A Dynamic Hybrid RANS/LES Modeling Methodology for Turbulent/Transitional Flow Field Prediction

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By

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A dynamic hybrid Reynolds-averaged Navier-Stokes (RANS)-Large Eddy Simulation (LES) modeling framework has been investigated and further developed to improve the Computational Fluid Dynamics (CFD) prediction of turbulent flow features along with laminar-to-turbulent transitional phenomena. In recent years, the use of hybrid RANS/LES (HRL) models has become more common in CFD simulations, since HRL models offer more accuracy than RANS in regions of flow separation at a reduced cost relative to LES in attached boundary layers. The first part of this research includes evaluation and validation of a dynamic HRL (DHRL) model that aims to address issues regarding the RANS-to-LES zonal transition and explicit grid dependence, both of which are inherent to most current HRL models. Simulations of two test cases—flow over a backward facing step and flow over a wing with leading-edge ice accretion—were performed to assess the potential of the DHRL model for predicting turbulent features involved in mainly unsteady separated flow. The DHRL simulation results are compared with experimental data, along with the computational results for other HRL and RANS
models. In summary, these comparisons demonstrate that the DHRL framework does address many of the weaknesses inherent in most current HRL models.

Although HRL models are widely used in turbulent flow simulations, they have limitations for transitional flow predictions. Most HRL models include a fully turbulent RANS component for attached boundary layer regions. The small number of HRL models that do include transition-sensitive RANS models have issues related to the RANS model itself and to the zonal transition between RANS and LES. In order to address those issues, a new transition-sensitive HRL modeling methodology has been developed that includes the DHRL methodology and a physics-based transition-sensitive RANS model. The feasibility of the transition-sensitive dynamic HRL (TDHRL) model has been investigated by performing numerical simulations of the flows over a circular cylinder and a PAK-B airfoil. Comparisons with experimental data along with computational results from other HRL and RANS models illustrate the potential of TDHRL model for accurately capturing the physics of complex transitional flow phenomena.
DEDICATION

I would like to dedicate this work to my parents M. Shamshul Alam and Khaleda Begum, along with my grandparents.
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\( d \) = wall distance

\( D_T \) = anisotropic (near-wall) dissipation term for \( k_T \)

\( D_L \) = anisotropic (near-wall) dissipation term for \( k_L \)

\( f_W \) = inviscid near-wall damping function

\( f_\omega \) = boundary layer wake term damping function in \( \omega \) equation

\( k_L \) = laminar kinetic energy

\( k_T \) = turbulent kinetic energy

\( k_{T,l} \) = effective “large-scale” turbulent kinetic energy

\( k_{T,s} \) = effective small-scale turbulent kinetic energy

\( k_{TOT} \) = total fluctuation kinetic energy

\( P_{KL} \) = production of laminar kinetic energy by mean strain rate

\( P_{KT} \) = production of turbulent kinetic energy by mean strain rate

\( R_{BP} \) = bypass transition production term

\( R_{NAT} \) = natural transition production term

\( t \) = time

\( \alpha \) = thermal diffusivity

\( \alpha_T \) = effective diffusivity for turbulence dependent variables

\( \omega \) = inverse turbulence time-scale

\( \nu \) = kinematic viscosity
CHAPTER I
INTRODUCTION

1.1 Background and Motivation

1.1.1 Turbulence Modeling Methods

Providing economic benefit and improved design reliability, computational fluid dynamics (CFD) is currently used in a wide variety of industries such as aeronautical and aerospace, biomedical, automotive, power generation, chemical processing, heating and cooling systems, meteorology, and marine systems. Despite significant progress in CFD regarding accurate geometrical representation, grid generation, robust numerical algorithms along with advanced computational resources; turbulence modeling still remains as one of the principal weaknesses in realistic CFD applications [1]. Direct Numerical Simulations (DNS), Large Eddy Simulations (LES), and Reynolds Averaged Navier-Stokes (RANS) are the three principal categories of strategies for turbulent flow simulation.

In the DNS method [2], the Navier-Stokes equations are solved directly without using any turbulence modeling. As all spatial and spectral scales of turbulence must be resolved, the DNS method provides accurate predictions but requires immense computational resources. Spatially, it requires resolving the smallest Kolmogorov scale up to the largest integral scale of flow domain. Hence, computational expense increases with increasing Reynolds number. Due to its computationally expensive nature, the DNS
method will not be feasible for industrial applications in the near future. Spalart [3] has provided an estimate that shows that the DNS method will be ready for a realistic industrial application (real world problem) around 2080.

LES models [4] apply filtering (averaging) operations to the Navier-Stokes equations to achieve resolved solutions of the large turbulent scales most responsible for momentum and energy transfer. In this approach, subgrid scale (SGS) models, based on the dependence of the smallest scale on the grid size, are required to model the unresolved smaller scales of the turbulence spectrum. LES models perform well in separated flow regions as they are capable of resolving the largest scales of turbulence that dominate momentum transfer in the flow field. Near wall performance of the LES model is problematic and depends on the amount of computational resources due to the very small length and time scales of the near wall region. As LES only resolves the larger turbulent scales and models the smaller scales, it requires significantly less computational resources than the DNS; but, this requirement is still prohibitively expensive for high Reynolds number flows [5].

In the RANS modeling approach [6], the Navier-Stokes equations are ensemble-averaged and all turbulent scales are modeled. Only the mean velocity is resolved in this approach. RANS is based on empirical or at least semi-empirical information and thus resolves less physics in comparison to DNS and LES models. As it models all turbulent scales, the RANS modeling approach requires the least computational resources and hence is widely used in industrial applications. RANS models perform well in the near-wall region due to the universality of the flow physics in the boundary layer region while
they have limitations in separated flow regions as the Reynolds-averaging process causes a loss of information [7].

The two principal challenges in turbulent flow predictions, as enumerated by Spalart [3], are: I) growth and separation of the boundary layer, and II) momentum transfer in the separated flow regions. It can be concluded from the above mentioned illustration that RANS models have capabilities to address challenge I and LES models are better suited for challenge II. However, neither type of model has thus far demonstrated the capability to address both challenges simultaneously and economically. As an alternative, Hybrid RANS/LES (HRL) modeling methodologies have been proposed, that combine the characteristics of both RANS and LES models and, thus, have the potential to resolve both challenges for turbulent flow prediction. In theory, the HRL modeling methodology offers more accuracy than RANS at a reduced cost relative to LES.

1.1.2 Current HRL Modeling

Current HRL models can be categorized as zonal or non-zonal. In a zonal model, a RANS model is employed in user-specified regions of the computational domain, and an LES model is employed in the remaining regions. The treatment of the interface between the characteristic RANS and LES regions of zonal models is problematic and remains an active area of research [8, 9]. In contrast, non-zonal methods are simpler to implement and exonerate the user from deciding where LES is to be applied in a given simulation. In general, in a non-zonal strategy, the eddy viscosity in the near-wall region adopts a value characteristic of a RANS model, while in the separated flow regions the eddy viscosity adopts a value characteristic of the subgrid stress (SGS) model in LES.
The Detached Eddy Simulation (DES) model of Spalart et al. [10] is the most widely used example of a non-zonal methodology. The switching between RANS and LES modes in the DES model is based on the local grid size, which has been shown to be problematic in an attached boundary layer [11]. Although some ad hoc modifications have been implemented to address limitations of the baseline DES model, these fixes only somewhat mitigate these issues without resolving them completely. In order to resolve the activation of grid-induced LES modes into the attached boundary layers, Spalart et al. [11] developed a modified version of the baseline DES model named the Delayed DES (DDES) model. The DDES model modifies the original definition of length scale in the baseline DES model based on the local flow and turbulence quantities. Shur et al. [12] proposed another modified version—Improved Delayed DES (IDDES)—to eliminate the “log layer mismatch” problem that occurs in the classical DES and the wall-modeled LES (WM-LES) approaches. The IDDES model acts like the DDES model except that it performs as a WM-LES type model in boundary layer regions when resolved turbulent quantities are present.

Other methods similar to the DES approach have been proposed in the literature [13-16]. Nichols and Nelson [13] developed a multi-scale model using both grid length and turbulent length scales instead of using only the grid length scale. The method incorporates Menter’s SST \( k-\omega \) [6] turbulence model. The SST model is solved using unfiltered turbulence quantities and the resulting turbulent eddy viscosity is then filtered and passed to the Navier-Stokes solver. Abe and Miyata [14] proposed an HRL modeling method using a non-linear eddy viscosity model (NLEVM) which addresses the near wall stress anisotropy issues. Girimaji [15] developed a slightly different method to bridge
RANS to DNS using two different controlling parameters, which are the ratios of the unresolved-to-total quantities for the turbulent kinetic energy and dissipation. These ratios range from zero to one, and determine the modes of the different methods to solve the flow field. In order to increase the applicability of RANS model for time dependent flow fields, Johansen et al. [16] incorporated a parameter based on the filter size for RANS based turbulence closures. In this method, the filter is used to construct the sub-filter stresses and is decoupled from the grid in such a way that the method can provide grid independent results.

1.1.3 Shortcomings of Current HRL Methods

The most critical challenge of non-zonal HRL modeling strategies is specifying the transition between RANS and LES behavior in the domain. Commonly, this zonal transition is defined such that the eddy viscosity varies between the Reynolds stress and the subgrid stress value. The Reynolds stress is based on an ensemble-averaging of all turbulent scales present in the flow field. In contrast, the subgrid stress models the turbulence scales that cannot be resolved on the grid used in the simulation. The Reynolds stress and subgrid stress are mathematically and physically different; hence, any effort to bridge these two separate effects using a single parameter (eddy viscosity) is prone to exhibit ambiguity and complexity. Several researchers identify the use of zonal transition based on only eddy viscosity as a major weakness of currently used HRL models [17-19]. Furthermore, many of the currently used HRL models adopt the local grid size as a model variable. This fact necessitates that great care be taken when building grids for HRL models, and in fact the grid must constructed with foreknowledge of the
model behavior, and used as a means of enforcing RANS-to-LES transition in the proper locations of the domain [17].

Spalart [17] denotes this transition from a purely modeled stress to a resolved dominating stress as a major concern. This problem becomes much more serious if the separation is triggered from a sharp point, and the RANS boundary layer lacks a significant level of LES content. Paterson and Peltier [18] investigated issues related to the RANS-to-LES transition in cases where no geometrically imposed separation point, such as a backward facing step flow, for example. They notice that a lag in the evolution of stress terms is introduced during the RANS-to-LES transition upstream of the separation point; hence, the resolved dominant (SGS) turbulent scales attain premature statistically averaged stress (Reynolds stress) scales. This effect, which occurs during the zonal transition, is termed “modeled-stress depletion” by Spalart et al. [17]. Nikitin et al. [19] clearly demonstrated the difficulties associated with calculating the correct grid resolution for the “gray region” where RANS and LES modes overlap in wall-bounded flows.

Several researchers have attempted to resolve this zonal transition issue [11, 12, 20-23], and some of these efforts have already been discussed [11, 12] developed the concept of the Scale-Adaptive Simulation (SAS) approach, which provides the potential to develop turbulence models that can be used in RANS and LES modes without any explicit grid-dependence. Hamba [22] suggests that the rapid variation of the filter width at the interface of RANS and LES zones is the reason for the velocity profile mismatch in the channel flow simulations, and that this issue can be resolved by incorporating an additional filter. In order to resolve the underlying issues of the a transition layer between
the RANS and LES regions, Piomelli et al [23] proposed the inclusion of a stochastic forcing function denoted as a “backscatter model” in the interface region. It must be again noted that all these attempts are properly viewed as ad hoc modifications rather than fundamental solutions to the modeling issues. Celik [24], in his turbulence modeling review, suggests that new criteria are required to resolve the RANS-to-LES transition issue in HRL models.

**Proposed Solution:** Based on the concept of a rigorous separation of the Reynolds stress and the subgrid stress, a new HRL methodology is proposed with the goal of eliminating the weakness of a zonal transition. This hybrid methodology works as a general framework to combine RANS model of any choice with any LES model.

1.1.4 Transition Modeling

Transitional flow phenomena are observed in various engineering applications including aerospace, aeronautics, biomedical, wind turbines, and aircraft turbomachinery. Transitional flow is of vital importance in aerodynamic simulations, which range from low-speed micro air vehicles to high-speed air vehicles. The inherent behavior of transitional phenomena is very complex and still remains unrevealed with respect to many physical aspects. In the advancement of CFD, extensive research has been performed in the areas of turbulence modeling and prediction. However, modeling research into transition sensitive CFD simulations is comparatively still very insignificant.

Several researchers have attempted to predict boundary layer transition using a wide variety of approaches that include Direct Numerical Simulations [25], simulations
using low Reynolds number eddy viscosity turbulence models [26-29], incorporation of an empirical correlation to a fully turbulent RANS model [30, 31], addition of transitional phenomena based transport equations to fully turbulent models [32-38] etc. For example, Kalitzin et al. [25] used a DNS method to predict the transitional phenomena for flow through a low pressure turbine blade (LPT) cascade. They attempted to attain transitional flow for different types of inlet flow conditions. Natural transition was observed near the trailing edge for turbulence free inlet flow conditions while upstream bypass transition was produced by grid turbulence and wake inlet conditions.

Due to wide range of applicability of RANS models, a number of researchers have introduced transitional flow analysis based on low Reynolds number eddy viscosity turbulence models [26, 27]. In this approach, the concept of “diffusion controlled” transition, i.e., transition triggered by the diffusion of freestream turbulence into the boundary layer, is employed [26]. Wilcox [27] used a two-equation turbulence model to predict the transitional flow behavior of incompressible flat plate simulation. In this method, two different transition specific closure coefficients were formulated using linear stability theory. Those coefficients were associated with the two-equation turbulence model for the computation. Although this transitional flow prediction approach achieved some degree of success, it has been noted that the description of such transitional flow prediction is dependent on initial condition and flow solution methods and thus this approach lacks the inherent transitional flow physics [28, 29].

Some researchers [30, 31] attempted to predict transitional flow field by coupling an empirical transition correlation to a fully turbulent RANS model. In this approach, the correlations are formulated based on experimental results. Generally, the correlations
relate turbulence intensity to the critical momentum thickness Reynolds number at which transition occurs. The method used by Dhawan and Narasimha [30] couples the transition correlation by adding a transition zone. Abu-Ghannam and Shaw [31] proposed an empirical correlation based on experiments for the transition onset location and transition length along with the boundary layer development. They coupled the correlation with the turbulence model based on the assumption that transition occurs instantaneously at a predicted onset location. Although this approach provides sufficient accuracy, its implementation is problematic in modern CFD codes. Such correlation-based transition models require the comparison between the momentum thickness Reynolds number and transition onset momentum thickness Reynolds number. In order to perform such a momentum thickness Reynolds number calculation, non-local information is required and this process becomes formidable for parallel computations of complex 3-D geometries using unstructured meshes.

Recent transition modeling approaches employ additional transport equations with the RANS-based turbulence models. Additional model terms may also be used to address the transitional behavior in the simulation. Within this recent transitional modeling approach, there are two categories: one is the physics-based modeling approach [32-35] and the other is the correlation-based modeling approach [36-38]. Edwards et al. [32] proposed a one equation transition/turbulence model that includes the blending of an eddy viscosity transport equation for non-turbulent fluctuation growth with a one equation turbulence model. An intermittency function based on the research of Dhawan and Narasimha[30] was incorporated to this method as well. Wang and Perot [33] applied additional equations for turbulence potential terms to formulate a single–point, physics-
based transition model. Walters and Laylek [34] developed a RANS-based, single point, elliptic transition model that addresses in-depth transitional flow physics and eliminates the incorporation of an intermittency factor. The newest version of this model is presented in the work of Walters and Cokljat [35]. Suzen and Huang [36] proposed a correlation-based transition model that includes a transport equation for calculation of an intermittency factor. This transport equation is incorporated into the turbulence model by modifying the definition of eddy-viscosity based on the intermittency factor. Steelant and Dick [37] developed a transport equation for the intermittency factor and incorporated it into conditioned Navier-Stokes equations. The transport equation was derived from the intermittency distribution of Dhawan and Narasimha [30]. Menter et al. [38] proposed a single-point, correlation-based transition model that includes two different transport equations: one for the intermittency factor and the other for the transition onset Reynolds number, i.e., critical momentum thickness Reynolds number. Most models within the categories of both physics- and correlation-based transition models require non-local information. As single-point transition models do not require non-local information in the simulation, they have enjoyed a wide acceptance in terms of implementation in modern CFD codes. To date, the single-point transition models of Wang and Perot [33], Walters and Laylek [34], and Menter et al. [38] have achieved wide acceptance due to their easy-to-implement nature.

1.1.5 Transition-Sensitive RANS Models in HRL Methods

Until now, HRL models have used fully turbulent models in the RANS part except a very few cases. Only one example [39] is found in the literature that used transition-sensitive HRL modeling methodology. Sorensen et al. [39] showed that an
HRL scheme with transition-sensitive RANS model performs better than an HRL scheme with a fully turbulent model in transitional flow simulations. Magagnato et al. [40] noted the importance of employing a transition-sensitive RANS model in an HRL scheme for successful prediction of transitional flows. The current trend of using fully turbulent RANS models limits the applicability of HRL models to transitional flow problems. The LES part of the HRL models is generally activated after flow separation and thus accurate prediction of the location of flow separation is highly dependent on the RANS part of the model. In general, the three categories of separated flows are: laminar separation with laminar reattachment, laminar separation with turbulent reattachment, and turbulent separation with turbulent reattachment [41]. In order to predict all types of flow separation accurately, the use of only fully-turbulent models in the RANS part is inadequate. Thus, a transition-sensitive turbulence model in the RANS part is a requirement.

