j and has at least 50x fewer grid points (4.8 million vs. 250 million). The present simulation method employs an inlet plane for the jet rather than a physical nozzle geometry. Disturbances are imposed at the inlet to excite the jet turbulence. Additionally, only the near-field region of the computational grid for Sim. A has sufficient resolution to capture noise data. Given the high level of resolution in Bogey and Marsden’s results and the computational limitations of the present simulations, identical replication of Bogey and Marsden’s results are not expected. However, general trends should be captured by Sim. A, and the noise spectra should agree well.

It is expected that Bogey and Marsden’s use of a nozzle geometry leads to differences when comparing the present simulations. In Sim. A, disturbances are imposed at the inlet to stimulate fluctuations at the inlet and produce a turbulent jet. The data from Sim. A are most similar to results from Bogey and Marsden for an initially laminar, transitioning jet. Given that a nozzle geometry is not included in Sim. A and that some small region of transition is expected when imposing a jet flow at an inlet plane, a comparison to an initially laminar jet is acceptable.
Combined contours of instantaneous vorticity and acoustic pressure provide an initial qualitative investigation of the results from Sim. A. Figure 5.17 is plotted in a style similar to results from Bogey and Marsden [34], with vorticity contours in the center (over the range y = ±2.5D_j) surrounded by the near-field pressure fluctuations. Vorticity is scaled by u_j/r_j and pressure is presented in non-dimensional form \( \frac{p}{\rho u_{\infty}^2} \). The potential core ends around 5D_j. Both large and small scale turbulent motions are observed prior to the merging of the shear layers. As the grid for Sim. A coarsens near x = 15D_j, the turbulence dies off sooner than in Bogey and Marsden’s results.

Figure 5.17
Vorticity and Pressure in the Near-Field for the Jet for Sim. A

138
In the pressure field, large peaks are evident in Figure 5.17. These peaks are related to the large-scale coherent structures [34]. The results from Bogey and Marsden show smaller, circular pressure waves radiating from vortex pairing in the early shear layer. These are not observed in Figure 5.17, most likely due to the fact that Sim. A shows the jet transitioning to a turbulent state just downstream of the inlet. The fine-resolution of Bogey and Marsden’s data shows many smaller-scale pressure fluctuations. Smaller waves are observed in Sim. A too, but they are clearly not resolved well by the computational grid.

For a more quantitative comparison, centerline flow properties are compared to replicated data from Bogey [25]. Mean axial velocity is plotted in Figure 5.18. The velocity decay matches Bogey’s data well, and the decrease in velocity around $x = 4.5D_j - 5D_j$ indicates the end of the potential core. More rapid turbulent development in this jet leads to a shorter potential core than is typically expected (around $x = 7D_j$ [34]), but Sim. A shows a slightly longer potential core than Bogey’s data. The intensity of roll-up and pairing of vortical structures in the transitioning shear layer causes the jet to develop more rapidly, shortening the potential core.

Root mean square values of axial and radial turbulence intensity from Bogey are plotted in Figure 5.19. For comparison, the square root of time-averaged TKE from Sim. A is plotted since the corresponding turbulence intensities were not recorded. Although the quantities are not identical, they are similarly derived from multiplications of the turbulent velocity fluctuations and demonstrate that Sim. A predicts a similar turbulence decay rate along the centerline. The rise in turbulence indicates the merging of the shear layers at the end of the potential core. Sim. A shows a slightly elongated potential core compared to Bogey’s data. Both centerline velocity and turbulence

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2 Data reprinted with permission from author.

139
intensity results demonstrate that the mean flow properties in Sim. A agree well with LES results from Bogey.

Two probe locations are used for comparison of the noise spectra from a time-history of pressure: Point B1 = (x = 0.0Dj, y = 7.5Dj, z = 0.0Dj) and Point B2 = (x = 10.0Dj, x = 7.5Dj, y = 0.0Dj). Figure 5.20 shows SPL spectra for the two points from Sim. A and also from Bogey and Marsden [34]. The pressure from Sim. A is taken from the Navier-Stokes solver, since the probe locations are still in the near-field of the jet where the grid resolution is fine enough to capture noise data. For both probe locations, the frequency spectra from St = 0.2 to St = 2 are matched. Higher frequency roll-off is observed above this frequency, and it is more pronounced in Probe B1, with an under-prediction of 20 dB at St = 5. The drastic roll-off in high frequency content for Probe B1 is due to coarser grid resolution near this probe location. The frequency decay is less obvious for Probe B2, where grid resolution is finer. The noise at Probe B2 under-predicts Bogey
Figure 5.19
Centerline Turbulence Intensity in the Jet

and Marsden’s data\textsuperscript{3} by about 5 dB at St = 5. The high-frequency roll-off is expected because fewer points were used in this computational grid compared to Bogey and Marsden’s grid.

With this favorable spectral comparison, it is expected that this grid and case setup can reliably resolve the jet spectra up to at least St = 2 at these probe locations, or up to St = 5 in locations where the grid spacing is finer. A perfect comparison with Bogey and Marsden’s data is not expected, given the differences in grid resolution and simulation technique. However, these results demonstrate enough quantitative and qualitative similarities to assume that the correct jet physics and noise mechanisms are captured by this computational setup.

\textsuperscript{3}Data reprinted with permission from author.
5.4.2.4 CLST Noise Spectra

The primary goal of CLST is to supplement the high-frequency content of LES. First, the noise spectra are analyzed, followed by an investigation of the turbulent properties of CLST in a following section. The same $M = 0.9$ jet was used to evaluate the CLST method. Conceptually, CLST works by adding synthesized smaller-scale fluctuations to under-resolved LES (referred to as VLES here). Filtering was applied to the LES jet results to obtain VLES, which can be seen as under-resolved LES on the current computational grid. The jet noise from both cases was captured using near-field pressure probes, with the filtering applied to the pressure field to generate the VLES noise spectra. The spectra from VLES can be considered the noise radiated from an under-resolved LES simulation. Simultaneously, synthetic turbulence generated by the CLST method was combined with resolved large-scale velocity fluctuations (filtered VLES velocities).
into the LEE source terms, giving the noise produced by the CLST method. The frequency content added by the synthetic turbulence was revealed by comparing noise spectra from LES, VLES, and CLST. This approach is representative of adding synthetic turbulence to an under-resolved LES simulation and recovering the “missing noise”, but in this case, the added noise is quantifiable and comparable to the “missing noise” because the “missing noise” has been removed by a filter with known frequency properties. The excellent agreement that was previously demonstrated between the NS and LEE pressure makes this is a valid comparison.

In the following analysis, the data were obtained from one simulation run, since the LES, VLES, and CLST flow-fields are saved simultaneously. Pressure histories for LES and VLES are taken directly from the NS solver, given that it showed excellent agreement with the LEE solver for several near-field probe locations in previous results. Pressure histories for CLST are taken from the LEE solver. For the CLST computation, a non-dimensional time-step size of $8.01 \times 10^{-4}$ was used, which equates to a CFL of 0.13. This was required for stability of the Adams-Bashforth time-marching algorithm.