1.1.6 Research Objectives

This research effort aims to develop a framework for a transition-sensitive RANS based HRL modeling methodology. As mentioned above, to date, only one effort [39] has been found which implemented a transition-sensitive RANS model in an HRL scheme. Although, this framework showed some degree of success, the transition model used in that framework itself is a correlation-based model. In that research, the correlation-based $\gamma-\tilde{R}e_\theta$ transition model of Menter et al. [38] is used with DES version of the $k-\omega$ SST model. In contrast, the present research will implement the physics based $k-k_L-\omega$ transition model of Walters and Cokljat [35] in the new HRL modeling framework. Several researchers [42, 43] demonstrated the supremacy of the $k-k_L-\omega$ transition model
over the Suzen and Huang transition model (precursor of $\gamma$-$R_e\theta$ transition model) [36], and the $\gamma$-$R_e\theta$ transition model; however, this supremacy is not universal but rather is based only on their test cases. Cutrone et al. [42] evaluated the $k$-$kL$-$\omega$ transition model as a better option than the Suzen and Huang transition model for turbomachinery flows. Genc [43] tested the performance of the $k$-$kL$-$\omega$ transition model and the $\gamma$-$R_e\theta$ transition model for the simulation of a thin airfoil in high Reynolds number flow and found that the $k$-$kL$-$\omega$ transition model performed better than the $\gamma$-$R_e\theta$ transition model for those flow problems. Moreover, it has already been shown that several issues are inherent to the DES model. The DHRL modeling methodology described earlier is designed to address those issues.

**Goal of Research:** The major motivation and goal of this research is to develop a framework that enables transition-sensitive RANS modeling in the RANS part of the new DHRL modeling methodology and, ultimately, to bridge this gap in current HRL modeling methodologies.

1.2 Contributions

A new dynamic HRL (DHRL) model proposed by Walters [44] is validated and investigated. One of the main purposes of this research is to determine if the DHRL model addresses the major issues inherent in most current hybrid RANS/LES (HRL) models, including RANS-to-LES zonal transition, explicit grid dependence, and delayed break down of separated shear layers. The newly developed DHRL model shows significant improvement toward the resolution of these issues. One of the key demonstrable benefits of the DHRL model is its potential for mitigating grid dependence issues inherent in most current HRL models with respect to the mean flow, while
allowing for increasing resolution of the turbulence in the LES mode as mesh spacing is reduced.

Furthermore, another noticeable issue associated with current HRL models is that they do not provide flexibility for transitional flow prediction, since these models were developed for fully turbulent flow simulations. A second aim of this research is to develop a transition-sensitive, RANS-based HRL modeling methodology. The transition-sensitive DHRL (TDHRL) modeling framework shows significant improvement relative to other turbulence models, including the most widely accepted HRL model and RANS model, in terms of capturing the flow physics accurately even in a highly unsteady, separated, adverse pressure-gradient-dominant flow field. In essence, the transition-sensitive DHRL (TDHRL) model demonstrates its efficacy as an alternative to hybrid RANS-LES simulations for complex problems that include flow separation and laminar-to-turbulent boundary layer transition.

Inclusion of the transition-sensitive RANS model helps the DHRL modeling framework achieve wider applicability as an HRL modeling framework for both turbulent and transitional flow field predictions. Consequently, this research effort contributes to the advancement of the state of the art in CFD simulation by providing a more flexible and universal method for hybrid RANS-LES than existing standalone models.

1.3 Publications and Presentations

A list of publications and presentations produced as the partial outcome of this research is given below.
1.3.1 Publications

1.3.1.1 Journal Publications


1.3.1.2 Conference Papers


1.3.2 Presentations


In addition to the above listed presentations, all conference publications were presented in the corresponding conferences as well.

1.4 Organization

Chapter II describes the methodology of the newly developed transition-sensitive DHRL modeling framework. The objectives of this study are briefly described in Chapter III. The investigation and validation of the DHRL model for turbulent flow using a canonical test case, the flow over a backward facing step, and a more geometrically
complex application, flow around wing with a leading edge ice accretion, are presented in Chapters IV and V, respectively. Chapter VI discusses the development and evaluation of a transition-sensitive DHRL (TDHRL) model for transitional flow field prediction. Chapter VII presents the final conclusions and major findings of this research.
CHAPTER II
METHODOLOGY

This section includes the brief description of the newly proposed Dynamic HRL (DHRL) modeling methodology which is employed in this research. The physics-based transition model, which is aimed to implement in the proposed transition-sensitive RANS modeling framework is described in this section as well.

2.1 Dynamic Hybrid RANS-LES (DHRL) Modeling Methodology [44]

For ease of presentation, the description of the DHRL model in this section focuses on single-phase, incompressible, Newtonian flow with no body forces. Applying an (undefined) filtering operation to the momentum equation yields:

\[
\frac{\partial \hat{u}_i}{\partial t} + \hat{u}_j \frac{\partial \hat{u}_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \hat{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left( 2\nu \hat{S}_{ij} \right) - \frac{\partial}{\partial x_j} \left( \tau_{ij} \right)
\]  

(2.1)

where \( u_i \) and \( \hat{u}_i \) are the instantaneous and filtered velocity, respectively. The last term on the right hand side represents the turbulent stress, corresponding in general to any residual stress obtained from either Reynolds averaging or filtering, and which can be expressed as:

\[
\tau_{ij} = u_i \hat{u}_j - \hat{u}_i \hat{u}_j
\]

(2.2)

This turbulent stress term requires modeling for closure of the momentum equation.
Generally, the hybrid RANS-LES models, including the popular DES model, incorporate a single term/parameter in the momentum equation to model the turbulent stress. This parameter takes the form of an eddy viscosity that attains a value characteristic of a modeled Reynolds stress in the RANS regions (near the wall) of the flowfield and a value characteristic of a modeled subgrid stress in the LES regions (away from the wall).

As mentioned earlier, bridging the effects of ensemble-averaged velocity fields (Reynolds stress) and spatially-filtered velocity fields (subgrid stress) with a single parameter introduces complexity and ambiguity. The DHRL modeling methodology seeks to avoid this ambiguity; and the mathematical formulation starts with the decomposition of velocity field in such a way that the effects of ensemble-averaged velocity fields and spatially-filtered velocity fields maintain a rigorous separation in the transitional or “mixed” zones.

The DHRL modeling methodology introduces a simulation-specific decomposition for the instantaneous velocity \( u_i \):

\[
  u_i = \frac{\bar{u}_i + u''_i + u'_{i}}{\bar{u}_i} \tag{2.3}
\]

where \( \bar{u}_i \) is the velocity resolved in the simulation, \( \bar{u}_i \) is the mean (Reynolds-averaged) velocity, \( u''_i \) is the resolved fluctuating velocity, and \( u'_{i} \) is the unresolved fluctuating velocity. Both the Reynolds-averaged velocity and resolved fluctuating velocity arise directly from the simulation, while the unresolved fluctuating velocity requires modeling through the turbulent stress/subfilter stress term. Substituting the decomposed instantaneous velocity \( u_i \) in Eq. (2.3) into Eq. (2.2), and assuming that the resolved and
unresolved velocity fluctuations are uncorrelated, the subfilter (residual) stress can be represented as:

$$\tau_{ij} = (\bar{u}_i \bar{u}_j - \bar{u}_i \bar{u}_j) + \bar{u}'_i \bar{u}'_j.$$  \hspace{1cm} (2.4)

The scale similarity concept has been followed to model both of the terms on the right-hand-side of Eq. (2.4), which yields an expression for the subfilter stress term as:

$$\tau_{ij} = \alpha(\bar{u}_i \bar{u}_j - \bar{u}_i \bar{u}_j) + \beta \bar{u}'_i \bar{u}'_j$$  \hspace{1cm} (2.5)

The first (both parts inside parenthesis) and the second terms on the right-hand-side of Eq. (2.5) are modeled as linear functions of the subgrid stress (SGS) and Reynolds stress, respectively, obtained from any suitable SGS and RANS model. The temporally and spatially varying proportionality constants $\alpha$ and $\beta$ are assumed to be complementary everywhere in the domain, such that the residual stress term can be modeled as the weighted average of both the SGS and RANS stress as follows:

$$\tau_{ij} = \alpha \tau_{ij}^{\text{SGS}} + (1 - \alpha) \tau_{ij}^{\text{RANS}}$$  \hspace{1cm} (2.6)

In order to determine the local value of weighing coefficient $\alpha$, a secondary filtering operation is applied, conceptually similar to the method of Lilly [4] for dynamic model coefficient evaluation. Based on the following:

$$\tau_{ij}^{\text{RANS}} = \bar{u}_i \bar{u}_j - \bar{u}_i \bar{u}_j$$  \hspace{1cm} (2.7)

$$\bar{u}'_i \bar{u}'_j = \bar{u}_i \bar{u}_j$$  \hspace{1cm} (2.8)

a secondary filter in the form of the Reynolds-averaging operation can be applied to Eq. (2.2) and combined with Eq. (2.7) to yield:

$$\tau_{ij}^{\text{RANS}} - \bar{\tau}_{ij} = (\bar{u}_i \bar{u}_j - \bar{u}_i \bar{u}_j) - (\bar{u}_i \bar{u}_j - \bar{u}_i \bar{u}_j) = \bar{u}_i \bar{u}_i - \bar{u}_i \bar{u}_j = \bar{u}'_i \bar{u}'_j$$  \hspace{1cm} (2.9)
Combining the Reynolds-averaged form of Eq. (2.6) with Eq. (2.9) to eliminate $\overline{\tau}_{ij}$, and taking the scalar product of the result with the mean (Reynolds-averaged) strain rate yields an expression for $\alpha$ as follows:

$$\alpha = \frac{\left( \frac{\overline{u_i' u_j'} S_{ij}}{\text{Resolved turbulent Production}} \right)}{\left( \frac{\tau_{ij}^{\text{RANS}} S_{ij}}{\text{RANS Production}} - \frac{\tau_{ij}^{\text{SGS}} S_{ij}}{\text{Inhomogeneous SGS Production}} \right)}$$

Equation (2.10)

The value of the coefficient $\alpha$ is based on the relative contribution to turbulence production due to the resolved scales, the mean (Reynolds-averaged) component of the subgrid model stress, and the RANS model stress. In practice, the value of $\alpha$ is limited such that $0 \leq \alpha \leq 1$. Eq. (2.10) indicates that the value of $\alpha$ becomes zero in regions with no resolved fluctuations, and thus a pure RANS mode is activated in those regions. However, if turbulent production via resolved fluctuations increases, the RANS stress contribution diminishes, and an LES subgrid stress contribution appears in the momentum equation maintaining a smooth variation of turbulent production. If the resolved turbulent production in any region is high enough, $\alpha$ obtains a value of 1, and a pure LES mode is recovered. It should also be noted that unlike most current HRL models, the DHRL methodology avoids any explicit mesh dependence in its formulation.

2.2 Transition-Sensitive RANS model

In this current research work, we are aiming to implement the $k-kL-\omega$ transition sensitive RANS model in the new HRL modeling framework. The physics based $k-kL-\omega$ transition model [35] incorporates an additional transport equation for laminar kinetic energy ($k_L$) modified forms of two-equation eddy viscosity turbulence models. The pretransitional boundary layer literally shows the nature of the laminar boundary layer based on the concept of mean velocity profile. For low freestream turbulence intensity
(less than 1%), the velocity fluctuation exhibits the nature of a Tollmien-Schlichting wave. As the freestream turbulence intensity increases, the instability increases with the high amplitude streamwise fluctuations and further increase in this fluctuation leads the termination of pretransitional boundary layer into bypass transition. This process of bypass transition from the pretransitional boundary layer is modeled through using the concept of laminar turbulent kinetic energy ($k_L$). In this $k$-$k_L$-$\omega$ transition modeling approach, the production of $k_L$ is defined as the interaction of Reynolds stresses that are associated with the pretransitional velocity fluctuations and mean shear. Laminar-to-turbulent transition is defined as the energy transfer process from laminar turbulent kinetic energy $k_L$ to turbulent kinetic energy $k_T$ and the transition onset location is based on local flow condition i.e. a single-point. The total fluctuation energy is described in the model as the sum of $k_L$ and $k_T$. A critical value of the ratio of turbulent production time-scale to the molecular diffusion time-scale regulates the onset of transition. The ratio of Tollmien-Schlichting time-scale to the molecular diffusion time-scale is defined as the criterion for natural transition.

The complete presentation of all model equations is not included here due to their availability in the literature. Three additional transport equations for $k_L$, $k_T$, and $\omega$ are solved in this model implementation:

$$\frac{Dk_T}{Dt} = P_{k_T} + R_{BP} + R_{NAT} - \omega k_T - D_T + \frac{\partial}{\partial x_j} \left[ \left( v + \frac{\sigma_T}{\sigma_k} \right) \frac{\partial k_T}{\partial x_j} \right] \quad (2.11)$$

$$\frac{Dk_L}{Dt} = P_{k_L} - R_{BP} - R_{NAT} - D_L + \frac{\partial}{\partial x_j} \left[ v \frac{\partial k_L}{\partial x_j} \right] \quad (2.12)$$
The total fluctuation kinetic energy is defined as:

\[ k_{TOT} = k_T + k_L \]  \hspace{1cm} (2.14)

The production terms for turbulent and laminar kinetic energy are defined as:

\[ P_{k_T} = \nu_{T,s}S^2 \]  \hspace{1cm} (2.15)
\[ P_{k_L} = \nu_{T,l}S^2 \]  \hspace{1cm} (2.16)

The implementation of this transition model is performed in the two-equation eddy viscosity RANS turbulence framework. When the Reynolds stress terms are very small, the flow field becomes laminar in the simulation. The eddy viscosity is computed by solving two additional transport equations for general two-equation models. In this approach, the \( k-\omega \) RANS form is applied where the additional transport equations for \( k \) and \( \omega \) are solved.
CHAPTER III

OBJECTIVES

The principal objective of this research effort is to develop a dynamic hybrid RANS/LES modeling framework, capable of improving turbulent and transitional flow field predictions. The particular objectives of this research are:

- Investigation of a new hybrid RANS/LES (HRL) modeling framework [44] that is capable of coupling any choice of RANS model with an arbitrary LES model.

- The new HRL modeling framework will be validated for fully turbulent flow predictions performing simulations for canonical and engineering-relevant test cases. The potential of the new HRL model will be assessed in comparison with other HRL and RANS models as well.

- Development of a transition-sensitive hybrid RANS/LES model that includes a physics-based transition-sensitive RANS model in the new HRL modeling framework.

- The new transition-sensitive HRL model will be tested and evaluated for transitional flow field predictions performing simulations for canonical and engineering-relevant test cases.
CHAPTER IV
INVESTIGATION AND VALIDATION OF A DYNAMIC HYBRID RANS/LES MODELING METHODOLOGY FOR SEPARATED FLOWS

The work reported in this chapter has been submitted for publication in the ASME Journal of Fluids Engineering.

4.1 Introduction

Accurate prediction of both attached and separated turbulent flows is important as they are commonly observed in a wide range of engineering applications. In general, Reynolds-averaged Navier-Stokes (RANS) models perform well in attached turbulent boundary layers [3] due to the somewhat universal nature of the turbulence observed in wall-bounded flows. However, RANS models generally perform more poorly in regions of separated flow, where adverse pressure gradients, reattachment of turbulent shear layers, strong three-dimensionality, and high levels of unsteadiness are present [7, 45-50]. In theory, Large Eddy Simulation (LES) models offer more accuracy than the RANS approach in separated flow regions, although their application to boundary layer regions are often problematic in terms of computational expense. Despite some degree of feasibility for industrial applications, the LES approach is still prohibitively expensive for high Reynolds number flows, especially those with wall-bounded effects [5]. The hybrid RANS/LES (HRL) [3] approach is potentially an attractive and viable alternative to
RANS and/or LES alone, since an HRL model seeks to combine the characteristics of both the RANS and LES models in an optimized fashion to effectively resolve both attached boundary layer and separated flows. In short, the HRL modeling methodology has the potential to be more accurate than RANS and less expensive than LES. For this reason interest in HRL methods has grown significantly over the last decade.

HRL models can be categorized as zonal or non-zonal. In a zonal model, a RANS model is employed in user-specified regions of the computational domain, and an LES model is employed in the remaining regions. The treatment of the interface between the characteristic RANS and LES regions of zonal models is problematic and remains an active area of research [8-9]. In contrast, non-zonal methods are simpler to implement and exonerate the user from deciding where LES is to be applied in a given simulation. In general, in a non-zonal strategy, the eddy viscosity in the near-wall region adopts a value characteristic of a RANS model, while in the separated flow regions the eddy viscosity adopts a value characteristic of the subgrid stress (SGS) model in LES. The Detached Eddy Simulation (DES) model of Spalart et al. [10] is the most widely used example of a non-zonal methodology. The switching between RANS and LES modes in the DES model is based on the local grid size, which has been shown to be problematic in the attached boundary layer [11]. Although some ad hoc modifications have been implemented to address limitations of the baseline DES model, these fixes mitigate these issues while not resolving them completely. In order to resolve the activation of grid-induced LES modes into the attached boundary layers, Spalart et al. [11] developed a modified version of the baseline DES model named the Delayed DES (DDES) model. The DDES model modifies the original definition of length scale in the baseline DES
model, based on the local flow and turbulence quantities. Shur et al. [12] proposed another modified version—Improved Delayed DES (IDDES)—to eliminate the “log layer mismatch” problem that occurs in the classical DES and the wall-modeled LES (WM-LES) approaches. The IDDES model acts like the DDES model except that it performs as a WM-LES type model in boundary layer regions when resolved turbulent quantities are present.

The most critical challenge of non-zonal HRL modeling strategies is specifying the transition between RANS and LES behavior in the domain. Commonly, this zonal transition is defined such that the eddy viscosity varies between the Reynolds stress and the subgrid stress value. The Reynolds stress is based on an ensemble-averaging of all turbulent scales present in the flow field. In contrast, the subgrid stress models the turbulence scales that cannot be resolved on the grid used in the simulation. The Reynolds stress and subgrid stress are mathematically and physically different; hence, any effort to bridge these two separate effects using a single parameter (eddy viscosity) is prone to exhibit ambiguity and complexity. Several researchers identify the use of zonal transition based on only eddy viscosity as a major weakness of currently used HRL models [17-19]. Furthermore, many of the currently used HRL models adopt the local grid size as a model variable. This fact necessitates that great care be taken when building grids for HRL models, and in fact the grid must be constructed with foreknowledge of the model behavior, and used as a means of enforcing RANS-to-LES transition in the proper locations of the domain [17].