In this evaluation of the CLST method, 8,000 eddies were inserted into the shear layer. In preliminary results, 8,000 eddies was found to strike a reasonable balance between increased computational cost and generating additional high-frequency noise. Eddies are imposed over the range $x = 1.8D_f$ to $x = 12.5D_f$. The eddy lifetime was $10.5 \sigma^n$, the eddy amplitude factor $\beta_{amp}$ was 8,000,000, the minimum eddy size was $\sigma_{min} = 3.2\Delta x_{avg}$, and the largest eddy size was $\sigma_{max} = 4.8\Delta x_{avg}$. A fourth-order filter was applied to the LES field twice for every time step to obtain the VLES field.
A qualitative comparison of the jet turbulence and near-field pressure spectra are shown in Figure 5.21 at the same instance in the simulation. Contours of vorticity (shown as non-dimensional quantities scaled by $D_j/U_j$) are shown for a range of $y = \pm 2D_j$. Pressure field contours are shown in units of non-dimensional pressure (scaled by $\frac{1}{\rho_\infty u_\infty^2}$). LES results in Figure 5.21 (a) reveal the developing turbulent jet, the merging of the potential core around $x = 5D_j$, and the slow dissipation of turbulence downstream past $15D_j$.

The pressure field displays several expected noise phenomena: Mach waves radiating at 30°, large-scale pressure fluctuations, and smaller, spherical waves radiating from the early shear layer. The VLES results in Figure 5.21 (b) reveal that filtering removes the fine-scale turbulence and leaves larger-scale turbulent structures untouched. The Mach waves and larger-scale pressure fluctuations are still evident in the VLES pressure field, but damping is observed and smaller-amplitude pressure fluctuations are not observed, especially upstream of the potential core. In Figure 5.21 (c), with the synthetic turbulence added to the VLES flow field, only the smallest increase in the intensity of the turbulent structures is observed upstream of the potential core. However, additional small-scale pressure waves are discernible in Figure 5.21 (c) due to the addition of synthetic eddies from CLST. Mach waves are also still observed in the angling of pressure waves toward the downstream. Slight visualization artifacts are present in the flow field in Figures 5.21 (a) and (c), but these do not contaminate the pressure spectra.

Pressure data were recorded from the CLST simulation every 15 time-steps over a total of 140,000 iterations. The probe locations are given in Table 5.4. The probes are divided into three categories. The first set is represented by six probes (P1 - P6) equally spaced along the line $y = 5D_j$, which is close to the jet but far enough from the shear layer to avoid contamination of the
Contours of Vorticity in the Shear Layer and Pressure in Near-Field for LES, VLES, and CLST acoustic data with hydrodynamic fluctuations. These points should give a good survey of the noise generated before and just after the potential core. The second set is represented by two probes (P7
and P8) along the line \( y = 7.5D_j \), which are the probe locations used to validate the case with data from Bogey and Marsden [34]. The third set corresponds to two probe locations (P9 and P10) along the arc \( r = 12D_j \). These probes cannot be considered far-field, which is usually at least \( 45D_j \) [1], but are the farthest distances from the nozzle at which data can be adequately resolved with this computational grid. Probe location P9 is at 90° to the jet downstream, which is representative of a sideline observation location. Probe location P10 is at 60° to the jet downstream.

Table 5.4

Point Locations of Pressure Data Probes for CLST Investigation

<table>
<thead>
<tr>
<th>Point #</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
<th>P6</th>
<th>P7</th>
<th>P8</th>
<th>P9</th>
<th>P10</th>
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</thead>
<tbody>
<tr>
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<td>1.0</td>
<td>2.0</td>
<td>3.0</td>
<td>4.0</td>
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<td>10.0</td>
<td>0.0</td>
<td>6.0</td>
</tr>
<tr>
<td>( y/D_j )</td>
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<td>5.0</td>
<td>5.0</td>
<td>5.0</td>
<td>5.0</td>
<td>5.0</td>
<td>7.5</td>
<td>7.5</td>
<td>12.0</td>
<td>10.39</td>
</tr>
<tr>
<td>( z/D_j )</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
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<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Pressure histories from the probe locations P1 through P6 in Table 5.4 are shown in Figure 5.22. Pressures, presented in Pascals, are plotted against simulation time for LES, VLES, and CLST. The time-histories show that no simulation transients are present in the data. The peaks observed in the LES data (black) are matched in amplitude by peaks from CLST (red). At Points P1 and P2, the CLST amplitudes appear slightly larger than LES, indicating that certain frequencies may be over-predicted.

The noise spectra from these pressure histories were obtained by a Fast Fourier Transform (FFT). The spectra for probe locations P1 through P6 in Table 5.4 are shown in Figure 5.23. Spectra from LES, VLES, and CLST are compared. Frequencies from \( St = 0.2 \) to \( St = 5 \) are present in the LES noise spectra. The noise produced by VLES filtering shows strong high frequency roll-off.
(a) Point P1 \((x = 0D_j)\)
(b) Point P2 \((x = 1D_j)\)
(c) Point P3 \((x = 2D_j)\)
(d) Point P4 \((x = 3D_j)\)
(e) Point P5 \((x = 4D_j)\)
(f) Point P6 \((x = 5D_j)\)

Figure 5.22
Pressure History for CLST Along the Line \(y = 5D_j\)
starting at frequencies of $St = 1$ to $St = 2$. At $St = 2$, VLES is around 10dB below LES for all six points. The VLES filtering also reduces portions of the low-frequency noise spectrum by up to 5dB in places. For all six probes, the addition of synthetic turbulence leads to the recovery of the filtered frequencies in the range $St = 1$ to $St = 5$. Several small differences (less than 5dB) are observed between the LES and CLST spectra at various points. Notably, at points P1 and P2 (Figure 5.23 (a) and (b)), there is an over-prediction around $St = 2$. At point P6 (Figure 5.23 (f)), there is an under-prediction around $St = 1$. Overall, at all six locations along the line $y = 5D_j$, the shape and level of the LES spectra is modeled well with CLST.

The spectra from points P7 and P8 are shown in Figure 5.24. Again, filtering removes frequencies above $St = 1$. Some evidence of roll-off from grid resolution can be seen around $St = 4$ in the SPL plot from point P7 (Figure 5.24 (a)). The noise from CLST again recovers the missing spectra and matches LES spectra. Point P7 shows slight over-prediction around $St = 2$.

Further from the nozzle, the spectra captured by points P9 and P10 are presented in Figure 5.25. The grid resolution at these two points is coarser than at the other probe locations, so the maximum $St$ was reduced to $St = 2$ and $St = 3$ for points P9 and P10, respectively. The cut-off from filtering appears to begin around $St = 0.6$ to $St = 0.7$. Again, the noise supplemented by CLST follows the LES spectra. There is a slight over-prediction for point P9 (Figure 5.25 (a)) above $St = 1$.

In several of the probe locations directly to the sideline of the nozzle, an over-prediction of 3-5 dB was observed around $St = 1$ and $St = 2$. These are probes P1, P2, and P7 (Figure 5.23 (a), Figure 5.23 (b), and Figure 5.24 (a), respectfully). Given that the influence of the SEM-generated noise is observed around $St = 1$, it is possible that excess noise is generated from the synthetic
Figure 5.23

SPL Spectra for CLST Along the Line $y = 5D_j$
edgies at these frequencies. The cause of these slight differences is unclear. It is possible that these differences may disappear with longer pressure time-histories.
The high frequencies were supplemented using only 8,000 eddies. It was expected from Hirai et al. [67] that up to 50,000 eddies could be required to predict the jet noise. The extra numerical cost incurred by calculating CLST with 8,000 eddies was approximately eight times that of the run-time for the LES + LEE solvers. For future applications of CLST, such as predicting noise to the jet far-field, this extra cost will be prohibitive. However, the cost is not necessarily due to the concept of the CLST method.

In fact, the excess cost is mostly due to the implementation of the CLST method in the code. The CLST equations from Section 5.3 were coded in a straightforward manner and no optimization was performed. Investigations into the code’s performance indicate that the loop responsible for summing the contribution of synthetic velocities at each grid point is the main reason for the cost increase. The reason for this increase has to do with allocation of memory when accessing arrays for the stored flow field velocities and the synthetic velocities. The current implementation incurs a large memory overhead as different parts of these arrays are continually accessed while looping over points in each computational block. This observation is crucial because it means that the CPU operations for the CLST method are not the reason for the cost increase.

Optimizing the code for CLST requires further research into computer science techniques for minimizing the memory required to transfer data between large arrays and is outside the scope of this work. However, if this problem can be solved, cost of CLST is expected to only be 50% to 100% increase over the cost of the LES + LEE solver, which is a much more reasonable cost trade-off. Additionally, the synthetic eddies are currently solved on only a small number of the total blocks in the computational grid. Increasing the number of eddy blocks and decreasing the number
of grid points per eddy block would also lead to a lower computational cost as the load balance is distributed more evenly between computational blocks.

While the cost of CLST is not ideal, the increase is likely far lower than the cost required to double the grid resolution of an LES + LEE solver. For an example, assume that the cut-off frequency of a given LES grid is $\text{St} = 1$. In order to double the frequency resolution to $\text{St} = 2$, it is simple to suggest that the grid spacing must be doubled, leading to an 8x increase (2x in each direction). To maintain the same stability and CFL condition ($\text{CFL} = \frac{u\Delta t}{\Delta x}$), this means that the time-step size must be halved, doubling the computational time. So to double the frequency resolution with LES only, at least 16x is needed. Practically, though, doubling the frequency spectra may require at least a 10x increase in grid resolution [46] so this option is more expensive.

Even with a moderate increase in computational cost, the CLST method recovered the noise removed by filtering at all ten probe locations. Despite small differences in the noise spectra, it is clear that CLST can supplement the missing noise spectra from under-resolved turbulence and also captures the correct noise spectra curves for near-field observer locations.

### 5.4.2.5 CLST Turbulence Investigation

The main goal of CLST is to supplement the missing noise field. A secondary concern is that the synthetic turbulent field produced by the CLST method is somewhat realistic. Synthetic turbulence is not meant to replicate realistic turbulence in every manner, but is meant to serve as a model. With this in mind, the turbulent properties of the synthetic velocity fluctuations are investigated in this section.
Velocity fluctuations are analyzed on the xy-plane $z = 0D_j$. Figure 5.26 shows the instantaneous fluctuating $y$-component of velocity, scaled by $U_j$ for LES (5.26 (a)), VLES (5.26 (b)), and CLST (5.26 (c)). The damping of smaller-scale turbulent structures is evident in Figure 5.26 (b) for the VLES velocity field. Figure 5.26 (c) displays the combined velocity fluctuations from VLES and the synthetic eddies produced by CLST, and the synthetic eddies only slightly enhance the strength of the turbulent fluctuations. The amplitude of the synthetic eddies appears to be quite small compared to the largest turbulent fluctuations, which is expected. Larger-scale fluctuations produced by the SEM field likely result from a superposition of eddies in space.

**Figure 5.26**

Y-Velocity Contours From LES, VLES, and CLST
For a more quantitative analysis of the imposed synthetic turbulence, instantaneous y-component velocities from LES, VLES, and CLST are plotted across the jet. The stations plotted are \( x = 3D_j \) (Figure 5.27 (a)), \( x = 4D_j \) (Figure 5.27 (b)), and \( x = 5D_j \) (Figure 5.27 (c)). The effects of filtering in VLES are observed at all three positions, mainly in a reduction of amplitude compared to LES. The inclusion of synthetic eddies from CLST shows larger-amplitude fluctuations compared to VLES, with amplitudes similar to LES. The larger-scale fluctuations are likely due to the superposition of synthetic eddies. The smallest turbulent fluctuations are not visible in these plots. It should also be noted that, while not pictured here, the LES TKE levels are not completely recovered by the addition of synthetic velocities.

A closer look at the synthetic turbulent field reveals that the fluctuations are small in amplitude. Figure 5.28 shows each velocity component of the synthetic field plotted on the xy-plane at an instantaneous time step. Figure 5.28 (a) shows the x-direction velocity fluctuations, Figure 5.28 (b) shows the y-direction velocity fluctuations, and Figure 5.28 (c) shows the z-direction velocity fluctuations. Both larger and smaller scale fluctuations are observed in the flow field. Larger structures occur when eddies overlap. Additionally, the eddies appear to be stretched and distorted by the shear layer and, it is assumed, by the large-scale LES turbulent field. This is evidence of sweeping and straining of the synthetic eddies.

Contours of velocity magnitude for the time-changing synthetic turbulent field are presented in Figure 5.29 to investigate sweeping qualitatively. The images are taken from the upper shear layer and show the development of eddies over time. Note that these are only the synthetic velocity fluctuations and do not include the VLES large-scale filtered structures.
Figure 5.27

Y-Velocity Plotted Across the Jet for LES, VLES, and CLST

Figure 5.29 (a) shows the turbulence at a time instance, t1. A black oval has been drawn around the region of interest containing an already stretched eddy. As time advances to Figure 5.29 (b) and to Figure 5.29 (c), and so on, the flow field is rotated counter-clockwise by the larger turbulent structures in the shear layer. The eddy structure rotates against the direction of the mean flow field, which is in the positive x-direction. This strong rotation present in the synthetic eddy field indicates that sweeping from a large-turbulent structure is occurring at this location. As time advances, a portion of the eddy structure decreases in amplitude as an eddy dies out before being recycled.
elsewhere. Stretching and straining of the eddies is also observed, as the large red structure at the location \( x = 3.5D_j \) in Figure 5.29 (a) slowly stretches and decays over time.