Spalart [17] denotes this transition from a purely modeled stress to a resolved dominating stress as a major concern. This problem becomes much more serious if the
separation is triggered from a sharp point, and the RANS boundary layer lacks a significant level of LES content. Paterson and Peltier [18] investigated issues related to the RANS-to-LES transition in cases where no geometrically imposed separation point as in a backward facing step flow, for example. They notice that a lag in the evolution of stress terms is introduced during the RANS-to-LES transition upstream of the separation point; hence, the resolved dominant (SGS) turbulent scales attain premature statistically averaged stress (Reynolds stress) scales. This effect that occurs during the zonal transition is termed “modeled-stress depletion” by Spalart et al. [11]. Nikitin et al. [19] clearly demonstrated the difficulties associated with calculating the correct grid resolution for the “gray region” where RANS and LES modes overlap in wall-bounded flows.

Several researchers have attempted to resolve this zonal transition issue [11, 12, 20-23], and some of these efforts have already been discussed [11, 12]. Menter et al. [20, 21] developed the concept of the Scale-Adaptive Simulation (SAS) approach, which provides the potential to develop turbulence models that can be used in RANS and LES modes without any explicit grid-dependence. Hamba [22] suggests that the rapid variation of the filter width at the interface of RANS and LES zones is the reason for the velocity profile mismatch in the channel flow simulations, and that this issue can be resolved by incorporating an additional filter. In order to resolve the underlying issues of the a transition layer between the RANS and LES regions, Piomelli et al. [23] proposed the inclusion of a stochastic forcing function denoted as a “backscatter model” in the interface region. It must be again noted that all these attempts are properly viewed as ad hoc modifications rather than fundamental solutions to the modeling issues. Celik [24], in
his turbulence modeling review, suggests that new criteria are required to resolve the RANS-to-LES transition issue in HRL models.

The motivation for developing the dynamic hybrid RANS-LES (DHRL) modeling methodology presented in this paper is to resolve the transition weaknesses and explicit grid dependence issues inherent in most current HRL models, assuming that these issues are fundamental in nature and not likely to be resolved by ad hoc modifications. The key features of this new DHRL modeling methodology are as follows: 1) this approach is a general framework that enables coupling of any given RANS model with any given LES model; 2) the approach is free from any explicit grid dependence in the model formulation; 3) the zonal transition between RANS and LES modes is based on the continuity of total turbulence production; and 4) this approach exactly reproduces the baseline RANS model simulation result in numerically steady-state flows.

In this study, a detailed evaluation of the DHRL model is performed using the finite-volume based commercial solver Ansys FLUENT® version 14.0. The DHRL model was coupled to the solver in a segregated fashion using FLUENT’s native User-Defined Function (UDF) capability. In the DHRL model described here, Menter’s SST $k$-$\omega$ model [6] was used as the RANS component, while the monotonically-integrated LES (MILES) model [51] was used as the LES component. In order to assess the viability of the DHRL model, the test case considered here was backstep flow simulations corresponding to the experimental study of Driver and Seegmiller [52]. The DHRL simulation results are compared with experimental data, along with the computed results of simulations using other RANS and HRL models available in the FLUENT flow solver.
4.2 Model Formulation

For completeness, the mathematical formulation of the DHRL model is presented in this section. The readers are also referred to Bhushan et al. [53] for a detailed description of the DHRL modeling methodology. In order to investigate the characteristics of DHRL it is compared to other turbulence models, including DDES and SST $k$-ω. Likewise, the SST $k$-ω model is used as the RANS component of the DHRL model and MILES is used as the LES component. The DDES and SST $k$-ω models were chosen for comparison purposes because these models are widely used HRL and RANS models, respectively. Because they have been well documented elsewhere, only a brief description, with appropriate references, is presented in subsections 4.2.1 to 4.2.4.

4.2.1 Dynamic Hybrid RANS-LES (DHRL) Methodology

For ease of presentation, the description of the DHRL model in this section focuses on single-phase, incompressible, Newtonian flow with no body forces. Applying an (undefined) filtering operation to the momentum equation yields:

$$\frac{\partial \hat{u}_i}{\partial t} + \hat{u}_j \frac{\partial \hat{u}_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left(2\nu \hat{S}_{ij}\right) - \frac{\partial}{\partial x_j} (\tau_{ij})$$

(4.1)

where $u_i$ and $\hat{u}_i$ are the instantaneous and filtered velocity, respectively. The last term on the right hand side represents the turbulent stress, corresponding in general to any residual stress obtained from either Reynolds averaging or filtering, and which can be expressed as:

$$\tau_{ij} = \hat{u}_i \hat{u}_j - \hat{u}_i \hat{u}_j$$

(4.2)

This turbulent stress term requires modeling for closure of the momentum equation.
Generally, the hybrid RANS-LES models, including the popular DES model, incorporate a single term/parameter in the momentum equation to model the turbulent stress. This parameter takes the form of an eddy viscosity that attains a value characteristic of a modeled Reynolds stress in the RANS regions (near the wall) of the flowfield and a value characteristic of a modeled subgrid stress in the LES regions (away from the wall).

As mentioned earlier, bridging the effects of ensemble-averaged velocity fields (Reynolds stress) and spatially-filtered velocity fields (subgrid stress) with a single parameter introduces complexity and ambiguity. The DHRL modeling methodology seeks to avoid this ambiguity; and the mathematical formulation starts with the decomposition of velocity field in such a way that the effects of ensemble-averaged velocity fields and spatially-filtered velocity fields maintain a rigorous separation in the transitional or “mixed” zones.

The DHRL modeling methodology introduces a simulation-specific decomposition for the instantaneous velocity \( u_i \):

\[
u_i = \frac{\bar{u}_i + u''_i}{\bar{u}_i} + u'_i\tag{4.3}\]

where \( \hat{u}_i \) is the velocity resolved in the simulation, \( \bar{u}_i \) is the mean (Reynolds-averaged) velocity, \( u''_i \) is the resolved fluctuating velocity, and \( u'_i \) is the unresolved fluctuating velocity. Both the Reynolds-averaged velocity and resolved fluctuating velocity arise directly from the simulation, while the unresolved fluctuating velocity requires modeling through the turbulent stress/subfilter stress term. Substituting the decomposed instantaneous velocity \( u_i \) in Eq. (4.3) into Eq. (4.2), and assuming that the resolved and
unresolved velocity fluctuations are uncorrelated, the subfilter (residual) stress can be represented as:

$$\tau_{ij} = (\bar{u}_i \bar{u}_j - \hat{u}_i \hat{u}_j) + \bar{u}_i' u'_j. \quad (4.4)$$

The scale similarity concept has been followed to model both of the terms on the right-hand-side of Eq. (4.4), which yields an expression for the subfilter stress term as:

$$\tau_{ij} = \alpha (\bar{u}_i \bar{u}_j - \hat{u}_i \hat{u}_j) + \beta \bar{u}_i' u'_j \quad (4.5)$$

The first (both parts inside parenthesis) and the second terms on the right-hand-side of Eq. (4.5) are modeled as linear functions of the subgrid stress (SGS) and Reynolds stress, respectively, obtained from any suitable SGS and RANS model. The temporally and spatially varying proportionality constants $\alpha$ and $\beta$ are assumed to be complementary everywhere in the domain, such that the residual stress term can be modeled as the weighted average of both the SGS and RANS stress as follows:

$$\tau_{ij} = \alpha \tau_{ij}^{SGS} + (1 - \alpha) \tau_{ij}^{RANS} \quad (4.6)$$

In order to determine the local value of weighing coefficient $\alpha$, a secondary filtering operation is applied, conceptually similar to the method of Lilly [4] for dynamic model coefficient evaluation. Based on the following:

$$\tau_{ij}^{RANS} = \bar{u}_i \bar{u}_j - \bar{u}_i \bar{u}_j \quad (4.7)$$

$$\bar{u}_i \bar{u}_j = \bar{u}_i \bar{u}_j \quad (4.8)$$

a secondary filter in the form of the Reynolds-averaging operation can be applied to Eq. (4.2) and combined with Eq. (4.7) to yield:

$$\tau_{ij}^{RANS} - \bar{\tau}_{ij} = (\bar{u}_i \bar{u}_j - \bar{u}_i \bar{u}_j) - (\bar{u}_i \bar{u}_j - \bar{u}_i \bar{u}_j) = \bar{u}_i \bar{u}_i - \bar{u}_i \bar{u}_j = \bar{u}_i' u'_j' \quad (4.9)$$
Combining the Reynolds-averaged form of Eq. (4.6) with Eq. (4.9) to eliminate $\bar{\tau}_{ij}$, and taking the scalar product of the result with the mean (Reynolds-averaged) strain rate yields an expression for $\alpha$ as follows:

$$
\alpha = \left( \frac{u_{i}''u_{i}''S_{ij}}{R_{i}^{\text{resolved turbulent production}}} - \frac{\tau_{ij}^{\text{RANS}}S_{ij}}{R_{i}^{\text{RANS production}}} - \frac{\tau_{ij}^{\text{SGS}}S_{ij}}{R_{i}^{\text{inhomogeneous SGS production}}} \right) - \bar{\tau}_{ij}
$$

(4.10)

The value of coefficient $\alpha$ is based on the relative contribution to turbulence production due to the resolved scales, the mean (Reynolds-averaged) component of the subgrid model stress, and the RANS model stress. In practice, the value of $\alpha$ is limited such that $0 \leq \alpha \leq 1$. Eq. (4.10) indicates that the value of $\alpha$ becomes zero in regions with no resolved fluctuations, and thus a pure RANS mode is activated in those regions. However, if turbulent production via resolved fluctuations increases, the RANS stress contribution diminishes, and an LES subgrid stress contribution appears in the momentum equation maintaining a smooth variation of turbulent production. If the resolved turbulent production in any region is high enough, $\alpha$ obtains a value of 1, and a pure LES mode is recovered. It should also be noted that unlike most current HRL models, the DHRL methodology avoids any explicit mesh dependence in its formulation.

The final aspect of the DHRL methodology concerns the computation of the RANS model component. In contrast to most other hybrid methods, the DHRL approach computes the RANS terms based solely on the Reynolds-averaged flowfield. In stationary flows, for example, the velocity field used to compute all RANS terms is obtained from a running time-average. Other appropriate averaging methods can be adopted for other flows. For the current study, stationary flows are considered and the RANS model is computed using the time-averaged flowfield.
4.2.2 Delayed Detached-Eddy Simulation (DDES)

As mentioned above, Spalart et al. [11] proposed the DDES model as a modified version of the baseline DES [10] model to mitigate the problem of grid-induced activation of the LES mode in attached boundary layers. This anomalous LES switching occurs in the DES model if a highly refined grid is used. Unlike the baseline DES model, the DDES model seeks to re-define the length scale \( \tilde{d} \) in such a way that it depends not only on the grid but also on the eddy-viscosity field:

\[
\tilde{d} = d - f_d \max \left( 0, d - c_{DES}\Delta \right) \tag{4.11}
\]

The function \( f_d \) attains a value of zero within the boundary layer ensuring the activation of RANS mode; however, the value of \( f_d \) equals unity outside the boundary layer recovering the baseline DES mode. The function \( f_d \) is expressed as:

\[
f_d = 1 - \tanh([8r_d]^{3}) \tag{4.12}
\]

where, the expression of the function \( r_d \) is as follows:

\[
r_d = \frac{v_t + v}{\sqrt{U_{ij}U_{ij}k^2d^2}}. \tag{4.13}
\]

4.2.3 Shear-Stress Transport (SST) \( k-\omega \)

The Shear-Stress Transport (SST \( k-\omega \)) model formulated by Menter [6] is based on the transport of the principal shear stress to facilitate the prediction of adverse pressure-gradient-dominant flows. It has been widely and successfully used for practical RANS CFD simulation of complex turbulent flows [54]. The eddy viscosity of the baseline \( k-\omega \) model is re-defined within the framework of the SST model as follows:

\[
v_t = \frac{a_1k}{\max(a_1\omega, Sr_2)} \tag{4.14}
\]
where, $F_2$ is a blending function, $\alpha_1$ is a constant, and $S$ represents an invariant measure of the strain rate magnitude. $F_2$ obtains a value of unity for boundary-layer flows, while it attains a value of zero for free shear layers. In this model, production of turbulent kinetic energy is larger than dissipation in the adverse pressure gradient boundary layer.

Two transport equations, one for the turbulent kinetic energy ($k$) and the other for the specific turbulence dissipation rate ($\omega$), are incorporated into this SST modeling framework as follows:

$$\frac{\partial \rho k}{\partial t} = \tau_{ij} \frac{\partial u_i}{\partial x_j} - \beta^* \rho \omega k + \frac{\partial}{\partial x_j} \left[ \left( \mu + \sigma_k \mu_t \right) \frac{\partial k}{\partial x_j} \right] \tag{4.15}$$

$$\frac{\partial \rho \omega}{\partial t} = \frac{\gamma}{\nu_t} \tau_{ij} \frac{\partial u_i}{\partial x_j} - \beta \rho \omega^2 + \frac{\partial}{\partial x_j} \left[ \left( \mu + \sigma_\omega \mu_t \right) \frac{\partial \omega}{\partial x_j} \right] + 2 \left(1 - F_1 \right) \rho \sigma_\omega \omega \frac{1}{\omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j} \tag{4.16}$$

The blending function $F_1$ plays a similar role as $F_2$, serving as an indicator function for near-wall and farfield regions of the flow. Near the wall, $F_1 = 1$, and a $k$-$\omega$ model form is recovered. Far from the wall, $F_1 \to 0$ and the model operates similar to a $k$-$\varepsilon$ model form. Readers are referred to [6] for further details. For the DHRL model implemented here, the above equations were used for the RANS component, with the difference that all terms containing constructions of the velocity field—including convection, production, and model coefficients—were computed using Reynolds-averaged values rather than resolved instantaneous values.
4.2.4 Monotonically-Integrated Large Eddy Simulation (MILES)

MILES is generally understood to refer to the approach to LES first proposed by Boris et al.[55], in which the dissipation inherent in the discretization error for the convective term serves as an implicit model for the subgrid stress tensor. It has been shown analytically that the numerical error is mathematically similar to an explicit eddy-viscosity model in such an approach [51], and a number of studies have appeared in the literature for which practical LES solutions have been successfully obtained. In the current study, the MILES method is used for reference LES simulations and as the LES component for the DHRL implementation. Two convective discretization schemes are investigated (discussed in the following section), both of which are designed to preserve monotonicity through upwinding and flux limiting, which meets the requirement for a MILES type approach.

4.3 Numerical Method

All simulations were run using the commercial finite-volume CFD solver Ansys FLUENT. The DHRL hybrid RANS-LES methodology was implemented using User-Defined Function subroutines. All other turbulence models used were available as built-in modeling options in FLUENT. Because the test cases considered are incompressible, the segregated pressure-based solver option was used with the SIMPLE method [56] for pressure-velocity coupling. A second-order implicit (three-point backward difference) method was used for the temporal discretization in all transient simulations. In all simulations, a second-order upwind, linear reconstruction scheme with Least-Squares gradient computation and slope limiting [57] was used as the baseline method for discretization of the convective terms in the momentum equations. Convective fluxes
were computed using momentum-weighted interpolation following Rhie and Chow [58].
For some cases, a less dissipative Bounded Central Differencing spatial reconstruction
scheme based on the Normalized Variable Diagram [59] was also used, in order to
evaluate the influence of discretization scheme on the results. Second-order centered
reconstruction was used for the pressure terms, and second-order central differencing was
used for the diffusion terms.

Steady RANS simulations were performed until a converged solution was
obtained, based on stabilization of all flow variables with respect to time, and a reduction
in the $L_2$ norm of all residuals to at least six orders of magnitude less than the initial
value. Furthermore, for the hybrid RANS-LES (HRL) models, the converged steady-state
result was used as the initial condition for the time-dependent HRL simulations. The
HRL model simulations were unsteady and were performed until a statistically steady-
state flow field was obtained. An appropriate time-step size for the transient simulations
was chosen to maintain an approximate maximum CFL number of unity, based on the
smallest streamwise mesh dimension and the freestream velocity. For the unsteady cases,
it was verified that the $L_2$ norm of all residuals were reduced by at least three orders of
magnitude during each time step. Furthermore, an HRL simulation with the time-step size
reduced by half was run and compared to the default time step results. No significant
change was seen in statistical quantities, which verified that the use of a maximum CFL
of approximately unity was appropriate.

4.4 Mesh Generation and Flow Conditions

Two different types of mesh were generated using the commercial meshing tool
Ansys GAMBIT® to investigate grid sensitivity issues inherent in most current HRL
modeling approaches [17], specifically the delay of shear layer breakup. All meshes used were three-dimensional. The same computational domain was used for each grid. The domain size extended 4 step heights (H) upstream of the step, 32H downstream of the step, 16H vertically from the wall in the downstream side of the step, and 6H in the spanwise direction. A structured multi-block meshing method was employed to generate each grid. The baseline coarse mesh contained 744,960 total cells, and the refined mesh consisted of a reasonably well magnified 7,946,400 total cells. An average $y^+$ value of less than unity was maintained to satisfy the recommended $y^+$ value for the RANS turbulence model used in this study. Isotropic quadrilateral cells were maintained in the LES region (from $x/H = 0.0$ to $x/H = 10.0$). The planar mesh for a 2D (x-y plane) slice of the refined mesh is shown in Figure 4.1.

![Figure 4.1: 2D planar mesh representation (refined mesh).](image)
Simulations using the DHRL, DDES, and SST $k-\omega$ models were performed matching all simulation parameters with the experimental data. As discussed above, since RANS models are currently the most popular for practical flow computations, steady simulations with the SST $k-\omega$ model were performed to evaluate the performance of the HRL models compared to the industry-standard steady RANS applications.

The profiles of inlet flow variables such as mean inflow velocity, turbulent kinetic energy and specific dissipation rate for the DHRL and RANS model simulations, and the modified eddy viscosity for the DDES model simulation were selected to match the experimental data. For the spanwise side boundaries, a periodic boundary condition was used in the transient simulations, while a symmetry boundary condition was used in the steady RANS simulation.

4.5 Results

The test case considered for the separated flow field investigation is backward facing step flow matching the experimental measurements of Driver and Seegmiller [52], which is a widely used benchmark test case for turbulence model validation.