These results are by no means a complete proof of sweeping in the synthetic field. In order to fully investigate the presence of sweeping in the turbulent flow field, space-time correlations are needed. However, correlations require long time-histories of data over spatial lines in the flowfield. These data were not saved for the current simulation, so this simple qualitative investigation must suffice.

From this limited set of turbulence investigations, it is clear that the turbulence produced by the synthetic eddies does not fully model all the turbulent fluctuations removed by filtering in the flow field. It should be noted that while 8,000 eddies appears to adequately cover the source domain,
additional eddies would probably lead to an increase in turbulent fluctuations. In fact, several preliminary test cases with 20,000 eddies were investigated, but the result was an over-prediction of noise. Additionally, the increase in computational cost became prohibitive, and the simulation progress was an order of magnitude slower.
The amplitude of the synthetic velocity fluctuations in CLST are not large enough to make a substantial difference and do not enhance the VLES turbulence enough to approach the LES solution. The intensity of the synthetic eddies was tuned to match the amplitude of the pressure waves, not the LES velocity fluctuations. This is an indication that the synthetic turbulence field generates more noise than expected, which could be the result of inserting unrealistic eddies into the flow. Alternatively, the distortion of eddies due to sweeping and straining could violate the divergence-free condition and radiate additional noise.

Ideally, matching the turbulent flow field first would provide an anchor point for further applications of the CLST method, eliminating the need to re-tune the amplitudes of the synthetic eddies to match the pressure waves for each simulation. It may be difficult to match both turbulent flow properties and the noise spectra. Hirai et al. [67], who also employed a synthetic turbulence model for jet noise predictions, had a similar issue with over-predicting the noise levels when first tuning their model to match turbulent statistics. However, given that the noise results match well in the previous section and meet the primary goal of the CLST method, the differences in the turbulent flow field are acceptable for the current stage of CLST development.

5.4.3 Jet at Mach 0.9 and a Reynolds Number of 3,600

A second simulation was performed to fully explore the turbulent properties of the synthetic field in the CLST method, as well as to further evaluate the method’s performance with different jet conditions. The jet parameters are taken from a set of well-known DNS results from Fruend for $M = 0.9$ and $Re = 3,600$ [58].
In the following, the first section describes the RANS simulation used to initialize the jet and calculate CLST synthetic eddy amplitudes. The second section provides a comparison of the LES results to DNS results. The final two sections evaluate the noise spectra generated by CLST and the turbulent properties of the synthetic turbulence generated by CLST.

### 5.4.3.1 RANS Flowfield

The LES flowfield was initialized by a RANS simulation using a $k - \epsilon$ closure model, with the same jet parameters, $M = 0.9$ and $Re = 3,600$. Figure 5.30 shows contour plots for velocity magnitude, turbulent kinetic energy (TKE), and turbulent dissipation rate ($\epsilon$). The flow domain and contours are scaled by the jet diameter and by the jet exit velocity, respectively. The velocity magnitude plot (Figure 5.30 (a)) shows a potential core and a shear layer that spreads as the jet grows. Similar to the high Reynolds number case, both TKE and $\epsilon$ show artificial peaks just after the inlet.

Figure 5.31 shows the development of velocity, TKE, and $\epsilon$ along the center line and “nozzle” lip line. The drop in velocity (seen in Figure 5.31 (a)) and the increase in TKE and $\epsilon$ (seen in Figures 5.31 (b) and (c)) indicate the end of the potential core around $7 D_j$. The increase in TKE and $\epsilon$ along the center line near $7-8 D_j$ shows the merging of the shear layers, which also indicates the end of the potential core. This agrees well with the DNS results of Freund [58] that estimates that the potential core ends around $x = 7 D_j$.

Figure 5.32 shows development of velocity, TKE, and $\epsilon$ profiles at several downstream stations across the jet. Figure 5.32 (a) also demonstrates the spreading of the jet downstream and the reduction in velocity after the potential core. The decay of velocity agrees with results shown in...
Freund [58], although in the present simulation there is a slight downturn towards the end of the jet core that does not seem physically accurate. The TKE profile in Figure 5.32 shows a similar development to the production of TKE from Freund [58]. This qualitative agreement with Freund’s results suggests that the RANS solution was sufficiently accurate.
5.4.3.2 DNS Comparison

Direct numerical simulations performed by a research collaborator, Professor Yuji Hattori from Tohoku University, provide another comparison to validate the capability of the LES solver at predicting jet noise (personal communication). The simulation results (referred to as Sim. B) are compared to DNS data from Professor Hattori and also to the DNS data published by Freund [58].
As with the higher Reynolds number jet, the nozzle was not directly simulated. A notable
difference between this case and the higher-Reynolds number case is that the disturbances imposed
at the nozzle inlet match those discussed by Freund [58]. Both the DNS results and Sim. B impose
these disturbances. The same grid from the moderately-high Reynolds number jet was used for
Sim. B.

Contours of vorticity (scaled by $\frac{\nu}{u_j}$) from the plane $z = 0D_j$ are shown in Figure 5.33, along with
near-field pressure contours (scaled by $\frac{1}{\rho_\infty a_\infty^2}$) from the LEE solver. Kelvin-Helmholtz instabilities
are observed in the transitioning shear layer. The potential core appears to end around $x = 7D_j$-
7.5D_j, which agrees with Freund [58]. The pressure field shows strong spherical waves radiating
from near the end of the potential core, which were observed in Freund’s analysis [58]. The waves
radiate in all direction, but appear stronger at an angle of 30° to the downstream.

Figure 5.34 shows the mean centerline velocity from Sim. B compared to results from Freund’s
DNS\textsuperscript{4}, again showing that the potential core is captured well in Sim. B. Downstream, near $x =
12.5D_j$, the mean velocity departs from Freund’s data as the jet begins to die out. This is likely due
to the difference in resolution between LES and DNS, with DNS resolving additional turbulent
mixing that sustains the jet downstream.

Figure 5.35 presents turbulent kinetic energy along the nozzle lip lines from Sim. B and from
the DNS provided by Professor Hattori. The overall shape of the TKE distributions are matched,
including the turbulent decay rate. However, the peak TKE levels are delayed one to two diameters
in Sim. B. As a result, the jet probably does not develop enough turbulence to sustain mixing far

\textsuperscript{4}Reprinted with permission from Cambridge University Press.
Contours of Vorticity and Pressure For The Low Reynolds Number Jet downstream. This is another possibly explanation for the jet decay observed in the previous two Figures.

Pressures from Sim. B and from Professor Hattori’s DNS results are shown in Figure 5.36. The probe locations are long the line $y = 10 D_j$. Point H1 is at $x = 0 D_j$ and point H2 is located at $x = 20 D_j$. The amplitude of pressure fluctuations from the two simulations agree well.

The noise spectra from points H1 and H2 are presented in Figure 5.37. Compared to the DNS results, the spectra predicted by Sim. B agrees well for frequencies up to $St = 2$. Above this
Figure 5.34

Centerline Mean Velocity Comparison to Freund [58]

Figure 5.35

Turbulent Kinetic Energy Along the Nozzle Lip Lines

frequency, roll-off is observed in the LES as expected. Above St = 3, artifacts from the FFT appear in the LES spectra in point H1, indicating the limit of frequency resolution at this location for
this case. For point H1, there is some over-prediction around St = 1. Overall, these results give confidence in the LES jet noise spectra up to St = 2 or 3, depending on the probe location.
5.4.3.3 CLST Noise Spectra

The CLST method is applied to the low Reynolds number jet. Noise spectra are evaluated in this section, and the turbulent properties are analyzed in the following section.

At this lower Reynolds number, the smallest turbulence eddies are larger than those from the higher Reynolds number jet and the need to impose many small eddies is not as significant. Due to this, the sizes of the synthetic eddies were increased and fewer eddies were imposed. For this investigation, 800 eddies were inserted into the shear layer from the range \( x = 4D_j \) to \( x = 11D_j \). The minimum eddy size was \( \sigma_{\text{min}} = 15\Delta x_{\text{avg}} \), and the largest eddy size was \( \sigma_{\text{max}} = 4.8\Delta x_{\text{avg}} \). The parameter \( \beta_{\text{amp}} \) was set to 400,000 and the eddy lifetime was \( 6\sigma^n \). A fourth-order filter was applied to the flow field four passes every time step to obtain VLES results.

The same procedure from the moderately-high Reynolds number jet was applied to analyze the CLST method. First, filtering was applied to the LES data to obtain an under-resolved VLES flow field. Then synthetic turbulence was added to VLES to obtain the CLST results. Contour plots of vorticity in the jet and pressure in the jet near-field for LES, VLES, and CLST are presented in Figure 5.38. Both small- and large-scale circular pressure waves are observed in the LES pressure field (Figure 5.38 (a)) and seem to radiate from the merging shear layers. In Figure 5.38 (b), the effects from the filter are seen both in a reduction of small-scale vortical structures in the jet and in a damping of the smaller-scale pressure waves from LES. The addition of synthetic eddies in Figure 5.38 (c) generates many smaller-scale pressure fluctuations. Additionally, stronger pressure fluctuations are observed moving back towards the nozzle. These are likely generated by the insertion of eddies into the flow field, which may come into play as a pulse. Even though there is a time-ramp for inserting each eddy, perhaps the ramp is too aggressive for this case. However, these
pressure waves appear highly directional and it is likely that they only influence the pressure field near the jet.

Figure 5.38

Contours of Vorticity in the Shear Layer and Pressure in Near-Field for LES, VLES, and CLST
Pressures histories were recorded at near-field probes from Table 5.4, with the addition of three data points, P11, P12, and P13 (see Table 5.5). Points P11 and P12 are probes along the line \( y = 10D_j \). Point P11 is sideline to the nozzle and P12 is more downstream. Probe P13 is grouped with probes P9 and P10 from Table 5.4, and exists at \( 45^\circ \) at a distance of \( 12D_j \) from the nozzle.

<table>
<thead>
<tr>
<th>Point Locations of Additional Pressure Data Probes</th>
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<tbody>
<tr>
<td>Point #</td>
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</tr>
<tr>
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<tr>
<td>( y/D_j )</td>
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<td>( z/D_j )</td>
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</table>

Data were recorded over 120,000 iterations at a times step of \( 7.4 \times 10^{-4} \) seconds. Pressures from points P11 and P12 are shown in Figure 5.39 (a) and Figure 5.39 (b), respectively. The CLST pressure fluctuations at P11 have a similar amplitude to the LES pressure fluctuations. At points P12, the CLST amplitudes are slightly lower than LES, which may indicate an under-prediction of SPL at certain frequencies at this point.

Sound spectra are shown in Figure 5.40, Figure 5.41, Figure 5.42, and Figure 5.43. Figure 5.40 shows noise spectra points P11 and P12, along the line \( y = 10D_j \). The VLES spectra roll-off occurs above \( St = 1 \). At these two points, the addition of turbulence from CLST supplements the high frequency noise spectra and recovers the LES spectra. At probe P11, the high-frequency spectrum is over-predicted by about 5-7 dB above \( St = 2 \). The CLST spectrum at probe P12 shows a slight under-prediction from the LES spectrum at the value of \( St = 1 \), which is likely related to the reduced pressure fluctuations observed in Figure 5.39 (b).
Figure 5.39

Pressure History for Low Reynolds Number CLST

Figure 5.40

SPL Spectra for CLST Along the Line $y = 10D_j$

Figure 5.41 shows the sound spectra captured by probes P9, P10, and P13, which are probes obtained at $r = 12D_j$ at various angles. The VLES high-frequency roll-off begins around $St = 0.7$ to $St = 1$ for these locations. These three probe locations are farther away from the nozzle where the
grid resolution is coarser, and artifacts of grid cut-off are observed in the spectra at points P9 and P10 above $St = 2$. However, despite this, at the point of VLES roll-off, the filtered high-frequency noise is recovered by CLST at all three probe locations. The CLST spectra follow the LES spectra well for the frequency range $St = 1$ to $St = 3$ for all points. An over-prediction of the highest frequencies in the CLST spectrum is observed at point P9.

Figure 5.41

SPL Spectra for CLST At A Distance of 12$D_j$
The noise captured at probes P5 and P6 are presented in Figure 5.42. The VLES spectra agree with LES up until around St = 1.3, then the filter leads to high-frequency roll-off. At the point of VLES roll-off, CLST supplements the VLES spectra and increases the high-frequency noise. However, the high-frequency spectra (above St = 2) are over-predicted by 5 to 10dB as compared to the LES noise spectra. This indicates that the CLST is producing excess noise, at least at these probe locations. This extra high-frequency content was not observed at probe locations farther from the jet and at more downstream angles (i.e. probes P12 and P13), so it is not clear how this extra noise would influence far-field nose predictions.

![SPL Spectra for CLST Along the Line y = 5D_j](image)

(a) Point P5 (x = 4D_j)  
(b) Point P6 (x = 5D_j)

Figure 5.42

SPL Spectra for CLST Along the Line y = 5D_j

The nearest probe locations show some contamination from the strong, rearward-moving pressure waves observed in Figure 5.38 (c). Spectra at probe P1 are presented in Figure 5.43. The large-scale pressure waves lead to an over-prediction of several dB of frequencies above St = 0.5.