Figures 4.2 (a) and 4.2 (b) illustrate the mean wall static pressure distribution along the streamwise direction, obtained from the baseline coarse mesh and the refined mesh, respectively. The pressure coefficient ($C_p$) values shown here were obtained from the spanwise-averaged mean pressure values. In the recirculation region ($x/H > 0$), the baseline coarse mesh computations in Figure 4.2 (a) show that both the DHRL and RANS computations exhibit a smooth pressure decrease and capture the negative peak pressure reasonably well in comparison to the experimental data. The DDES results show an overpredicted pressure decrease and an offset in negative peak pressure. The
streamwise pressure rise in the separated flow region predicted by both the DHRL and RANS computations shows a similar behavior to that of the experimental measurements, while the wall pressure in the DDES computation shows a delayed pressure recovery. The pressure distribution after the flow reattachment shown by all computations is similar to the experimental behavior, except for a small overprediction shown by the DDES model in the region $10.0 < x/H < 16.0$. Figure 4.2 (b) shows that the refined mesh computations using all three models agree similarly with the experimental data. Results obtained using the refined mesh show a significant improvement relative to the baseline mesh in the DDES model computations. These differences demonstrate the mesh sensitivity issue inherent in the DDES model. In contrast, both the baseline and refined mesh predictions made using the DHRL model show similar results, which implies that the DHRL model is relatively insensitive to mesh resolution, in agreement with the results from the attached flow test case above.
Figure 4.2  Wall static pressure distribution for (a) coarse baseline mesh and (b) refined mesh.
Figures 4.3 (a) and 4.3 (b) show the mean wall skin friction coefficient (Cf) distribution along the streamwise direction, computed on the coarse baseline mesh and the refined mesh, respectively. The predicted Cf values shown here are spanwise-averaged, though little spanwise variation was observed in the converged results. Figure 4.3 (a) shows that the wall skin friction in the separated flow region predicted by the DHRL model and the RANS model agrees with the experimental data. The negative peak Cf values captured by the DHRL and RANS models are quite close to the experimental value. In comparison, the DDES model prediction shows a larger flow separation region along with an overpredicted value of Cf. The flow reattachment location is defined by the region downstream of the step, where the negative Cf value indicating the separated flow subsequently attains a positive value. The measured flow reattachment location obtained using a linear-interpolation of the oil-flow laser skin-friction measurements is approximately x/H = 6.38 [52]. The DHRL model computation shows an earlier flow reattachment and predicts the reattachment location at approximately x/H = 5.60, an underprediction of 9%. The RANS model predicts the flow reattachment location at x/H = 6.30, which is closer to the experimental measurements. The DDES model computation shows a delayed flow reattachment at x/H = 9.23. The mesh is apparently too coarse for the DDES model to resolve the Reynolds stress contribution and eventually produces the delayed flow reattachment. The Cf value is predicted reasonably well by the DHRL model downstream of the flow reattachment location. Additionally, the prediction in the farthest downstream region indicates a steady decay in the turbulent shear-stress, which is also demonstrated by the experimental measurements. In contrast, the DDES model computation shows a clear mismatch in the Cf prediction after the flow reattachment.
Figure 4.3  Skin-friction distribution for (a) coarse baseline mesh and (b) refined mesh.
and an overprediction in the farthest downstream region. The RANS model predicts $C_f$ well just after the flow reattachment but shows an underprediction in the subsequent downstream regions. The behavior of the DHRL model as compared to the SST model, including shorter prediction of reattachment length and increased wall shear stress downstream of reattachment, can be attributed to the presence of resolved turbulent fluctuations that lead to more rapid mixing of the separated shear layer and transport of momentum towards the wall. This is more clearly seen in the mean velocity predictions presented below.

The mean wall skin-friction distribution obtained using the refined mesh, depicted in Figure 4.3 (b), demonstrates that all computed results agree quite well with the experimental data. However, in the context of mesh sensitivity, not all of the models show similar qualitative or quantitative features for the coarse and refined meshes. An apparently mesh-independent behavior is evident in the DHRL model predictions, since the $C_f$ calculations for both the coarse and refined meshes are very similar. The reattachment location ($x/H = 5.70$ approx.) calculated using the refined mesh is less than 2% more than coarse mesh calculation. The RANS model predictions of $C_f$ values for both the coarse and refined meshes show excellent agreement in all regions along with the flow reattachment location. This is similar to the attached flow test case above, for which the steady RANS results were nearly essentially mesh independent. In contrast, a substantial mesh sensitivity is observed in the DDES model predictions. $C_f$ values in almost all regions show significant differences between the coarse and fine mesh calculations. Mesh refinement improves the flow reattachment prediction ($x/H = 6.31$) of the DDES model significantly as well.
The literature [45, 60] reports that delayed shear layer breakup and poor resolution of the Reynolds stresses in separated flow regions are evident in the DDES model predictions, especially when the grid is insufficiently refined. One of the advantages of the DHRL model is that it addresses these issues inherently. Figures 4.4 (a) and 4.4 (b) show the instantaneous spanwise vorticity (z-vorticity) contours computed from the coarse mesh DHRL and DDES model simulations, respectively. The DHRL model results show more potential to capture turbulent eddies than the DDES model. Moreover, a more conspicuous shear layer breakup is present in the DHRL model predictions than in the DDES results. This explains the poor wall static pressure and skin friction predictions produced by the DDES model and the much better performance of the DHRL model. Figures 4.5 (a) and 4.5 (b) show the instantaneous spanwise vorticity (z-vorticity) contours obtained from finer mesh simulations for the DHRL and DDES models, respectively. Both models capture turbulent scales well and consequently predict $C_p$ and $C_f$ well. In the context of mesh sensitivity, the DHRL model contours are qualitatively similar on both the coarse and finer mesh computations except for the resolution of smaller scales on the refined mesh. In contrast, the DDES model contours show significant qualitative differences between coarse and fine meshes in both the turbulent scales that are resolved and the shear layer breakup. Figures 4.6 (a) and 4.6 (b) show the spanwise vorticity (z-vorticity) contours obtained from the RANS simulations on the baseline coarse mesh and refined meshes, respectively. Clearly, these contours show no turbulent scale prediction in the flow field and only show very insubstantial fluctuation in shear layers close to the recirculation region. This indicates that actual
unsteady behavior of the flow field is not present in the RANS simulation, which is evident in the HRL model simulations.

Figure 4.4  Contours of instantaneous z-vorticity computed from baseline coarse mesh using (a) DHRL and (b) DDES model.
Figure 4.5  Contours of instantaneous z-vorticity computed from refined mesh using (a) DHRL and (b) DDES model.
To minimize the dissipation errors in the solution, low numerical diffusion schemes are generally recommended for the discretization of the convective terms in the momentum equations for LES and HRL modeling methodologies. The bounded central differencing (BCD) [59] method is one such low-diffusion scheme that provides stability as well. Simulations on the refined mesh using the BCD scheme were performed, and the results were compared with the baseline second-order upwind simulation results to investigate the effect of discretization on HRL model solutions. Figure 4.7 shows the $C_p$ distribution along the streamwise direction, computed on the refined mesh using both the BCD and second-order upwind discretization schemes. Apparently identical $C_p$ predictions are observed in the DHRL model simulations for both discretization schemes. In contrast, the DDES simulation results for both schemes show a small but clear
deviation in the recirculation region. Compared to the upwind scheme predictions, the BCD discretization method produces a slightly overpredicted negative $C_p$ peak and a delayed pressure recovery for the DDES model.

![Figure 4.7](image)

**Figure 4.7** Wall static pressure distribution computed from refined mesh using BCD and second-order upwind discretization scheme.

Figure 4.8 shows the $C_f$ predictions for both HRL models using the refined mesh for both discretization schemes. Results from both the BCD and upwind schemes show similar behavior for the DHRL simulations, except a small improvement is observed in the BCD scheme predictions in the downstream region after flow reattachment. The flow reattachment location ($x/H = 5.75$) predicted by the BCD scheme simulation is also close to the upwind scheme simulation prediction ($x/H = 5.70$). Compared to the upwind scheme prediction, the BCD scheme simulation for the DDES model shows a streamwise
offset in $C_r$ prediction in the separated flow regions. The BCD scheme also exhibits a much-delayed flow reattachment prediction ($x/H = 6.80$ approx.) compared to the upwind scheme ($x/H = 6.31$ approx.). In summary, the effect of using the BCD discretization scheme instead of a second-order upwind scheme in the DHRL simulations is minimal although the BCD scheme does improve the solution slightly. In the DDES simulations, using the BCD scheme instead of the second-order upwind scheme deteriorates the results, which was not expected. The reason behind this unexpected behavior might stem from the violation of the convection boundedness criterion of the BCD method due to the very low sub-grid scale turbulent diffusivity produced by the DDES model simulations.

Figure 4.8  Skin-friction distribution computed from refined mesh using BCD and second-order upwind discretization scheme.
Figures 4.9 (a-i) show the mean streamwise velocity profiles at different stations along the streamwise direction obtained using the refined mesh. Velocity profiles in the flow reversal region (at stations $x/H = 1.0$, 1.5, 2.0, and 3.0) show that characteristics such as the boundary layer growth close to the wall and the separation-bubble size in the wall normal direction are well captured by the DHRL model and RANS model simulations relative to the experimental measurements. In contrast, the DDES model simulation underpredicts the negative velocity peak in the near wall region and overpredicts the separation-bubble size. All model predictions agree well with the experimental data away from the wall. At station $x/H = 5.0$, just before the flow reattachment location, computational results for the DHRL and RANS model simulations show reasonably good agreement with the experimental measurements in terms of the negative velocity peak and separation-bubble size (in the wall normal direction) predictions, while the DDES model overpredicts the flow behavior. It is evident that in the separated flow region near the wall, the DHRL model results show good performance and a RANS-like behavior, which demonstrates the potential for obtaining accurate RANS contributions in the DHRL modeling framework. Interestingly, the experimental velocity profile at station $x/H = 6.0$ shows a complete flow reattachment, which is early relative to the experimental reattachment location (at $x/H = 6.38$ approximately) obtained using a linear-interpolation of the oil-flow laser skin-friction measurements [52]. The reason for this apparent discrepancy in the measured data is not clear. Computed DHRL results also predict reattached flow at station $x/H = 6.0$, while both the RANS and DDES model results still show separated flow at this station. At station $x/H = 7.0$, both the RANS and DDES model results show reattached flow. At this station, all computational
results underpredict velocity near the wall. In the far downstream (at stations x/H = 16.0 and 20.0), both the DHRL and DDES model predictions agree well with the experimental data while the RANS model results show an underpredictive behavior in the near wall region.
Figure 4.9  Comparison of mean-velocity profiles at different streamwise stations (a-i).
Figures 4.10 (a-i) compare the experimental turbulent kinetic energy data with the computed turbulent kinetic energy profiles obtained using the refined mesh at the same stations used for the mean velocity profiles. The computed turbulent kinetic energy values used in this investigation were normalized by the square of the mean inflow velocity to match the experimental data. In the flow separation region very close to the step at station $x/H = 1.0$, the DHRL results show a sudden rise in turbulent kinetic energy similar to the experimental data, which demonstrates the potential of the DHRL model to capture the resolved turbulence scales rapidly during the occurrence of flow separation, i.e., the rapid evolution of stress contributions from RANS to LES in the DHRL model framework. On the other hand, the DDES model prediction shows an extremely small rise in turbulent kinetic energy, which indicates a delayed shear layer breakup and the inability to resolve the turbulence scales rapidly. This is reflected in the delayed mixing of mean momentum shown in Figure 16. The RANS model simulation captures the turbulent kinetic energy rise quickly after the flow separation. Both the DHRL and RANS model simulation results exhibit a steady rise in turbulent kinetic energy similar to the experimental data until $x/H = 5.0$. Results from the DDES simulation show a very slow rise at stations $x/H = 1.5$ and $2.0$, but a substantial increase at $x/H = 5.0$ compared to the experimental measurements. Around the flow reattachment regions at stations $x/H = 6.0$ and $7.0$, the computed turbulent kinetic energy profiles decay in a manner similar to the experimental results, although the peak value of turbulent kinetic energy is underpredicted by both the DHRL and RANS simulation results and substantially overpredicted by the DDES model simulation. In the far downstream regions at stations
x/H = 16.0 and 20.0, all computed results agree qualitatively well with the experimental data.
Figure 4.10  Comparison of turbulent kinetic energy profiles at different streamwise stations
In summary, compared to the results of the DDES simulations, the DHRL model shows a more rapid shear layer breakup in the separation zone, a stronger mixing of the wake downstream of flow reattachment, and an eventual faster evolution towards a turbulent boundary layer. Inclusion of the RANS stress in the momentum equation as a source term enhances the mixing of shear layer, while still allowing rapid growth of fluctuating velocities, which is one of the major attributes of the DHRL formulation. Because the RANS eddy viscosity is applied only to the mean flow, small flow perturbations that arise due to instabilities are not quickly damped out, facilitating the growth of the instabilities and eventual breakdown and transition to resolved turbulence.

In the case of steady RANS simulations, even though the overall results match reasonably well with the experimental data, actual unsteady separated flow characteristics (strong vortex rolling, turbulence fluctuations, and growth of instabilities) are not evident in the simulations.

4.6 Conclusions

A dynamic hybrid RANS/LES modeling framework (DHRL) has been tested for the backward facing step flow, to evaluate the potential of the model to capture the turbulence characteristics inherent in separated flows. Specifically, the test case examined the behavior of the DHRL model, versus more traditional hybrid model, with regard to mesh refinement level and discretization scheme.

The DHRL model result produced a rapid breakdown of the shear layer in the LES region even for relatively coarse mesh resolution, and qualitatively similar behavior for coarse and fine meshes and two different discretization schemes. The DDES model, in contrast, showed delayed shear layer breakdown and significant dependence on mesh
resolution level. The DHRL mean flow results are comparable to the SST model in the recirculation region, and show improvement downstream of shear layer breakdown and reattachment. DHRL results were also comparable to SST results, and superior to DDES results, for predicted turbulent kinetic energy downstream of the step, although in the SST model the turbulence was completely modeled while for the DHRL model the turbulence was resolved as unsteady fluctuations. One of the key identifiable benefits to the DHRL model is its potential for mitigating grid dependence issues inherent in most current HRL models with respect to the mean flow, while allowing for increasing resolution of the turbulence in LES mode as mesh spacing is reduced.
CHAPTER V

EVALUATION OF HYBRID RANS/LES MODELS FOR PREDICTING FLOW AROUND AN ICED AIRFOIL

The work reported in this chapter has been submitted for publication in the AIAA Journal of Aircraft.

Nomenclature

\[ c = \text{chord} \]
\[ C_D = \text{coefficient of drag} \]
\[ C_L = \text{coefficient of lift} \]
\[ C_P = \text{coefficient of pressure} \]
\[ \alpha = \text{angle of attack} \]
\[ M = \text{freestream Mach number} \]
\[ Re = \text{Reynolds number} \]
\[ u = \text{time-averaged (mean) streamwise velocity} \]
\[ U = \text{freestream velocity} \]
\[ u-rms = \text{root mean square of streamwise velocity fluctuation} \]
\[ y_{surf} = \text{normal location of airfoil surface} \]
\[ Z_{domain} = \text{spanwise domain size} \]
5.1 Introduction

Flow separation on a lifting surface can significantly degrade aerodynamic performance by producing a reduction in lift, an increase in drag, a decrease in stall angle of attack, and in severe cases, a complete loss of aircraft control. One source of such flow separation that is particularly dangerous is the result of ice accreting near the leading edge of a wing. Although major ice accretion occurs on the wings, every exposed aircraft surface such as propellers, windshields, antennas, vents, intakes, and cowlings can accumulate substantial amounts of ice during the flight as well. Statistics show that ice accretion was identified as the cause of 12% of the total number of weather related accidents from 1990 to 2000 [61]. The loss of life attributed to icing led the National Transportation Safety Board (NTSB) to mark icing as a “MOST WANTED Aviation Transportation Safety Improvement” since 1997 [62, 63]. Wind tunnel and flight test reports show that even seemingly inconsequential ice accretions (no thicker than a piece of coarse sandpaper) on the leading edge or suction surface of a wing can cause a reduction in lift of 30% and an increase in drag of 40% [61]. The scenario is worse if the accretion is larger, as it can result in an even greater lift reduction and a substantial drag increase that can surpass 80% [61].

The ice shapes due to in-flight icing can produce a variety of complex flow phenomena depending on the characteristics of the ice shape. Generally, ice shapes are classified as one of four types: roughness, horn, streamwise, and spanwise ridge [64]. These classifications are based on the geometric characteristics of the ice shape and their resulting effects on the flow. Not surprisingly, atmospheric conditions contribute to the characteristics of the ice shape. Transparent glaze ice occurs under conditions of freezing
drizzle while opaque rime ice occurs under colder conditions. Of particular interest here are horn ice shapes, which are typically produced under glaze conditions.

Consider a horn ice accretion on the upper surface of a wing. Starting from the stagnation point, the flow accelerates around the horn until it encounters an adverse pressure gradient and, eventually, separates. The vorticity present in the separated shear layer causes it to break up and form discrete vortices. These shear layer vortices enhance flow mixing that eventually induces reattachment in the downstream region. Highly unsteady flow occurs due to the presence of adverse and favorable pressure gradients upstream and downstream of the reattachment location, respectively [65]. Even a relatively small horn can generate a large separation bubble downstream that is the dominant characteristic of the flow.

5.1.1 Simulating Unsteady Separated Flows

Computations of viscous, unsteady, separated flow fields encounter numerous difficulties and require carefully-selected turbulence models. The three primary categories of turbulent flow prediction methods are: direct numerical simulations (DNS), large eddy simulations (LES), and Reynolds Averaged Navier-Stokes simulations (RANS). Although the DNS method resolves all spatial and temporal scales of turbulence and thus provides accurate predictions, this approach is generally too costly to be performed using current computing hardware for configurations of engineering interest. LES models apply filtering (averaging) operations to the Navier-Stokes equations to achieve resolved solutions of the turbulent scales in the spectrum that are above a certain threshold. In this approach, the unresolved smaller scales are modeled using subgrid stress (SGS) models, based on the dependence of the smallest scale on the grid size. LES
models perform well in separated flow regions as they are capable of resolving the largest scales of turbulence that dominate momentum transfer in the flow field. Near wall performance of LES models is problematic and depends on the available computational resources due to the very small length and time scales of the near wall turbulence. As LES only resolves the larger turbulent scales and models the smaller scales, it requires significantly less computational resources than the DNS; but, this requirement is still excessive for high Reynolds number flow [5]. The RANS modeling approach only resolves mean velocity and models all turbulent scales. RANS models are the least computationally intensive of the three and remain the most widely used method for industrial flow applications. RANS models perform well in the boundary layer region but often show poor capabilities in separated regions due to the loss of information in the Reynolds-averaging process and the lack of universality in separated flows. These limitations of RANS models in separated flow regions have been noted by several researchers [46-49, 66, 67] in studies of massively separated unsteady flow around airfoils with ice shapes. They showed that RANS predictions degrade when the flow separates fully on the suction surface of an iced airfoil.

5.1.2 Hybrid RANS/LES Models

An attractive alternative to RANS that is currently practical from a computational standpoint is the Hybrid RANS/LES (HRL) approach. HRL methods combine the characteristics of both RANS and LES models and, thus, have the potential to effectively resolve both boundary layer and separated flows. In theory, the HRL modeling methodology offers more accuracy than RANS at a reduced cost relative to LES. Current HRL models are based on the concept of zonal modeling where unsteady RANS methods
are employed near the wall while LES methods are used in the separated flow region. This zonal definition can be either explicit [68] or implicit. An implicit zonal description that is based on a single parameter (eddy viscosity) is the most commonly used method. According to this implicit zonal description, the eddy viscosity near the wall region adopts a value which is consistent with a RANS modeling approach, i.e., characteristic of the Reynolds stress, while in separated flow regions the eddy viscosity adopts a value characteristic of the subgrid stress (SGS) model of an LES. The Detached Eddy Simulation (DES) model of Spalart et al. [10] is an example of implicit zonal description. The segregation of RANS and LES modes in this model is based on the local grid spacing, which is problematic in the attached boundary layer. Some ad hoc modifications have been implemented to resolve deficiencies of the baseline DES model. Using dynamically defined detached shear layer regions, Slimon [68] introduced a zonal DES (ZDES) method to prevent the blending of RANS regions with LES regions. In this method, the definition of a detached shear layer region is based on the local flow and turbulence quantities during the computation. Spalart et al. [11] also developed a modified version of the baseline DES model that is named the Delayed DES (DDES) to resolve the introduction of grid-induced LES modes into attached boundary layers. DDES modifies the original definition of length scale based on local flow and turbulence quantities. Shur et al. [12] proposed another modified version named Improved Delayed DES (IDDES) to eliminate the “log layer mismatch” that occurs in the classical DES and the wall-modeled LES (WM-LES) approaches. The IDDES model mostly acts like the DDES model except that it appears as the WM-LES model in the boundary layer regions where resolved turbulent quantities are present. It should be noted that Thompson et al.
and Kumar et al. [69] showed that flow field predictions around ice accreted wings improved compared to steady RANS methods when the DES model was used. However promising, most HRL models including the popular DES model, still have several unresolved issues. Commonly, the zonal transition/separation is defined such that the eddy viscosity varies between a value to define the Reynolds stress and a value to define the subgrid stress. The Reynolds stress is based on the concept of ensemble averaging of all turbulent scales while the subgrid stress models the turbulence scales that are too small to be resolved in the simulation. Clearly, these stresses are mathematically and physically different and, hence, any effort to bridge these two will exhibit ambiguity. The use of zonal separation based on only eddy viscosity is identified as a major weakness of current HRL models by several researchers [17-19]. Besides zonal transition, explicit grid dependence is a second major issue inherent in most hybrid RANS/LES models, since those models include terms that are functions of the local mesh size.

The DHRL modeling framework developed by Walters et al. [44, 53, 70-80] seeks to resolve those issues and has shown a significant degree of success. It is here hypothesized that the DHRL model may be a better choice than the existing HRL models for ice accreted aircraft flow field predictions. The main motivation of this research is to evaluate the potential of the DHRL model for prediction of such highly unsteady and massively separated complex flow fields in comparison with other HRL and RANS models along with experimental data.
5.2 Model Formulation

For completeness, this section briefly describes the formulation of the newly
developed DHRL model. Readers are referred to Walters et al. [44,53] for the detailed
description of this model. Applying the filtering operation to the momentum equation for
single-phase, incompressible, Newtonian flow with no body forces yields:

\[
\frac{\partial \hat{u}_i}{\partial t} + \hat{u}_j \frac{\partial \hat{u}_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left(2\nu \hat{S}_{ij}\right) - \frac{\partial}{\partial x_j} \left(\tau_{ij}\right)
\]

(5.1)

where the instantaneous and filtered velocity are given by \( u_i \) and \( \hat{u}_i \), respectively, and the
last term on the right hand side is the turbulent stress/subfilter stress which is defined as:

\[
\tau_{ij} = u_i \hat{u}_j - \hat{u}_i \hat{u}_j
\]

(5.2)

This term requires modeling for closure of the momentum equation.

In the DHRL modeling methodology, the instantaneous velocity \( u_i \) is
decomposed as:

\[
u_i = \bar{u}_i + u''_i + u'_i
\]

(5.3)

where, \( \bar{u}_i \) is the mean (Reynolds-averaged) velocity, \( u''_i \) is the resolved fluctuating
velocity, and \( u'_i \) is the unresolved fluctuating velocity. Both the mean and resolved
fluctuating velocity arise directly from the simulation, while the unresolved fluctuating
velocity requires modeling through the turbulent stress term, \( \tau_{ij} \). Substituting the
instantaneous velocity \( u_i \) shown in Eq. (5.3) into Eq. (5.2), and assuming that the
resolved and unresolved velocity fluctuations are uncorrelated, the expression for the
subfilter stress can be decomposed as:

\[
\tau_{ij} = \alpha \tau_{ij}^{SGS} + (1 - \alpha) \tau_{ij}^{RANS}
\]

(5.4)
where, \( \tau_{ij}^{SGS} \) is the subgrid stress predicted by the LES model and \( \tau_{ij}^{RANS} \) is the Reynolds stress computed using the RANS model. The weighting coefficient \( \alpha \) is computed based on the turbulence production due to the resolved scales, mean (Reynolds-averaged) component of subgrid stress, and the Reynolds stress:

\[
\alpha = \left( \frac{\overline{u''u''} \overline{S_{ij}}}{\text{Resolved turbulent Production}} \right) / \left( \frac{\overline{\tau_{ij}^{RANS}}}{\text{RANS Production}} - \frac{\overline{\tau_{ij}^{SGS}}}{\text{Inhomogeneous SGS Production}} \right) \quad (5.5)
\]

In practice, the value of the coefficient \( \alpha \) is limited to vary from 0 to 1. In flow regions with no resolved fluctuations, \( \alpha \) obtains a value of zero, and thus the pure RANS mode is activated in those regions. If the production of resolved fluctuations increases in any region, the RANS stress contribution diminishes, and an LES stress contribution appears in the momentum equation maintaining a smooth variation of turbulent production. If the resolved turbulent production is very high, the value of \( \alpha \) becomes unity, and hence the pure LES mode will be recovered.

The key features of the DHRL modeling methodology are as follows: 1) coupling of any given RANS model with any given LES model is enabled in this hybrid modeling framework; 2) the approach is free from any explicit grid dependence; 3) the zonal transition between RANS and LES modes is based on the continuity of total turbulence production; and 4) this approach exactly reproduces the baseline RANS model simulation result in steady flows.

The DHRL model has been tested for several different cases [44, 53, 72, 78, 81] including channel flow, backward facing step, cardiovascular device features relevant to biomedical applications, surface combatant, and suboff geometries. For these cases, the
DHRL model showed a significant degree of success compared to other RANS and HRL models.

To investigate the feasibility of the DHRL model compared to other commonly used turbulence models, simulations were also performed with the SST $k$-$\omega$, S-A, and DDES models available within the finite-volume based commercial solver Ansys FLUENT®. The DDES and SST $k$-$\omega$ models were chosen since these models are widely accepted HRL and RANS models, respectively. The S-A RANS model was chosen, because this model was designed specifically for aerodynamic applications. A brief description of these models is presented in the following sub-sections.

### 5.2.1 Shear-Stress Transport (SST) $k$-$\omega$

Menter [6] proposed the Shear-Stress Transport (SST $k$-$\omega$) model that is based on the transport of the principal shear stress to facilitate the prediction of adverse pressure-gradient-dominant flows. The eddy viscosity of the baseline $k$-$\omega$ model is re-defined within the framework of the SST model and attains the following expression:

$$v_t = \frac{a_1 k}{\max(a_1 \omega, S F_2)} \quad (5.6)$$

where, $F_2$ is a blending function, $a_1$ is a constant, and $S$ represents an invariant measure of the strain rate magnitude. The value of $F_2$ equals unity for the boundary-layer flows, whereas it attains a value of zero for free shear layers. In this SST modeling framework, production of turbulent kinetic energy is larger than dissipation in the adverse pressure gradient boundary layer.

The SST modeling framework incorporates two transport equations for the turbulent kinetic energy ($k$) and the specific turbulence dissipation rate ($\omega$):
\[
\frac{d \rho k}{dt} = \tau_{ij} \frac{\partial u_i}{\partial x_j} - \beta^* \rho \omega k + \frac{\partial}{\partial x_j} \left[ (\mu + \sigma_k \mu_t) \frac{\partial k}{\partial x_j} \right] \quad (5.7)
\]

\[
\frac{d \rho \omega}{dt} = \frac{\gamma}{\nu_t} \tau_{ij} \frac{\partial u_i}{\partial x_j} - \beta \rho \omega^2 + \frac{\partial}{\partial x_j} \left[ (\mu + \sigma_\omega \mu_t) \frac{\partial \omega}{\partial x_j} \right] + 2 \left( 1 - F_1 \right) \rho \sigma_\omega \frac{1}{\omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j} \quad (5.8)
\]

The blending function \(F_1\) plays a similar role as \(F_2\), serving as an indicator function for near-wall and farfield regions of the flow. Near the wall, \(F_1 = 1\), and a \(k-\omega\) model form is recovered. Far from the wall, \(F_1 \to 0\) and the model operates similar to a \(k-\varepsilon\) model form. Readers are referred to [6] for further details. For the DHRL model implemented here, the above equations were used for the RANS component, with the difference that all terms containing constructions of the velocity field—including convection, production, and model coefficients—were computed using Reynolds-averaged values rather than resolved instantaneous values.

### 5.2.2 Spalart-Allmaras (S-A) RANS Model

The Spalart-Allmaras (S-A) [82] model is a one-equation turbulence model in which the Reynolds stresses are evaluated using the Boussinesq approach as in the SST \(k-\omega\) model. The S-A model solves a transport equation to compute an eddy viscosity variable in the flow domain:

\[
\frac{D \bar{\nu}}{Dt} = P - D + T + \frac{1}{\sigma} \left[ \nabla \cdot \left( (\nu + \bar{\nu}) \nabla \bar{\nu} \right) + c_{b2} (\nabla \bar{\nu})^2 \right] \quad (5.9)
\]

where, \(\nu\) and \(\bar{\nu}\) are the kinematic viscosity and S-A working variable, respectively. The eddy viscosity is computed as follows:

\[
\nu_t = \bar{\nu} f_{v1} \quad , \quad f_{v1} = \frac{\chi^3}{\chi^2 + c_{b1}} \quad , \quad \chi = \frac{\bar{\nu}}{\nu} \quad (5.10)
\]
Turbulence production, destruction, and trip terms in Eq. (5.11) are given as:

\[ P = c_{b1} (1 - f_{t2}) S \tilde{\nu}, \quad D = \left( c_{\omega 1} f_{\omega} - \frac{c_{b1}}{\kappa^2} f_{t2} \right) \left[ \frac{\bar{v}}{d} \right]^2, \quad T = f_{t1} (\Delta u)^2 \quad (5.11) \]

where, \( \tilde{S} \) is the modified vorticity expressed as:

\[ \tilde{S} \equiv S + \frac{\bar{v}}{\kappa^2 d^2} f_{v2} \quad f_{v2} = 1 - \frac{\chi}{1 + \chi f_{v1}} \quad (5.12) \]

The model constants are

\[ c_{b1} = 0.1355, \quad c_{b2} = 0.622, \quad c_{v1} = 7.1, \quad \sigma = \frac{2}{3}, \quad \kappa = 0.41 \]

\[ c_{\omega 1} = \frac{c_{b1}}{\kappa^2} + \left( 1 + \frac{c_{b2}}{\sigma} \right) \]

Unlike the baseline S-A model, Ansys FLUENT® implements the S-A model in such a way that wall function is enabled when mesh resolution is not sufficiently fine to resolve the mean velocity accurately all the way to the wall [83].

### 5.2.3 Delayed Detached-Eddy Simulation (DDES)

The original DES model [10] is based on the Spalart-Allmaras one-equation RANS model, in which an alternate version of the length scale is incorporated, equal to the minimum of the local mesh size (multiplied by a model coefficient) and the distance to the nearest wall. The DDES model is one of the subsequent versions of the baseline DES model, modified to mitigate problems observed in DES. Spalart et al. [11] developed the DDES model to resolve the grid-induced activation of the LES mode in the attached boundary layer. This inconsistent LES switching occurs in the baseline DES model if a highly refined grid is used. In contrast to the baseline DES model, the DDES model redefines the effective turbulent length scale \( \bar{d} \) in such a way that it depends not
only on the local grid size ($\Delta$) and distance to the nearest wall ($d$), but also on the eddy-viscosity field:

$$\bar{d} \equiv d - f_d \max (0, d - C_{DES}\Delta) \quad (5.13)$$

When the function $f_d$ is equal to one, the baseline DES model is recovered, such that the effective length scale is the minimum of that determined by the wall distance and grid scale.

The function $f_d$ is evaluated as:

$$f_d \equiv 1 - \tanh([8r_d]^3) \quad (5.14)$$

where the expression of the function $r_d$ is:

$$r_d \equiv \frac{v_{r+} + \nu}{\sqrt{u_{ij}u_{ij} \kappa^2 d^2}} \quad (5.15)$$

The value of the function $f_d$ becomes zero within the boundary layer, ensuring the activation of RANS mode, and equals unity outside the boundary layer recovering the baseline DES mode.

5.3 Results

A computational investigation of massively separated, unsteady flow over a wing with a leading-edge ice accretion was performed, and selected results are presented in this section. The specific configuration is the 944 ice shape, which is a horn-type accretion on a GLC-305 airfoil produced by a 22.5-minute exposure to glaze-ice conditions [84]. The resulting flow exhibits a highly unsteady, three-dimensional region of separated flow in the region downstream of the horn. Simulations using RANS (S-A and SST $k$-$\omega$) and HRL (DHRL and DDES) models were performed to evaluate the efficacy of each
turbulence model. The computed results of all simulations are compared with experimental measurements obtained by Addy et al. [85] and Broeren et al.[86].

5.3.1 Experimental Studies of Addy et al. [85] and Broeren et al.[86]

The experimental effort was conducted using the NASA Langley Research Center Low-Turbulence Pressure Tunnel (LTPT) to investigate icing effects on a business jet airfoil (GLC-305 airfoil with the 944 glaze-ice shape). The tunnel has a test section 36 inches wide by 90 inches high by 90 inches long and was designed for testing airfoils with chord lengths up to 36 inches. The GLC-305 airfoil model used in this investigation had a 36 inch chord and 36 inch span. The airfoil model was mounted horizontally across the width of the test section. The traversing apparatus used for data collection was set above the airfoil. Very low freestream turbulence intensity levels of about 0.1% or less were maintained throughout the experiments. The flow field measurements were carried out for a variety of ice shape with varying Reynolds number, Mach number, and angle of attack.

5.3.2 Mesh Generation

In order to investigate the mesh sensitivity of the various turbulence models, two different, three-dimensional meshes were generated using the commercial software package Ansys/GAMBIT®. Initially, a planar, hybrid mesh was generated around the iced airfoil. An unstructured mesh, which occupied the bulk of the domain, was used in the region outside of the boundary layer to reduce the total number of cells in the domain. A structured mesh was generated in the viscous boundary layer region of the airfoil with the ice shape. The distance to the first point off the wall was defined in such a way that an
average $y^+$ value of less than 0.5 was maintained, which is within the recommended $y^+$ value for the RANS and HRL models used here. The three-dimensional mesh was generated by extruding the two-dimensional mesh in the spanwise direction. The edge length in the extruded direction was selected to make the cells in the region of separated flow downstream of the horn as isotropic as possible. Based on a chord length of $c = 0.9144$ m, the computational domain was generated so that the inflow and outflow boundaries were located approximately 20 chord lengths upstream and downstream of the airfoil, respectively, and the top and bottom boundaries were located approximately 16 chord lengths above and below the airfoil, respectively. These dimensions were chosen to minimize any blockage effects caused by the domain boundaries. The spanwise extrusion of the two-dimensional surface was extended to $0.5c$. The total number of computational cells in the three-dimensional coarse baseline mesh was 10M, while the refined mesh contained a total of 15M computational cells. Figure 5.1 (a) shows the planar mesh of the entire 2-D domain. Figures 5.1 (b) and 1(c) show the mesh in the vicinity of the boundary layer.

5.3.3 Flow Solution

The finite-volume based commercial CFD solver ANSYS/Fluent® version 14.0 was used for the simulations. The computations were performed using a pressure-based scheme (SIMPLE [56]) and the meshes (coarse baseline and refined) described above. The DHRL model was implemented in the flow solver using the User-Defined Function capability in ANSYS/Fluent®. The candidate RANS and LES models for the DHRL model were the SST $k-\omega$ model and the monotonically-integrated large-eddy simulation (MILES) model [51], respectively. Simulations of RANS (S-A and SST $k-\omega$) and DDES
models were performed using Fluent’s resident versions of those models. The RANS simulations were steady, while the DDES and DHRL simulations were time accurate. The steady simulations were run until a converged solution was achieved, while the time accurate simulations were run until a statistically stationary flow was attained. The time step size was chosen in such a way that the maximum CFL number in the simulation was approximately unity. An implicit, second-order temporal discretization was used for all

![ Computational domain: (a) full 2-D domain, (b) closer view around the airfoil, (c) closer view of region near horn showing structured mesh in the viscous boundary layer region.](image)
simulations. A second-order upwind (linear reconstruction) spatial discretization scheme was used for the convective terms in the RANS simulations. It should be noted that low numerical diffusion schemes are widely recommended [59] for discretization of the convective terms in the momentum equations for LES and HRL modeling methodologies to minimize the dissipation and dispersion errors in the solution. The bounded central differencing (BCD) [59] method is one such low-diffusion scheme that provides stability as well, and therefore, was chosen for the spatial discretization of HRL model simulations. For the side boundaries, a periodic boundary condition was used for the HRL simulations while a symmetry condition was used for the steady RANS simulations. A velocity-inlet and a pressure-outlet boundary condition were used for all simulations at the inlet and outlet boundaries, respectively.