171
The shape of the high-frequency spectrum at this location also indicates extra high-frequency noise content.

![Figure 5.43](image)

**Figure 5.43**

Contaminated SPL Spectra at Probe P1 (x = 0D_j, y = 5D_j)

As in the moderately-high Reynolds number jet, pressure probes directly to the sideline of the nozzle showed an over-prediction of high-frequency content. This over-prediction was more accentuated in the results from the low-Reynolds number jet. The stronger influence of the synthetic turbulence in this case could mean that the CLST method needs to be tuned better for this specific case. It is possible that fewer eddies are needed for this case since the Reynolds number is so low. Alternatively, the smallest synthetic eddies could be too small and generate too much high-frequency content. This could also just be an artifact of imposing synthetic eddies onto the flow field that requires further investigation.

The strong pressure waves radiating back towards the inlet are most likely caused by large-amplitude eddies being inserted into the flow field. The large amplitudes of the eddies probably
come from turbulence intensity levels in the upstream shear layer that are over-predicted by the RANS simulation.

Despite the over-prediction of high frequencies observed at several probe locations and the presence of stronger pressure waves radiating back towards the inlet, these results continue to demonstrate that adding synthetic fluctuations to the filtered VLES field leads to a recovery of high-frequency content.

5.4.3.4 CLST Turbulence Investigation

The turbulent flow field of the SEM method is investigated for the low Reynolds number jet. Figure 5.44 displays contours of instantaneous y-velocity on the zy-plane \( z = 0D_j \). In Figure 5.44 (a), only medium to large scale eddies are observed. The VLES filtering removes most of these fluctuations, but the intensity of the synthetic eddies is not large enough to make a substantial difference in enhancing the VLES solution. As a result, the velocity fluctuations from CLST do not approach the level of fluctuations from the LES solution. The amplitude of the synthetic eddies was tuned to match the amplitude of the pressure waves. The resulting low-intensity fluctuations indicate that the synthetic field generates more noise than expected. Reasons for this have been discussed previously.

The velocities from Figure 5.44 are plotted across the jet at \( x = 6D_j \) and \( x = 7D_j \) in Figure 5.45. Eddy superposition does lead to slight differences in the CLST velocities when compared to the filtered VLES velocities, but in general, the synthetic eddies are not strong enough to recover the turbulence lost to filtering.
Figure 5.44

Y-Velocity Contours From LES, VLES, and CLST

Figure 5.46 shows instantaneous x-component, y-component, and z-component velocity fluctuations from the synthetic eddy method. The amplitudes of the synthetic velocities are small and less influential in the low Reynolds case, when comparing to LES fluctuations. Note that the contours are plotted for $u = \pm 0.04U_j$. Mostly medium and large scale fluctuations are present in the flow field due to the low Reynolds number of the jet, but some smaller scale fluctuations are observed. The eddies are sheared by the mean flow. Some evidence of sweeping can be seen in the x-velocity plot (Figure 5.46 (a)) in the blue eddy structure in the lower half of the shear layer near the inlet.
The “s” shape indicates that the eddy is distorted by some large-scale underlying turbulent motion and not just by the shear layer.

Figure 5.45

Y-Velocity Across Jet for LES, VLES, and CLST

Figure 5.46

Velocity Contours From the SEM Method
As in the moderately-high Reynolds number jet, the noise spectra is enhanced in the high frequencies by CLST, but the turbulence properties of the method require further investigation to produce a more realistic model of velocity fluctuations.

5.5 Shortcomings and Issues with CLST

The results presented in the previous sections revealed several issues with the CLST method that should be addressed in future development.

- Optimizing the SEM method to reduce costs: The synthetic eddy method used in CLST requires a modest increase in computational expense to simulate the convection of thousands of eddies in the shear layer. However, the main reason for the added expense is that the implementation of the code that leads to a large memory overhead. Since the loop over the synthetic eddies lies inside the loop over the grid points in a block, data exchange between the two loops is very time consuming due to the manner in which the data are accessed in the cache memory. Further research into optimizing how the data are accessed for CLST calculations could drastically reduce the expense, such that CLST would only cost 50% or 100% more than a baseline LES + LEE calculation. Additionally, load balancing of the computational blocks for eddy calculations would further reduce the computational expense.

- Matching turbulent flow properties: In this work, the turbulent properties of the flow field were not modeled well by the synthetic turbulence despite matching the noise spectra. This is an indication that the synthetic eddy field generates more noise than expected, which is possibly the result of inserting unrealistic eddies or violating the divergence-free condition due to the distortion and straining of eddies by the mean flow. Ideally, in addition to the noise, the
turbulent flow field properties should be modeled well by the synthetic eddies. This would make it easier to tune the method to a general range of jet noise conditions. A further investigation is needed into exactly how much TKE is required to resolve the LES accurately enough to capture large scales for the low-frequency noise. Additionally, knowing the balance of the energy scale is important, as is knowing whether the synthetic eddies add too much or too little energy. A thorough investigation of turbulent properties requires plotting space-time correlations, which can be time-consuming to obtain. A future goal would be to model more realistic turbulence without increasing the noise content further.

- CLST frequency content: The only control over the frequencies produced by CLST is through the maximum and minimum size of the eddies. In the simulation process, these often had to be adjusted to achieve the desired spectral content. Some differences in spectra were observed when comparing CLST to LES, and it is difficult to adjust the method to account for all of these differences without changing the entire spectra. The presence of large pressure waves radiating back toward the nozzle affected the near-field pressure and increased low-frequency content. There is currently no way to damp such waves. Additionally, in general, noise at sideline probes showed an over-prediction of higher frequency noise. The exact reason needs to be investigated, but it could be related to the eddies generating more noise than they should or to a violation of the divergence-free condition.

- Sweeping effects on the noise field: Although sweeping was observed in the synthetic turbulence, the effect of including sweeping instead of employing a constant convection velocity (such as $0.6U_j$) was not investigated. It is possible that the idea of sweeping imposed in
CLST is not correct or that it leads to an over- or under-prediction in the noise field. To fully investigate sweeping, space-time correlations are needed.

5.6 Summary

The CLST method was applied to two Mach 0.9 jets: a moderately-high Reynolds number jet and a low-Reynolds number jet. The results from investigating the SEM-based CLST method are summarized in this section.

The LEE solver was validated against the LES noise field with excellent agreement. For the moderately-high Reynolds number jet (Re = 100,000), the computational setup was shown to match LES results by Bogey and Marsden [34]. The CLST method was evaluated by using a filter to produce an under-resolved VLES flow field. Synthetic fluctuations generated by CLST were added to the VLES flowfield. A comparison with LES revealed that the CLST method works well in supplementing the high frequency noise spectra for under-resolved LES jet simulations. The synthetic velocity field did add turbulent fluctuations to the VLES flowfield, but the turbulent properties of the flow field did not match those of the LES results well. Tuning the method to predict the noise spectra led to an under-prediction of velocity fluctuations compared to LES. This is an indication that the synthetic field generates excessive noise. Evidence of sweeping and straining in the synthetic flow field was observed in the synthetic velocity field.