5.3.4 Comparisons with Experimental Data

Figures 5.2 (a-i) show comparisons of the computed mean streamwise velocity profiles (obtained from the refined mesh simulations) with experimental data at different chordwise stations along the suction surface. All computations show strong flow reversal at station $x/c = 0.12$ (Figure 5.2 (a)), which is located just aft of the ice horn. Both the DHRL and SST models underpredict the negative velocity peak in the near-wall region, while the S-A model overshoots, and the DDES computation shows a large underprediction. All models overpredict the extent of the separation bubble in the wall normal direction with the overprediction by the DDES model being the largest. In the outer boundary layer, far away from the wall, all models underpredict the experimental data, which show an acceleration significantly above freestream. This underprediction by all models in the freestream and outer boundary layer persists over all chordwise
measurement stations, though the reason for it is unclear. The velocity profiles computed by all of the models at stations $x/c = 0.15$ (Figure 5.2 (b)) and $x/c = 0.20$ (Figure 5.2 (c)) show similar characteristics to those at station $x/c = 0.12$ (Figure 5.2 (a)). Similar to the experimental data, the DHRL and RANS models exhibit a gradual increase in reversed flow magnitude (negative velocity peak) at stations $x/c = 0.15$ and $x/c = 0.20$, whereas the DDES model simulations predict a nearly constant negative velocity peak for these stations.

As shown in Figures 5.2 (d) and 5.2 (e), at farther downstream locations close to the mid-chord region ($x/c = 0.40$ and $x/c = 0.45$), the DHRL model simulations predict flow reversal similar to the experimental measurements, showing a gradual decrease in the strength and extent of the reversed flow region. The DHRL model results closely agree with the experimental measurements in the near-wall region at those stations showing an indication of imminent boundary layer reattachment. At those stations, both of the RANS simulations show an approximately constant size of the reversed flow region and the peak negative values in the near-wall boundary layer are overpredicted in comparison with experiments. The DDES model simulations predict a massive increase in the size of reversed flow region, and the near-wall boundary layer prediction is overshot by a large margin from the experimental data.

The boundary layer prediction by the DHRL model at station $x/c = 0.50$ (Figure 5.2 (f)) shows that the flow is almost reattached, which is in close agreement with the experimental data. Both of the RANS and the DDES simulations overpredict the separated boundary layer behavior (negative velocity peak) and do not provide any indication of an imminent flow reattachment. At station $x/c = 0.55$ (Figure 5.2 (g)), the
DHRL model simulation shows a boundary layer reattachment similar to the experimental data. Neither of the RANS or the DDES models predict boundary layer reattachment at this station. Figure 5.2 (h) illustrates that downstream of the experimental flow reattachment location (at station $x/c = 0.60$), both of the RANS and the DDES simulations still show a separated boundary layer. However, the DDES model prediction at this location shows a significant reduction in the extent of the reversed flow region indicating an imminent boundary layer reattachment in the subsequent downstream locations. Farther downstream at station $x/c = 0.75$ (Figure 5.2 (i)), the DDES model predicts boundary layer reattachment; however, the RANS models still predict separated flow. All models quantitatively underpredict the streamwise velocity through most of the boundary layer at this station.

It is apparent from the velocity profile predicted by the HRL models that the LES-like behavior in the DHRL model activates near after the flow separation (aft of the horn), which eventually helps predict the flow reversal characteristics qualitatively well in comparison with the experiments. Close to the flow reattachment region, the contribution of LES-like behavior reduces and the RANS-like behavior increases in the DHRL model, and consequently the boundary layer prediction using the DHRL model agrees well with the experimental data. In contrast, the LES-like behavior activates much farther downstream of the flow separation (aft of the horn) in the DDES model and remains dominant even after the experimental flow reattachment location.
Figure 5.2  Comparison of mean streamwise velocity profiles at different stations computed using refined mesh (a-i).
Figure 5.2 (continued)
To illustrate the mesh sensitivity of the turbulence models used in this study, comparisons of the computed mean streamwise velocity profiles (obtained from the baseline coarse mesh simulations) with the experimental data are shown in Figures 5.3 (a-i). The coarse and refined mesh simulation results are presented in separate plots due to the limitations of plotting styles, symbols, and spacing required to exhibit the different results in a single plot. In the regions of reversed flow just aft of the glaze-ice horn (at stations \( x/c = 0.12, 0.15, \) and 0.20) shown in Figures 5.3 (a), (b), and (c), all model results, except for the DDES results, show flow characteristics similar to the refined mesh simulations. As shown in Figures 5.3 (d) and (e), around the mid-chord regions (at stations \( x/c = 0.40 \) and 0.45), computations on the baseline mesh qualitatively produce similar near-wall and wake-region boundary layer behavior that is shown in the refined mesh simulation results. However, the baseline mesh simulation results show some degree of offset from the refined mesh simulation results, though the difference is not significant. In the vicinity of the flow reattachment region (at stations \( x/c = 0.50, 0.55, \) and 0.60) shown in Figures 5.3 (f), (g), and (h), the DHRL model simulations predict better quantitative velocity profiles than other models in comparison with the experimental data. Only the DHRL model predicts flow reattachment at station \( x/c = 0.55 \) (the flow reattachment location measured in the experiment). Farther downstream (at station \( x/c = 0.75 \) (Figure 5.3 (i)), the DDES model results show flow reattachment, while the profiles computed by the RANS models remain separated. Overall, at each station, the baseline simulation results are qualitatively similar to the profiles obtained from the refined mesh simulations. The profiles computed on both meshes show some differences, but they are not quantitatively significant.
Figure 5.3 Comparison of mean streamwise velocity profiles at different stations computed using coarse baseline mesh (a-i).
Figures 5.4 (a-i) show a comparison of the experimentally measured turbulence intensity with the turbulence intensity profiles computed using the HRL models. These profiles were obtained from both the baseline coarse and refined mesh simulations on the suction surface at the same stations for the mean velocity profiles. Turbulence intensity profiles are calculated using the root-mean-square (RMS) of the fluctuations in the streamwise velocity ($u$-velocity) component normalized by the freestream velocity. The details of computed turbulence intensity profiles obtained from the refined mesh
simulations are discussed first. At station x/c = 0.12 (Figure 5.4 (a)), close to the glaze-ice horn, both the experimental data and the DHRL model simulation results indicate a strong production of turbulent fluctuations in the separated shear layer. The peak turbulence intensity computed by the DHRL model simulation at this station is approximately 0.24, which closely compares with the experimental peak turbulence intensity value of approximately 0.25. The pattern of turbulence production in the separated shear layer regions and wake regions predicted by the DHRL model agrees well with the experimental measurement. However, the DHRL model simulation overpredicts the turbulence intensity in the near-wall boundary layer regions. In comparison with the experimental data, the DDES model simulation produces a weaker turbulent fluctuation in the separated shear layer, along with a much lower peak turbulent intensity value of approximately 0.12. Turbulence production in the separated shear layer predicted by the DDES model simulation shows an inconsistent pattern compared to the experiments. However, the DDES model computation predicts the turbulence intensity well in the outer boundary layer; a substantial turbulence intensity underprediction is noticed in all other regions.

In the subsequent downstream regions (at stations x/c = 0.15 shown in Figure 5.4 (b) and x/c = 0.20 shown in Figure 5.4 (c)) with a more pronounced flow reversal, the DHRL simulations predict a gradual rise in the peak values of turbulence intensity closely matching the experimental values, whereas the DDES model simulations once again significantly underpredict the peak values. In the DDES model results, the anomalous behavior of the production of turbulent fluctuations in the separated shear layer is evident at these stations similar to the station x/c = 0.12. In the downstream
stations (at x/c = 0.40, 0.45, 0.50, 0.55 and 0.60) shown in Figures 5.4 (d-h), the gradients of experimental turbulence intensity profiles in the wall normal direction show a comparatively more smooth variation than those of upstream stations (x/c = 0.12, 0.15, and 0.20). Although the DHRL model simulations overpredict the results relative to the experimental data, the turbulence intensity profiles qualitatively agree well at those stations. There is good qualitative agreement between the DDES prediction and experimental data in the far-wall boundary layer region. However, the DDES model computations substantially overpredict the turbulent fluctuations in the near-wall region. Such significant production of turbulent fluctuations in the near-wall region by the DDES model indicates a premature transition to the LES mode in the boundary layer. At station x/c = 0.75 (Figure 5.4 (i)), both the DHRL and DDES simulations qualitatively agree with the experimental data.

In the coarse mesh simulation results, in the region just aft of the ice-horn (at stations x/c = 0.12, 0.15, and 0.20) shown in Figures 5.4 (a-c), the DHRL model computations agree well with the experimental data. The baseline mesh results are very similar to the refined mesh simulations for the DHRL model at these stations. However, compared to the refined mesh simulation results, the DDES baseline coarse mesh results show a substantially higher production of turbulent fluctuations throughout the boundary layer, especially in regions containing flow reversal. In subsequent stations (x/c = 0.40, 0.45, 0.50, 0.55, 0.60, and 0.75) shown in Figures 5.4 (d-i), the baseline coarse mesh predictions using the DHRL model show reasonably good qualitative agreement with the experiments and the quantitative agreement in the wake region is very good. At these stations, the difference between the coarse and refined mesh results using the DHRL
model is not significant. In contrast, the baseline coarse mesh simulation using the DDES model shows substantially overpredicted turbulent intensity profiles for these stations compared to the experimental data. However, the DDES results show qualitative agreement with the experiment in the wake region. A comparison of the coarse and refined mesh simulations using the DDES model shows a significant difference, except in the near-wall region. Overall, computations of turbulent intensity profiles using both coarse and refined meshes demonstrate a relative insensitivity to mesh refinement for the DHRL model, whereas the DDES model simulation results indicate substantial mesh sensitivity.
Figure 5.4  Comparison of turbulent intensity (streamwise-component) profiles at different stations computed using coarse baseline mesh and refined mesh (a-i).
Figure 5.4 (continued).
Table 5.1 shows a comparison of computed mean lift and drag coefficients (obtained from refined mesh simulations) with experimental data. The HRL models underpredict the drag by 12.3% (DDES) and 16.2% (DHRL), while the SST model shows the best agreement, underpredicting drag by only 6.7%. These results are surprising given that the SST model failed to properly resolve shear layer development and reattachment, as shown in Figure 5.2. The HRL models more accurately predict the lift coefficient, however. RANS and DDES model computations underpredict the $C_L$ value from 3.3% (DDES) to 16.2% (SA), while the DHRL model simulation overpredicts $C_L$ by 2.1%.

Table 5.1  Comparison of computed CL and CD values with experimental data

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<tr>
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<th>$C_L$</th>
<th>$C_D$</th>
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<tbody>
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<td>Experimental</td>
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<td>0.105</td>
</tr>
<tr>
<td>DHRL</td>
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<td>0.088</td>
</tr>
<tr>
<td>DDES</td>
<td>0.638</td>
<td>0.092</td>
</tr>
<tr>
<td>SST $k$-$\omega$</td>
<td>0.620</td>
<td>0.098</td>
</tr>
<tr>
<td>S-A</td>
<td>0.553</td>
<td>0.087</td>
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Figure 5.5 shows comparisons of computed, mean surface pressure coefficients (obtained using the refined mesh) with experimental data. Results predicted by all of the models agree reasonably well with experimental data in the region near the stagnation point. However, the predicted suction peak, where flow acceleration occurs (near the tip of the ice horn), overshoots the experimental peak. The region of relatively constant pressure on the suction surface provides an estimate of the extent of the separation
bubble. In the experimental studies [85, 86], this nearly constant pressure region extends to approximately x/c = 0.25. In comparison to experimental measurements, the DHRL model predicts a slightly smaller separation bubble based on the pressure distribution, while the steady RANS and DDES model simulations predict much larger separation bubbles. Presumably, this is the reason behind the overprediction of the lift coefficient by the DHRL model simulation and underprediction by the other models. All model simulations overpredict the suction surface pressure in the downstream region. Thompson et al. [66, 67] also noted a similar suction surface Cp overprediction for the computations using the S-A model and the baseline DES model. It is not clear if this is due to a fundamental discrepancy in computational predictions, measurement uncertainty, or small but important differences in the computational and experimental geometry. With regard to the latter, it is expected that subtle details in the ice horn shape will influence the boundary layer separation location and in turn the pressure in the recirculation region on the upper surface. Cp distributions of the lower surface of the wing produced by all computations show good agreement with experimental data with only a small offset between the different models.
5.3.5 Predicted Flow Features

Figures 5.6 (a-b) show instantaneous velocity (u-component of velocity) isosurfaces (25 m/s) colored by pressure for the HRL models. These isosurfaces qualitatively depict the shear layer roll up and the complex, unsteady, and three-dimensional nature of these massively separated flow fields. The blue shading represents low pressure regions which indicate the presence of vortex cores. The DHRL model predictions better capture the three-dimensional structures and much smaller turbulent scales than do the DDES model predictions. It is clearly shown from the isosurfaces that the DHRL model produces earlier shear layer breakup and a strong rollup in the region just aft of the ice horn, which is an expected physical phenomenon where flow separation is induced by the sudden change in surface geometry, such as a backward facing step.
flow [52, 87, 88]. In contrast, the DDES model shows a delayed shear layer breakup well aft of the horn. It has been reported in literature [60] that one of the most common behaviors of the DES model is the delay of the shear layer breakup that commonly occurs in regions of separated flow. Thompson et al. [66, 67] also noted a similar delay in flow separation by the DES model. Although the DDES model mitigates some of the DES model’s inherent deficiencies, the velocity isosurface clearly demonstrates that delayed shear layer breakup still remains in the DDES model.

Figure 5.6 Comparison of instantaneous velocity (u-velocity component) isosurface colored by pressure and computed by (a) DHRL model and (b) DDES model.

Figure 5.7 shows the instantaneous z-vorticity (spanwise vorticity component) contours of the two HRL model simulations at three different spanwise locations (DHRL on the left, DDES on the right). At each spanwise location (z/zdomain = 0.25, z/zdomain = 0.50, and z/zdomain = 0.75) the DHRL model predictions exhibit strong vorticity shedding and rollup patterns just aft of the ice horn, while the DDES model predictions show a
delayed shear layer rollup. The DHRL model predictions show irregularities in the vortex structures with spanwise variation, both of which are indicative of a well-developed three-dimensional flow field patterns. Although the DDES model predictions show spanwise variation in the vortical structures, the three-dimensionality of the flow field is much more conspicuous in the predicted contours of the DHRL model than those of the DDES model.

<table>
<thead>
<tr>
<th>DHRL</th>
<th>DDES</th>
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<tr>
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Figure 5.7 Comparison of instantaneous z-vorticity contours at different spanwise locations.
5.4 Conclusion

In this study, flow around an airfoil with glaze-ice shape was simulated to assess the efficacy of applying a newly developed hybrid RANS-LES (HRL) model for predicting massively separated unsteady flow fields. The Dynamic HRL (DHRL) model is based on two physically and mathematically different parameters to define the transition of RANS to LES mode, and vice versa. The DHRL model simulation results are compared with experimental data along with other RANS (SST $k-$ω and S-A model) HRL (DDES) models.

All computed mean wall static pressure profiles are overpredicted on the suction surface; however, RANS and DHRL model predictions exhibit qualitative agreement with the experiment. In contrast, the qualitative agreement between the DDES pressure prediction and experiment is inconsistent. Comparison of mean streamwise velocity profiles shows that the DHRL model simulation results agree with experimental data better than the RANS and DDES model predictions. The major attribute of the DHRL model is the prediction of flow reattachment very near the location measured in the experiments. In contrast, the DDES model simulation shows a significantly delayed flow reattachment (at the farthest downstream measurement station) and the RANS models predict no flow reattachment. Each turbulence model employed in this study shows insubstantial mesh sensitivity for mean velocity predictions for the range of mesh sizes considered. Turbulence intensity profiles predicted by the DHRL model simulation show better agreement with the experiment compared to the DDES model predictions. The DHRL model simulations show significantly less mesh sensitive behavior than the DDES model simulations for turbulence intensity profile predictions. The nature of the growth,
breakup, and rollup of the vortical structures shows that the DHRL model simulations qualitatively yield the expected physical phenomena in such a configuration where flow separation is induced by a sudden change in surface geometry, e.g., backward facing step flow. On the contrary, the inherent nature of delayed shear layer breakup and rolling reported in literature [60, 66, 67] is still present in the DDES model predictions of this study.

Based on these assessments, it can be concluded that the DHRL model shows promise in capturing the massively separated unsteady flow fields present in the engineering-relevant applications, specifically flow over an ice accreted wing. Further investigation, including comparison to alternative test cases and further investigation of the effect of grid refinement, is warranted.
CHAPTER VI

A TRANSITION-SENSITIVE HYBRID RANS/LES MODELING METHODOLOGY
FOR CFD APPLICATIONS

The work reported in this chapter will be submitted to the ASME Journal of Turbomachinery.