A low Reynolds number jet was also simulated with CLST with a similar outcome: that CLST modeled the high-frequencies well with synthetic turbulence. However, the highest frequencies were over-predicted at several probe locations, mostly those in the sideline direction. Additionally, large pressure waves radiating back toward the nozzle contaminated several probe locations, but it is
doubtful these waves will influence the far-field noise due to their highly directional nature. Again, tuning the CLST method to match the noise spectra penalized the amplitude of the synthetic field to the point that the added velocity fluctuations did not significantly enhance the VLES turbulence and the LES solution was not matched.

The cost increase from CLST for these two cases was moderate, but it is cheaper than simply doubling the grid resolution of the LES solver. Furthermore, more efficient code optimization could significantly reduce the cost of CLST. Although not without shortcomings, the SEM-based CLST method shows promise at modeling a more complete jet noise spectra with only a moderate increase in cost.
CHAPTER 6
CONCLUSIONS

This dissertation investigates the commonly observed problem that LES-based jet noise simulations under-predict the highest frequency range in the jet noise spectrum. By reviewing literature on jet noise and computational noise prediction, a case is made that increasing the grid resolution of LES is too costly to resolve the missing high frequency content and that alternative, more efficient models should be developed to address this issue. A majority of this research focuses on developing and testing a framework to couple resolved velocity fluctuations with synthetic velocity fluctuations. To this end, the dissertation introduces and tests a new model called the Coupled LES-Synthetic Turbulence method (CLST).

The main idea of CLST is to resolve large and medium scale turbulent fluctuations with LES (or VLES) and to model smaller-scale turbulent fluctuations via a synthetic turbulence method (SNGR or SEM). Since fine-scale turbulence gives rise to the highest frequencies of the jet noise spectra, the synthetic turbulence serves as a model for the high-frequency noise that is often under-predicted by LES. In other words, in the CLST framework, synthetic turbulence is used to supplement the missing noise spectra. Given that synthetic turbulence methods are usually computationally inexpensive, the CLST method models the high-frequency noise spectra at a reduced computational cost compared to resolving the same frequencies with LES. In CLST, the
noise field is predicted using a formulation of the Linearized Euler equations (LEE), where the acoustic waves are generated by source terms from combining the fluctuations from VLES and the stochastic fields.

Two synthetic turbulence models were evaluated in the proposed framework. The first is a Fourier mode-based stochastic turbulence model, identified in the literature as the SNGR method (Stochastic Noise Generation and Radiation). The second is an SEM-based approach (Synthetic Eddy Method).

Large eddy simulations of Mach 0.9 jets were performed at conditions corresponding to moderately-high Reynolds numbers. Filtering the LES flow fields produced a VLES flowfield that intentionally damped the high-wavenumber fluctuations from the flow which correspond to the high-frequency range from the acoustic spectrum. Comparing LES spectra to VLES spectra quantified the missing noise that VLES did not predict. Synthetic velocities generated by either SNGR or SEM were imposed on the VLES flowfield and the resulting noise was compared to the unfiltered LES noise spectra to determine whether the higher-frequencies were supplemented by the synthetic turbulence models. Results from both the SNGR-based method (for Re = 120,000) and SEM-based method (Re = 100,000 and Re = 3,600) show that the CLST framework can supplement high frequency content in the jet and that the modeled noise spectra agree well with LES in general.

Each method was not without drawbacks and shortcomings. The SNGR-based method (called VPST in previous sections) is computationally inexpensive, but does not correctly handle convection and sweeping of the synthesized turbulence. The Fourier modes in SNGR are global and do not allow for local variations in the synthetic field that might result from sweeping. This means that adding fluctuations of a specific size to a specific region of the flow (adding small turbulent
fluctuations to the initial shear layer, for instance) is impossible. Accounting for sweeping with this method would involve solving additional equations to convect a time-weighted synthetic field, which would certainly increase the computational effort of this method. Additionally, the global Fourier modes are truncated at the edge of the synthetic source region, which could produce spurious noise. Yet another issue with this method is that it relies on the assumption of isotropic turbulence to generate divergence free turbulence. This assumption may not be valid in the jet shear layer, which would result in this method generating noise that violates the divergence-free condition on which it is built.

The main issue with the SEM-based method (called CLST in previous sections) was that the eddies appeared to generate more noise than anticipated when the intensity of the synthetic turbulent field was matched to the LES turbulence intensity. In other words, since the method was tuned to predict noise levels well, the intensity of the imposed synthetic velocities was not significant enough, and the velocity fluctuations from CLST did not approach the levels observed in the LES solution. Therefore a major issue that should be addressed in future investigations is how to match the turbulent flow quantities of LES without over-predicting the noise. The extra synthetic eddy noise could result from inserting unrealistic synthetic eddies into the flow. Additionally, distortion of the eddies due to sweeping and stretching may violate the divergence-free condition, leading to extra noise. For the SEM-based method, the prediction of higher frequencies was more consistent at various probe locations for the higher Reynolds number jet (Re = 100,000). Over-prediction of higher frequencies was observed at some sideline probe locations for the SEM-based method. Additionally, some larger-scale waves were observed radiating back towards the nozzle in results from the lower Reynolds number case (Re = 3,600).
The SEM-based method has an advantage over the SNGR-based method in that it accounts for eddy convection and sweeping (which is important to jet noise) in a more realistic manner. However, it does so at a moderate increase in the computational cost. This cost may be mitigated by more optimally managing the memory associated with calculating the synthetic velocities. While the reason for the extra cost is known, further research is required to identify a solution.

Future work will focus on further development of the SEM-based CLST method, mainly because this method includes sweeping in the synthetic velocity field. In addition to the noted shortcomings that need to be addressed, to fully evaluate the CLST method, far field noise must be investigated. This would require implementing interpolation into the CLST method so that separate NS and LEE grids could be used, with the LEE grid extended to a far field distance of 45D_j or 75D_j, which is typical for far field noise [1]. Alternatively, LEE + FW-H or Kirchhoff could be used to extend the noise predictions to the far-field. Even a volume integral acoustic analogy method, such as Lighthill’s method, could be employed because the synthetic field could be accounted for in the volume integration. A follow-up investigation would be required to determine what LEE grid resolution is needed to capture the high frequencies modeled by the synthetic turbulence. Additionally, a systematic investigation should be performed with CLST to determine how each parameter of the method influences the turbulent field and noise spectra. Parameters such as eddy lifetime, placement of eddies in the jet, the number of eddies, and the eddy amplitudes could be analyzed and optimized for a more hands-free approach. Yet another future work is to modify the random eddy sizing and placement in the flow domain in an attempt to place smaller eddies in the upstream region of the shear layer and larger eddies more downstream. Another modification could
be to consider incorporating the generation of anisotropic turbulence using Reynolds stresses from RANS [63].