**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s$</td>
<td>streamwise coordinate, distance from leading edge</td>
</tr>
<tr>
<td>$L_S$</td>
<td>suction surface length</td>
</tr>
<tr>
<td>FSTI</td>
<td>freestream turbulence intensity</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>$U_e$</td>
<td>exit velocity</td>
</tr>
<tr>
<td>$U$</td>
<td>mean velocity</td>
</tr>
<tr>
<td>$C_P$</td>
<td>$2 \left( \frac{P_T - P}{\rho U_e^2} \right)$</td>
</tr>
<tr>
<td>$P_T$</td>
<td>upstream stagnation pressure</td>
</tr>
<tr>
<td>$P$</td>
<td>pressure</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density</td>
</tr>
<tr>
<td>$y$</td>
<td>cross-stream coordinate, distance from wall</td>
</tr>
</tbody>
</table>
6.1 Introduction

Providing economic benefit and improved design reliability, computational fluid dynamics (CFD) is currently used in a wide variety of application areas such as aeronautical and aerospace, biomedical, automotive, power generation, chemical processing, heating and cooling systems, meteorology, and marine systems. Despite significant progress in CFD regarding accurate geometrical representation and grid generation, robust numerical algorithms, and advanced computational resources, turbulence modeling still remains as one of the principal weaknesses in realistic CFD applications [1]. Direct Numerical Simulations (DNS), Large Eddy Simulations (LES), and Reynolds Averaged Navier-Stokes (RANS) are the three principal categories of turbulent flow simulation. Of these three categories, each has advantages as well as limitations. For example, DNS provides the most accurate predictions but requires immense computational resources. While LES is less expensive than DNS and performs well in separated flow regions, it is very expensive in the near-wall region. RANS is the least expensive and generally shows good near-wall prediction capabilities but exhibits poor performance in regions of separated flow. Hybrid RANS/LES (HRL) modeling [3] is an attractive alternative approach to combine the advantages of both RANS and LES models in an optimized fashion, in which a RANS modeling approach is employed in the near-wall region and an LES modeling approach is used in separated flow regions. In recent years, HRL models for turbulent flow simulations have received increased attention due to their effective use of computational resources to provide accurate predictions.
It is well known that a flow is not always fully turbulent. Transition-to-turbulent phenomena occur due to the effects of Reynolds number, geometry, roughness, curvature, freestream turbulence intensity (FSTI), etc. Transitional phenomena are observed in flow fields associated with various engineering disciplines. Transitional flow is of vital importance in aerodynamic simulations which can range from low-speed micro air vehicles to high-speed air vehicles. The fundamental characteristics of transitional phenomena are very complex and still remain unrevealed with respect to many physical aspects.

Until now, HRL methods have primarily used fully-turbulent models in the RANS component of the model. To the author’s knowledge, only one example [39] was found in the literature that used a transition-sensitive HRL modeling methodology. Since fully-turbulent RANS models have limitations in transitional flow field prediction [89-91], the current trend of using fully-turbulent RANS models limits the applicability of HRL models for flow problems in which transition is an important phenomenon. The LES component of an HRL model is generally only activated after flow separation; therefore, accurate prediction of the location of flow separation is highly dependent on the RANS component of the model.

In general, separated flows can be characterized as one of the following three types: laminar separation with laminar reattachment, laminar separation with turbulent reattachment, and turbulent separation with turbulent reattachment [41]. In order to predict all three types of flow separation accurately, the use of only fully-turbulent RANS models is inadequate. Therefore, a transition-sensitive turbulence model in the RANS component is a requirement. Sorensen et al. [39] showed that an HRL scheme with a
transition-sensitive RANS model performs better than an HRL scheme with a fully turbulent model in transitional flow simulations. Magagnato et al. [40] noted the importance of employing a transition-sensitive RANS model in an HRL scheme for successful prediction of transitional flows.

The motivation of this research effort is to develop a framework for a transition-sensitive, RANS-based HRL modeling methodology. The physics-based transition-sensitive RANS model of Walters and Cokljat [35] is implemented into a newly developed dynamic hybrid RANS/LES (DHRL) modeling framework (Walters et al. [44, 53]). As mentioned above, to date, only one effort [39] has been found that implemented a transition-sensitive RANS model in an HRL scheme. In that study, the correlation-based $\gamma-\tilde{R}e_\theta$ transition model of Menter et al. [38] was used with a DES version of the $k$-$\omega$ SST model. In contrast, the new transition-sensitive HRL (TDHRL) model of the present research implements the physics based $k$-$k_L$-$\omega$ transition model in the RANS part of the HRL model. Some studies [42,43] have shown improved performance of the $k$-$k_L$-$\omega$ transition model relative to the Suzen and Huang transition model (precursor of $\gamma-\tilde{R}e_\theta$ transition model) [36], and the $\gamma-\tilde{R}e_\theta$ transition model; however, this improvement is not universal but rather is based on specific test cases. Cutrone et al. [42] evaluated the $k$-$k_L$-$\omega$ transition model as a better option than the Suzen and Huang transition model for turbomachinery flows. Genc [43] tested the performance of the $k$-$k_L$-$\omega$ transition model and the $\gamma-\tilde{R}e_\theta$ transition model for the simulation of a thin airfoil in high Reynolds number flow and found that the $k$-$k_L$-$\omega$ transition model performed better than the $\gamma-\tilde{R}e_\theta$ transition model. Moreover, it has been reported that several issues, including treatment of RANS-to-LES zonal transition, are inherent to the DES model. In contrast, the DHRL
modeling framework of Walters et al. [44, 53, 72, 78] has shown a significant degree of success in mitigating those issues. It is therefore hoped that the combination of the physics-based transition model within the dynamic hybrid modeling framework may lead to improvement in the predictive capabilities for this class of modeling.

6.2 New Transition-Sensitive HRL Method

This section includes a brief description of the DHRL modeling methodology employed here. The physics-based transition model, which is implemented in the new transition-sensitive HRL modeling framework, is briefly described as well.

6.2.1 Dynamic Hybrid RANS/LES (DHRL) Modeling Methodology

Since a detailed description of the DHRL modeling methodology is available in the literature [44, 53], only a brief description of the formulation is included here. Applying the filtering operation to the momentum equation for a single-phase, incompressible, Newtonian flow with no body forces yields:

\[
\frac{\partial \tilde{u}_i}{\partial t} + \hat{u}_j \frac{\partial \tilde{u}_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left( 2\nu \hat{S}_{ij} \right) - \frac{\partial}{\partial x_j} \left( \tau_{ij} \right)
\] (6.1)

where, the instantaneous and filtered velocities are given by \( u_i \) and \( \tilde{u}_i \), respectively. The last term on the right hand side is the turbulent stress/subfilter stress, which is defined as:

\[
\tau_{ij} = u_i' \hat{u}_j - \hat{u}_i \hat{u}_j
\] (6.2)

This turbulent stress term requires modeling for closure of the momentum equation.

In the DHRL modeling approach, the instantaneous velocity \( u_i \) is decomposed as:

\[
u_i = \frac{\tilde{u}_i}{\hat{u}_i} + u''_i + u'_i
\] (6.3)
where, $\bar{u}_i$ is the mean (Reynolds-averaged) velocity, $u''_i$ is the resolved fluctuating velocity, and $u'_i$ is the unresolved fluctuating velocity. Both the mean and resolved fluctuating velocities are computed directly from the simulation, while the unresolved fluctuating velocity requires modeling through the turbulent stress term, $\tau_{ij}$. Substituting the instantaneous velocity ($u_i$) shown in Eq. (6.3) into Eq. (6.2), and assuming that the resolved and unresolved velocity fluctuations are uncorrelated, the expression for the subfilter stress can be decomposed as:

$$
\tau_{ij} = \alpha \frac{\tau_{ij}^{SGS}}{\bar{S}_{ij}} + (1 - \alpha) \frac{\tau_{ij}^{RANS}}{S_{ij}}
$$

where, $\tau_{ij}^{SGS}$ is the subgrid stress predicted by the LES model and $\tau_{ij}^{RANS}$ is the Reynolds stress computed using the RANS model. The weighting coefficient $\alpha$ is computed based on the turbulence production due to the resolved scales, mean (Reynolds-averaged) component of subgrid stress, and the Reynolds stress:

$$
\alpha = \left( \frac{\overline{u''_i u''_j S_{ij}}}{\text{Resolved turbulent production}} \right) / \left( \frac{\tau_{ij}^{RANS} S_{ij}}{\text{RANS production}} - \frac{\tau_{ij}^{SGS} S_{ij}}{\text{Inhomogeneous SGS production}} \right)
$$

In practice, the value of the coefficient $\alpha$ is limited to vary from 0 to 1. In flow regions with no resolved fluctuations, $\alpha$ obtains a value of zero, and thus the pure RANS mode is activated in those regions. If the production of resolved fluctuations increases in any region, the RANS stress contribution diminishes, and an LES stress contribution appears thereby maintaining a smooth variation of turbulent production. If the resolved turbulent production is very high, the value of $\alpha$ becomes unity, and hence the pure LES mode will be recovered.
The final key aspect of the DHRL approach is that the RANS stress, as well as the solution of the RANS model equations, is based solely on the Reynolds averaged velocity field, e.g.:

\[
\tau_{ij}^{\text{RANS}} = 2\nu_f \overline{S_{ij}}
\]  

(6.6)

6.2.2 Transition-Sensitive RANS Model

The \(k-k_L-\omega\) transition sensitive RANS model [35] was implemented in the DHRL modeling framework as the RANS component. The model incorporates an additional transport equation for laminar kinetic energy \((k_L)\), in addition to a modified form of a two-equation eddy viscosity turbulence model. For low freestream turbulence intensity (less than 1%), the velocity fluctuations in the pretransitional boundary layer are modeled as Tollmien-Schlichting waves. As the freestream turbulence intensity increases, pretransitional instability increases with high-amplitude streamwise fluctuations, and a downstream increase in the energy of these fluctuations leads to the initiation of bypass transition. The kinetic energy of the fluctuations in the pretransitional boundary layer is modeled using the concept of laminar kinetic energy \((k_L)\). In the \(k-k_L-\omega\) transition modeling approach, the production of \(k_L\) is defined as the interaction of Reynolds stresses that are associated with the pretransitional velocity fluctuations and mean shear. Laminar-to-turbulent transition is defined as the energy transfer process from laminar kinetic energy \(k_L\) to turbulent kinetic energy \(k_T\) and the transition onset location is based on local flow conditions, i.e., the model is single-point. The total fluctuation energy is defined as the sum of \(k_L\) and \(k_T\). A critical value of the ratio of turbulent production time-scale to the
molecular diffusion time-scale regulates the onset of transition. The ratio of the Tollmien-Schlichting time-scale to the molecular diffusion time-scale is defined as the criterion for natural transition.

The complete presentation of all model equations is not included here due to their availability in the literature [35], but the key aspects of the model formulation are reproduced. Three additional transport equations for $k_L$, $k_T$, and $\omega$ are solved in this model implementation:

\[
\frac{Dk_T}{Dt} = P_{k_T} + R_{BP} + R_{NAT} - \omega k_T - D_T + \frac{\partial}{\partial x_j} \left[ \left( v + \frac{\sigma_T}{\sigma} \right) \frac{\partial k_T}{\partial x_j} \right] \tag{6.7}
\]

\[
\frac{Dk_L}{Dt} = P_{k_L} - R_{BP} - R_{NAT} - D_L + \frac{\partial}{\partial x_j} \left[ v \frac{\partial k_L}{\partial x_j} \right] \tag{6.8}
\]

\[
\frac{\omega}{Dt} = C_{\omega_1} \frac{\omega}{k_T} P_{k_T} + \left( \frac{C_{\omega_2}}{f_w} - 1 \right) \frac{\omega}{k_T} (R_{BP} + R_{NAT}) - C_{\omega_2} \omega^2 + C_{\omega_3} f_\omega \alpha_T f_w \sqrt{\frac{k_T}{d^3}} + \frac{\partial}{\partial x_j} \left[ \left( v + \frac{\sigma_T}{\sigma_\omega} \right) \frac{\partial \omega}{\partial x_j} \right] \tag{6.9}
\]

The total fluctuation kinetic energy is defined as:

\[
k_{TOT} = k_T + k_L \tag{6.10}
\]

The production terms for turbulent and laminar kinetic energy are defined as:

\[
P_{k_T} = v_{T,S} S^2 \tag{6.11}
\]

\[
P_{k_L} = v_{T,L} S^2 \tag{6.12}
\]

### 6.3 Test Cases

In this section, simulation results from a canonical flow problem, flow over a circular cylinder, and a more engineering-relevant problem, flow around a PAK-B airfoil, are presented. To demonstrate the characteristics of the new transition-sensitive DHRL
(TDHRL) method in comparison with other turbulence models, simulations using a fully
turbulent RANS (SST $k-\omega$) model [6], the DDES [11] model, and the DHRL [44] model
are presented. The purpose of performing simulations using both the TDHRL and DHRL
models is to evaluate the contribution of the transition-sensitive RANS model
incorporated into the baseline DHRL modeling framework. The finite-volume based
commercial software ANSYS/Fluent® version 14.0 was used for all simulations. Both
the new transition-sensitive DHRL (TDHRL) model and the baseline DHRL model were
implemented in the flow solver using User-Defined Functions. Simulations employing
RANS (SST $k-\omega$) and DDES models were performed using versions of those models
native to Fluent. The commercial software package ANSYS/Gambit® was used to
generate three-dimensional structured grids for each configuration.

6.3.1 Flow Over a Circular Cylinder

The flow over a circular cylinder is considered to be a canonical test case for
assessment of any transition/turbulence model in transition-to-turbulent flow predictions.
All computational results are compared with the experimental measurements of Perrin et
al.[92]. The experimental measurements were performed at $Re = 140,000$ and $FSTI =
1.5\%$. It should be noted that the experimental conditions differ somewhat from the
conditions employed in the simulations ($Re = 100,000$ and $FSTI = 0.7\%$) performed in
this study.

6.3.1.1 Mesh Generation

Figure 6.1 (a) shows the planar mesh for the two-dimensional computational
domain for cylinder diameter, $D = 1.0$ m. Based on the cylinder diameter, the
computational domain was extended to a total of 30D in both the streamwise and the wall normal directions. The two-dimensional mesh was extruded in the spanwise direction to a total width of 5D to generate a three-dimensional mesh. Mesh density was increased in the wake region aft of the cylinder to capture vortex shedding and breakup accurately. Figure 6.1 (b) shows a closer view of the mesh in the wake region. The total number of computational cells was 2,073,600.

![Figure 6.1](image)

Figure 6.1  Mesh employed for circular cylinder simulations, (a) 2D planar mesh representation, (b) closer view of mesh in wake region

6.3.1.2  Flow Solution

The Reynolds number for this case is $\text{Re} = 100,000$; based on the cylinder diameter and the freestream velocity, $U_\alpha = 1.46$ m/s. The value of freestream turbulence intensity, $\text{FSTI} = 0.7\%$, was used in all simulations. The candidate LES model for both the DHRL and TDHRL simulations was the monotonically-integrated large-eddy simulation (MILES) model [51]. The RANS model for the DHRL simulations was the
fully turbulent SST $k-\omega$ model, while the $k-\omega_{\text{L}}$ transition model was used in the RANS part of the TDHRL model. A second-order implicit method was used for temporal discretization in all simulations. A second-order upwind method was used for spatial discretization in the SST model simulations. The bounded central difference (BCD) scheme [59], which is recommended for LES model simulations due to its low dissipative nature, was used to discretize the convective terms in the momentum equations in all HRL model simulations. For the side boundaries, a periodic boundary condition was used for all computations. A velocity-inlet boundary condition and a pressure-outlet boundary condition were used at the inlet and outlet boundaries, respectively, for all simulations. Since the experimental flow conditions are slightly different than the computations, only a qualitative assessment is presented at this point. Since the mesh used for these simulations had already been tested and demonstrated by Chitta et al.[93], no further grid independence tests were performed.

6.3.1.3 Results

Figure 6.2 (a-e) shows a comparison of computed streamline patterns with the experimental measurements. Similar to the experimental data, all computational results except the DDES model show a pattern of two topologically-symmetric eddies in the near-wake region. In the DDES prediction, the eddy located above the wake center line contains a secondary eddy. The extent of predicted recirculation region in the cross-stream direction for all models is narrower than the experimental wake passage. The recirculation length measured in the experimental study extended a distance of $x/D = 1.25$ in the streamwise direction. The TDHRL model simulation predicted the recirculation length of $x/D = 1.15$, which is the predicted value that most closely matches the
experimental measurement. The recirculation length computed using the DHRL model is the shortest \((x/D = 1.0)\) and the DDES model is the longest \((x/D = 1.50)\). The SST simulation predicts a value of \(x/D = 1.10\).

Figure 6.2    Comparison of streamlines (a-e).
Figure 6.3 (a-e) and Figure 6.4 (a-e) show the comparison of iso-contours of mean streamwise velocity and mean cross-stream velocity, respectively. Similar to the experimental data, the computed streamwise velocity contours show a nearly symmetric pattern while the cross-stream velocity contours exhibit a nearly antisymmetric pattern. Noting that the experimental and computational flow conditions differ in several key aspects (high blockage, freestream turbulence intensity, and mainly the Reynolds number), the computational predictions can be considered qualitatively reasonable.
Figure 6.3  Comparison of iso-contours of mean streamwise velocity (a-e).
Figure 6.4  Comparison of iso-contours of mean cross-stream velocity (a-e).
6.3.2  **Flow Over a PAK-B Airfoil**

A computational investigation of flow around a PAK-B airfoil was performed and computed results are presented in this section. Transient simulations using the DDES model, DHRL model, and the TDHRL model were performed, while steady simulations were performed using the SST $k-\omega$ model. The computed results of all simulations are compared with the experimental measurements of Volino [94].

6.3.2.1  **Experimental Study of Volino [94]**

Volino conducted an experimental investigation of flow around a PAK-B airfoil by varying both the Reynolds number and the FSTI. The experiments included Reynolds numbers based on suction surface length and exit velocity ranging from $Re = 25,000$ to $Re = 300,000$ at low FSTI (0.5%) and high FSTI (9%) conditions. The results reported from the experimental study included mean and fluctuating velocity, turbulent shear stress, and intermittency profiles at different stations (shown in Table 6.1) on the suction surface.

<table>
<thead>
<tr>
<th>Station</th>
<th>s/Ls</th>
<th>Station</th>
<th>s/Ls</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0.528</td>
<td>9</td>
<td>0.777</td>
</tr>
<tr>
<td>7</td>
<td>0.611</td>
<td>10</td>
<td>0.861</td>
</tr>
<tr>
<td>8</td>
<td>0.694</td>
<td>11</td>
<td>0.944</td>
</tr>
</tbody>
</table>
6.3.2.2 **Mesh Generation**

Figure 6.5 (a) shows the planar mesh of the entire 2-D domain. The first cell distance from the wall was defined in such a way that an average $y^+$ value of less than unity was maintained. To obtain a three-dimensional mesh, the 2-D mesh shown was extruded in the spanwise direction a distance of 0.5 chord length, with a uniform distribution of 150 cells over the spanwise extent. Figures 6.5 (b) and 6.5 (c) show a close up view of mesh in the vicinity of leading edge and trailing edge, respectively. The total number of computational cells in the three-dimensional mesh is 4,227,000. Figure 6.5 (d) shows the stations, from 6 (first from the left) to 11 on the suction surface, at which measurements were made.
Figure 6.5  Mesh and geometry employed for PAK-B airfoil simulations, 2D planar mesh representation (a), closer view of mesh in the vicinity of leading edge (b) and trailing edge (c), the measurement stations (d).
6.3.2.3 Flow Solution

Reynolds numbers of Re = 25,000 and Re = 100,000 were considered for the low FSTI case while Re = 100,000 was chosen for the high FSTI case so that computational investigations would match the experiments of Volino [94]. The LES model employed for both the TDHRL and DHRL simulations was the monotonically-integrated large-eddy simulation (MILES) model [51]. The RANS component of the DHRL model was the fully turbulent SST $k-\omega$ model, while the $k-k_L-\omega$ transition model was used in the RANS part of the TDHRL model. A second-order upwind method was used for spatial discretization in the steady SST model simulations. The bounded central difference (BCD) scheme was used to discretize the convective terms in the momentum equations in the HRL model simulations. A second-order implicit (three-point backward difference) method was used for temporal discretization in all HRL model simulations. A periodic boundary condition was used for the spanwise side boundaries for the airfoil cascade flow.

The transient simulations were run until a statistically stationary flow field was achieved. Since the mesh used for the simulations of this study had already been tested by Dhakal et al.[95], no further grid independence tests were performed.

6.3.2.4 Results

Figures 6.6 (a-c) show a comparison of computed wall pressure distribution profiles with experimental data. For the low FSTI, low Reynolds number (Re = 25,000) case, experimental boundary layer separation is measured near $s/L_s = 0.6$ on the suction surface, and once the boundary layer is separated, reattachment does not occur [94]. The TDHRL model simulation results show that boundary layer separation appears to occur
slightly upstream of \( s/L_s = 0.6 \) and that reattachment is completely absent on the suction surface of the airfoil. The DHRL model simulation results exhibit the same behavior as the TDHRL results. In contrast, the DDES simulation predicts delayed boundary layer separation (at approximately station 8, i.e., \( s/L_s=0.7 \)) and flow reattachment by station 10. The SST model simulation predicts earlier boundary layer separation (at \( s/L_s = 0.55 \)) than the experiment and the computed boundary layer remains separated on the suction surface. At the locations upstream of the suction surface peak pressure, both the TDHRL and DHRL model simulation results exhibit good agreement with experimental data while the DDES model computation shows an overpredicted pressure distribution. The SST model simulation result underpredicts the suction surface peak pressure, while showing good agreement with experimental data at the locations starting from the leading edge to slightly prior to the location of experimental suction peak pressure. In the regions of separated flow, both the TDHRL and DHRL predictions are in reasonable agreement with the experimental data with the exception of an underprediction of the pressure close to the trailing edge. An overpredicted pressure distribution is observed in the DDES model results and the overprediction is exacerbated in the trailing edge region. An underpredicted suction surface pressure distribution is exhibited by the SST results in the regions of separated flow. It should be noted that in this low Re low FSTI case, there is no appreciable difference between the predictions made using the TDHRL model and the DHRL model.

For the case of low FSTI, high Reynolds number (\( Re = 100,000 \)) conditions, the experimental measurements show that boundary layer separation occurs at approximately \( s/L_s = 0.6 \) and reattachment occurs prior to station 11 [94]. The computational results
obtained using the TDHRL model show a similar boundary layer separation location but earlier reattachment in comparison with experimental measurements for this case. On the other hand, all other model predictions exhibit fully-attached flow over the entire suction surface. In terms of a quantitative comparison, the TDHRL predictions show reasonably good agreement with experimental data from the upstream stations to the location of flow separation; although after separation, the flow reattaches rapidly and underpredicts the $C_P$ values. Both the DHRL and SST simulation results show similar behavior when compared to the TDHRL prediction, except in the region of flow separation. The DDES model simulation overpredicts the $C_P$ values until approximately $s/L_S = 0.7$ and the overprediction becomes significant, close to the trailing edge.

For the high FSTI, high Reynolds number (Re = 100,000) case, the experimental data shows that boundary layer separation occurs at approximately $s/L_S = 0.7$ (close to station 8) and reattachment is observed upstream of station 10 [94]. The TDHRL simulation predicts boundary layer separation around station 8 and shows a rapid flow reattachment upstream of station 9. All other computational results show an attached boundary layer that does not separate. Pressure predictions by the TDHRL, DHRL, and SST model simulations agree reasonably well with the experimental data with the exception of an underprediction aft of the separation region of the boundary layer. Computational results from the DDES simulation show an overprediction of the experimental data over the entire suction surface.

For the high Re cases, it is clear that the only difference between the TDHRL simulation and DHRL simulation is the contribution of the transition-sensitive RANS model that predicts boundary layer separation. On the other hand, using the fully-
turbulent RANS model in the RANS part of the DHRL model fails to produce boundary layer separation.

Figure 6.6  Comparison of $C_P$ distribution (a-c).
Figures 6.7 (a-f) show comparisons of mean velocity magnitude profiles between the computed results and experimental data at different stations on the suction surface for the low FSTI, low Reynolds number (Re = 25,000) case. Recall that the stations at which experimental measurement were made, are shown in Table 6.1 and Figure 6.5 (d). The experimental result shows attached laminar flow at station 6. Boundary layer separation occurs close to station 7 and the height of the separation bubble increases gradually for the farther downstream stations. Experimental mean velocity profiles also exhibit a large flow separation with bubble burst at the trailing edge and no flow reattachment [94]. All computational results show attached laminar flow at station 6, which is similar to the experimental data. The TDHRL, DHRL, and SST simulation results match the experimental boundary layer separation location (at station 7); however, the SST computation shows a much larger separation bubble compared to the experimental data. In contrast, the DDES results show a delayed boundary layer separation (at station 8). The predicted bubble size for both the TDHRL and DHRL computations increases continuously through the downstream stations and overpredicts the bubble size. Similar to the experimental data, both the TDHRL and DHRL model results show no flow reattachment. Computational results obtained using the DDES model show a reattached boundary layer upstream of station 10. The bubble size predicted using the SST model shows a gradual increase through subsequent downstream stations and a significant overshoot at the stations close to the trailing edge. Similar to the experiment, the boundary layer predicted by SST simulation remains separated. In terms of a quantitative comparison, all of the computational results, except those predicted by the SST model, agree well with the experimental data at station 6. The SST computation underpredicts
the near-wall velocity profile at this station. Both the TDHRL and DHRL simulation results underpredict the mean velocity in the near-wall region for stations farther downstream but match reasonably well with experimental velocity profiles away from the wall at stations 7, 8, and 9. In contrast, both the DDES and SST simulation results exhibit an under-predictive behavior that is exacerbated at the downstream stations. The degree of mean velocity underprediction is large for the SST computations at the stations close to the trailing edge (station 10 and 11).
Figure 6.7 Comparison of mean velocity profile of low FSTI low Re case at different stations (a-f).
Figures 6.8 (a-f) show mean velocity profiles for the low FSTI high, Reynolds number (Re = 100,000) case. The experimental results show that the boundary layer is near separation at station 7 and separates by station 8. The separation bubble grows until station 10 and the boundary layer reattaches by station 11 [94]. Similar to the experimental measurements, the TDHRL simulation predicts boundary layer separation just after station 7 but before station 8. At station 8, the boundary layer is separated but it reattaches rapidly slightly upstream of station 9. A completely attached boundary layer is predicted by all other models. In terms of a quantitative comparison to the experimental data, all computational results are in reasonably good agreement with experimental results at stations 6 and 7. At station 8, results computed using the TDHRL model underpredict the mean velocity in the near-wall region but match well with experimental data away from the wall. The DDES model results are over-predictive in the near-wall region while under-predictive away from the wall at station 8. At this station, both the DHRL and SST computations overpredict the near-wall velocity profile and slightly underpredict the mean velocity away from the wall. All of the computational results are over-predictive in the near-wall region and under-predictive in the far-wall region at stations 9, and 10. At station 11, both the DHRL and SST simulation results show better agreement than the TDHRL and DDES models when compared to the experimental data.

It should be noted that the mean velocity profiles predicted by the DHRL and SST simulations are nearly identical for this high Reynolds, number low FSTI case. Such similarity in the mean velocity profiles indicates that the fully turbulent RANS model (SST) used in the DHRL model remains activated in the entire flow regime on the suction
surface since the very low FSTI value contributes insufficient resolved turbulent fluctuations to trigger the LES mode in the DHRL model.
Figure 6.8  Comparison of mean velocity profile of low FSTI high Re case at different stations (a-f).
Figures 6.9 (a-f) show the mean velocity profiles at different stations on the suction surface for the high FSTI, high Reynolds number (Re = 100,000) case. It is observed from the experimental data that boundary layer separation, which is measured upstream of station 8, is relatively insensitive to the variation in FSTI, but reattachment occurs earlier (by station 10) for the higher value of FSTI. The size of the separation bubble is also decreased for the high FSTI case [94]. Similar to the experimental data, simulation results using the TDHRL model also exhibit boundary layer separation before station 8, but a comparatively more rapid reattachment by station 9. In contrast, a fully attached boundary layer is predicted by all other models used in this study. Results from the TDHRL simulation quantitatively agree well with the experimental data at all stations other than 9 and 11. Computational results predicted using the DDES model show quantitatively poor agreement with experimental data except at station 6. The DHRL and SST simulation results quantitatively agree well with the experimental data at stations 6 and 7; however, the velocity profile predictions at the remaining stations are mostly poor.

Unlike the high Reynolds number, low FSTI case, the velocity profile predictions by the DHRL and SST model simulations are not identical in the high Reynolds number high, FSTI case. For both cases, the Reynolds numbers are the same but significant differences exist in the FSTI values. Eventually, the higher value of FSTI contributes to the degree of resolved turbulent fluctuation in the flow regime (especially in the farther downstream regions) that facilitates the LES stress development in the DHRL model simulation.
Figure 6.9  Comparison of mean velocity profile of high FSTI high Re case at different stations (a-f).
It has been noted in the literature [96] that the performance of the baseline DES model can be poor in a flow field in which flow separation is not either geometry or grid imposed due to “modeled stress-depletion”, which is inherent to the DES modeling approach. This behavior is exhibited by the DDES model in this study where flow separation is not induced by a sudden change in the surface geometry, such as occurs in a backward facing step for example. Although the DDES model mitigates this deficiency to some extent, the issue is not completely resolved. The performance of the DDES model for predicting both boundary layer separation and reattachment locations is therefore relatively poor in comparison to experimental results. The delayed shear layer breakup issue inherent in the DDES modeling methodology is possibly another reason behind this poor performance. Like the DDES results, the DHRL and SST model simulations also fail to capture boundary layer separation for the high Reynolds number cases. In contrast, the TDHRL predictions show reasonable performance in comparison to experimental data. These differences in performance between the TDHRL and other models can be at least partly explained by observing each model’s ability to resolve turbulence fluctuations in the flow field as shown in Figures 6.10 (a-l). The instantaneous spanwise velocity contours (z-velocity component normalized by exit velocity) predicted by the HRL models and the RANS model for all three cases are shown in these figures. It is clear that the TDHRL model captures the turbulent fluctuations much better than other HRL (DDES and DHRL) models. In fact, for the two high Reynolds number test cases, these HRL (DDES and DHRL) models show zero resolved fluctuations, which indicates that the DHRL and DDES models effectively operate entirely in a steady-RANS mode. The results shown in this study demonstrate the ability of a transitional RANS component to
improve the prediction of the separation behavior, prior to the development of resolved, LES-like fluctuations in the simulations.

Figure 6.10  Comparison of normalized instantaneous z-velocity contours of HRL models and normalized z-velocity contours of SST model (a-l).
Figure 6.10 (continued)
6.4 Conclusion

In this study, an initial version of a new transition-sensitive RANS based HRL modeling framework (TDHRL) that incorporates the physics-based \( k-k_L-\omega \) transition model into a newly developed dynamic hybrid RANS/LES (DHRL) modeling framework is described. A canonical test case – flow over a circular cylinder – and an engineering-relevant test case of flow around a PAK-B airfoil are employed for a feasibility study for the TDHRL model. The TDHRL model simulation results are compared with the experimental data along with the computed results of other models (the DDES model, the DHRL model, and the SST \( k-\omega \) model).

In the case of flow around a circular cylinder, comparisons of predicted near-wake flow structures (streamlines, mean streamwise and cross-stream velocity components) with experimental data are presented. Since the experimental flow conditions (Reynolds number, FSTI, and blockage effect) are somewhat different from the computational flow conditions, only a qualitative assessment of the computational models can be performed in this study. None of the computational model results can be claimed to provide the best match with the experimental data; however, all model predictions can be considered reasonable. Rather than using a MILES model, use of an explicit eddy-viscosity model, e.g., dynamic Smagorinsky model as the LES component, might improve the flow field prediction capability of the TDHRL model.

For the PAK-B airfoil, mean wall static pressure profiles show that the TDHRL, DHRL, and SST model results agree reasonably well with the experimental measurements except for showing some degree of pressure underprediction after boundary layer separation. In contrast, the DDES model simulation results are mostly
over-predictive throughout the entire suction surface and this over-predictive behavior is exacerbated close to the trailing edge. Mean wall static pressure profiles also show that the boundary layer separation and reattachment predicted by the TDHRL simulations can be claimed to be the best match with the experimental measurements among all computational models. Mean velocity profiles at different stations on the suction surface show that the TDHRL model results agree qualitatively better with the experimental measurements in comparison to the all other model predictions. The TDHRL model simulation captures the boundary layer separation location similar to the experimental measurements for all three cases. The predictions of the TDHRL model also show no flow reattachment for the low FSTI low Re case, similar to the experimental results. For the low FSTI, high Re and high FSTI, high Re cases, the TDHRL model predicts earlier boundary layer reattachment in comparison to experimental measurements. The DDES model simulation for the low FSTI, low Re case predicts a delayed boundary layer separation and, unlike the experimental results, shows boundary layer reattachment. Both the DHRL and SST models predict boundary layer separation locations similar to the experimental data for the low Re, low FSTI case. Unlike the experimental results, the DDES, DHRL, and SST model simulations predict fully attached boundary layer for low FSTI, high Re and high FSTI, high Re cases.

The results presented here demonstrate the feasibility of the TDHRL model as an alternative for hybrid RANS-LES simulations of complex problems that include flow separation and laminar-to-turbulent boundary layer transition. Further efforts are warranted to validate this modeling approach in detail using canonical and applied test cases, and to investigate implementation strategies for improving the model performance.
CHAPTER VII

FINAL CONCLUSIONS

The main purpose of this research was to investigate and further develop a dynamic hybrid RANS/LES modeling framework for turbulent and transitional flow field prediction. In the first part of this research, a dynamic HRL (DHRL) modeling framework was presented that is a general HRL formulation framework to couple any choice of RANS model with an arbitrary LES model. The DHRL model was evaluated for fully turbulent flows using a canonical test case of flow over a backward facing step and an engineering-relevant test case of flow around a wing with a leading-edge ice accretion. In order to address the limitations of HRL models in transitional flow prediction, a transition-sensitive dynamic HRL (TDHRL) model was developed in the second part of this research. The TDHRL model incorporates a physics-based transition-sensitive RANS model in the DHRL framework. For transitional flow, the TDHRL model was evaluated by performing numerical simulations of flow over a circular cylinder and PAK-B airfoil. The major findings of this research can be summarized as:

- The DHRL model showed its potential by improving some of the major deficiencies inherent in most current HRL models including explicit grid dependence and delayed breakdown of separated shear layers.
• The mesh sensitivity study performed for both the backward facing step flow and the flow around an iced wing showed the DHRL model’s potential for mitigating explicit grid dependence as evidenced by the rapid breakdown of the shear layer in the LES region even for relatively coarse mesh resolution.

• The nature of the growth, breakup, and rollup of the vortical structures showed that the DHRL model simulations qualitatively yield the expected physical phenomena for configurations in which flow separation is induced by a sudden change in surface geometry, similar to backward facing step flow.

• The separated boundary layer (around near-wall and wake-region) and flow reattachment behavior exhibited by the DHRL model simulation results also showed its capability to capture unsteady separated turbulent flow field better than other HRL and RANS models.

• The transition-sensitive DHRL (TDHRL) model showed superior predictive capability over the DDES, baseline DHRL, and RANS models to capture boundary layer separation and reattachment behavior. For high Reynolds number cases, none of the computational models except the TDHRL model predicted boundary layer separation similar to the experiment in the PAK-B airfoil flow case.

• One of the key identifiable attributes of the TDHRL model was its potential for capturing the flow physics more accurately than the other HRL models (including the baseline DHRL model itself) in the PAK-B
airfoil flow field, where flow separation is not induced by a sudden change in the surface geometry. Generally, in such non-geometry induced flow fields, most current HRL models exhibit deficiencies in evolving quick enough to produce LES-like resolved turbulent structures due to “modeled stress-depletion”. For the high Reynolds number PAK-B airfoil cases, this behavior was pronounced in the other HRL (DDES and DHRL) model simulations, since those models showed zero resolved fluctuations, which indicated that the model was operating in a steady-RANS mode. In contrast, the TDHRL model results captured resolved fluctuations showing its ability to improve the prediction of the separation behavior, prior to the development of resolved, LES-like fluctuations in the simulations.

Overall, this research demonstrates the potential of the DHRL modeling framework for both turbulent and transitional flow field predictions. A significant improvement in computed results and the wider applicability relative to current HRL and RANS models have been exhibited by the DHRL modeling framework, especially with the inclusion of a transition-sensitive RANS model. Further investigation and improvement of the existing DHRL modeling framework will provide a more flexible and universal method for hybrid RANS-LES than existing standalone models.
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