In summary, a new method was introduced and tested to model high-frequency content for jet noise predictions. The results presented in this dissertation demonstrate that the missing high-frequencies can be modeled with synthetic turbulence and show that the novel concept of coupling resolved and modeled turbulence scales is a valid approach to noise prediction. The CLST method is an efficient and viable alternative to high resolution LES or DNS because it can predict the high frequency range in the acoustic noise spectrum at a reasonable expense.
REFERENCES


185


186


188


APPENDIX A

LINEARIZED EULER EQUATION VALIDATION
This section discusses the evaluation and validation of the LEE formulation as described in the Methods section. This testing was to ensure that the code was properly implemented and that the desired results were produced. Two two-dimensional cases serve as simple test cases for the propagation mechanics and source terms.

### A.1 Simple Dipole

While the LES code was validated on various test cases before, the LEE algorithm needs validation. To this end, here a 2D dipole oscillating in the x-direction only was chosen, following the implementation of Lafitte et al. [85]. The source term for the LEE x-momentum equation was as follows:

\[
S = B \cos\left(\frac{\pi x}{10}\right) \exp^{-\alpha y^2} \sin(\omega t) \tag{A.1}
\]

The LEE equations were used to propagate the sources generated by the above source. Results can be seen in Figure A.1 in the same style as shown in Figure 1 from Lafitte et al. [85]. Showing isocontours of pressure, Figure A.1 makes it clear that the implementation of the LEE propagates waves correctly. On closer inspection, the waves generated in the present simulation have a slightly shorter wavelength than those from Lafitte et al. [85]. Since the grid resolution was matched between the example case and the present simulation, there must be some slight difference in the frequency of the source term. This is confirmed in Figure A.2, which shows the pressure along the line \( y = 0 \). in a similar style to Figure 2 Lafitte et al. [85]. Counting the peaks relative to the x-axis shows that the wavelength of the present simulation is slightly shorter, further indicating
some difference in our source term implementation. However, this was deemed adequate validation since the proper qualitative wave propagation behavior was observed.

**Figure A.1**
Dipole Validation Results: Pressure Isocontours

**Figure A.2**
Dipole Validation Results: Pressure Along \( y = 0 \)
A.2 Co-Rotating Vortex System

The case of a co-rotating vortex system was investigated numerically by Bogey et al. [30]. The velocity distribution of each vortex, from the center is given as [30, 87]:

\[ V_\theta(r) = -\frac{\Gamma r}{2\pi (r_c^2 + r^2)} \]  (A.2)

Figure A.3 shows the initial velocity distribution\(^1\). Conceptually, the co-rotating case produces a quadrupole sound distribution [98, 108]. Since quadrupoles were theorized as a jet noise-generation source by Lighthill, this is an appropriate validation case for the acoustic field generated by shear layers and jet flows [87]. Furthermore, the work of Bogey et al. [30] employs the same source terms as the current formulation, providing an example case for validating the source terms, if only in two dimensions. The entire code for this validation case is simply a two-dimensional version of the three-dimensional code used for coupled LES/LEE jet noise simulations, including using the same source terms. Similar numerical techniques, such as grid stretching and sponge layers are used near the boundaries. The NS and LEE equations are solved on the same computational grid. Note that the formulation of the LEE used in this example is based on primitive variables [60], while Bogey et al. [30] use a weak conservative form of the equations.

The initial velocity distribution was set as initial conditions of the two-dimensional Navier-Stokes simulation, producing the rotating vortices. The resulting velocity field was input into the LEE source terms, and a sound field was generated. As discussed in several papers [30, 87, 98], the vortices co-rotate and produce a quadrupole sound distribution [98, 108] with a predictable acoustic frequency based on the angular rotation of the vortex pair. Following several rotation periods, the vortices then merge. Merging was not observed in the present simulation, but this was viewed as

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acceptable since it provided a longer period of time from which to extract the far-field pressure fluctuations and analyze the resulting noise.

Figure A.4 shows the LEE-calculated dilatation field from Bogey et al., \( \Theta = -1/(\rho_0 c_0^2) \frac{\partial p}{\partial t} \), with levels from -15 to +15 and results obtained from the present LEE code. The double-spiral structure from Bogey et. al’s results\(^2\) is observed in the present simulation. Slight differences exist between the two contour plots, likely due to the incipient merging of the vortices in Bogey et al.’s results. Additionally, the effects of grid stretching toward the boundaries and a sponge layer in the present simulation can be seen in the reduced intensity of the dilatation decays with increasing values of \( r/r_0 \).

Figure A.5 (a) shows dilatation profiles from Bogey et al. [30] calculated by DNS and LEE at a line through the initial starting position after around 4.7 periods of rotation\(^3\). Results from

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\(^3\)Reprinted with permission from the authors.
the present simulation are displayed for comparison in Figure A.5 (b), with results from both the Navier-Stokes simulation and the LEE simulation. Both contours display similar trends, including the number of positive and negative peaks, as well as the amplitude of those peaks. The effects of the grid stretching and sponge layer are more clearly seen in the results of the present simulation, as the dilatation smooths out for values of $x$ greater than $100r_0$. Within the results of the present simulation, with the exception of the first peak, only slight differences are observed in the NS and LEE plots.

Using pressure time-history taken at the point $(x = 25, y = 0)$, the acoustic frequency of the radiated noise was calculated. Figure A.6 shows the pressure time-history of the signal and spectral content resulting from an FFT. The time-history has been truncated at the beginning to remove the initial transient. The dominant frequency predicted by the present simulation, scaled by Strouhal
number, is 0.32 as taken from Figure A.6. This frequency closely matches the frequency predicted by the overall rotation, \( f_a = \Gamma / (4\pi^2 r_0^2) \). The pressure history also confirms the radiated frequency by the period of the pressure peaks.

Figure A.6

Sound Calculation For a 2D Co-Rotating Vortex at the Point \((x, y) = (25, 0)\)
The frequency of the smaller amplitude second peak in Figure A.6 is roughly double that of the maximum peak, so it is likely that this is a harmonic or may be an artifact of the FFT process since the pressure time-history appears fairly consistent. A closer investigation of the pressure time-history shows that the NS fluctuations have slightly lower amplitudes than the LEE fluctuations and that small phasing differences occur between the NS and LEE results within each pressure pulse. These small differences do not appear to effect the dominant acoustic frequency. The LEE source terms may be responsible for these slight differences in the pressure histories.

Overall, the present implementation of LEE accounts for the expected acoustic wave propagation behavior and predicts the correct frequency response for a co-rotating vortex pair in two dimensions. These validation results build confidence in the present LEE source terms and the accuracy of the overall LEE coding strategy.