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A Multiscale Modeling Methodology for Composites that includes Fiber Strength Stochastics

Trenton M (Trenton Mitchell) Ricks

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A multiscale modeling methodology for composites that includes fiber strength stochastics

By

Trenton Mitchell Ricks

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Mississippi State University
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Mississippi State, Mississippi

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A multiscale modeling methodology for composites that includes fiber strength stochastics

By

Trenton Mitchell Ricks

Approved:

Thomas E. Lacy
Professor of Aerospace Engineering
(Major Professor)

Brett A. Bednarcyk
Committee Participant of Aerospace Engineering
(Committee Member)

Hossein Toghiani
Associate Professor of Chemical Engineering
(Committee Member)

Charles U. Pittman
Emeritus Professor of Chemistry
(Committee Member)

Rani W. Sullivan
Associate Professor of Aerospace Engineering
(Committee Member)

J. Mark Janus
Associate Professor of Aerospace Engineering
(Graduate Coordinator)

Lori M. Bruce
Associate Dean for Research and Graduate Studies
A modified Weibull cumulative distribution function, which accounts for the effect of fiber length on the probability of failure, was used to characterize the variation in fiber tensile strength in a SCS-6/ TIMETAL 21S material system and was implemented within the framework of the NASA code MAC/GMC. A parametric study investigating the effect of repeating unit cell architecture and fiber strength distribution on the RUC-averaged ultimate composite strength and failure was performed. Multiscale progressive failure analyses of a tensile dogbone specimen were performed using FEAMAC/ ABAQUS to assess the effect of local variations in fiber strength on the global response. The effect of the RUC architecture, fiber strength distribution, and microscale/ macroscale discretization on the global response was determined. The methodology developed in this work for accounting for statistical variations in microscale properties that feed into macroscale progressive failure analyses can readily be applied to other composite material systems.
DEDICATION

This work is dedicated to my mother, Patti Finley. Her steadfast love, devotion, and instruction helped me become the person I am today and taught me the truly important things in life.
ACKNOWLEDGEMENTS

Many people have poured into my life and invested in me to help make this work a reality. First of all, I would like to thank my family and friends for their love and support during this time. I would also like to thank the numerous undergraduate and graduate students who have given insight in their respective research areas and help make research lively and enjoyable. I also want to acknowledge all of the colleagues at NASA Glenn Research Center for providing me with the opportunity to do this work as well as invest in me during my time there, particularly Steve Arnold, Brett Bednarcyk, and Evan Pineda. Much thanks are also due to my committee, Drs. Tom Lacy, Brett Bednarcyk, Charles Pittman, Hossein Toghiani, and Rani Sullivan for their insight, direction, and guidance on this work. Most of all, I would like to thank my advisor, Dr. Tom Lacy for the countless hours spent in discussion, the constant involvement in my research, and for teaching and showing me countless academic, professional, and life principles that I will carry with me the rest of my career.
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<td><em>Cumulative Distribution Function</em></td>
</tr>
<tr>
<td>MAC/GMC</td>
<td><em>Micromechanics Analysis Code with Generalized Method of Cells</em></td>
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<td>RUC</td>
<td><em>Repeating Unit Cell</em></td>
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<td>FE</td>
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CHAPTER I
INTRODUCTION

1.1 Composite Materials

Composite materials are increasingly being utilized for aerospace and automotive applications due to an increase in specific properties (i.e., specific stiffness/strength) over monolithic metallic materials [1-2]. In general, a composite material can be produced by combining two or more distinct materials (e.g., a reinforcing phase and a matrix phase) to produce a new material with more desirable characteristics and properties than either of the individual constituents [2]. The reinforcing phase typically consists of continuous fibers (monofilaments or tows), discontinuous fibers (whiskers or short fibers), or particulates. The matrix is typically polymeric, metallic, or ceramic in nature and is often used to classify composite materials. For instance, a metal matrix composite (MMC) material system could consist of a silicon carbide fiber as the reinforcing phase with a titanium matrix. Additionally, nanocomposite materials can be produced by combining nanoscale reinforcements such as carbon nanotubes/nanofibers with a matrix material.

1.2 Multiscale Modeling

Recently, a dramatic increase in research focusing on the development of multiscale modeling techniques has resulted due a significant increase in computational capabilities [3]. One goal in multiscale modeling is to provide a unified framework to
account for observed phenomena across multiple disparate length and temporal scales and relating the effects at one scale to those of the other(s). In general, this is accomplished through the use of homogenization and/or localization procedures. Homogenization procedures are used to calculate effective properties at one scale (e.g., microscale) and pass the information to a higher scale (e.g., macroscale). As a result, the individual constituents are smeared out and replaced by an effective volume of material. Conversely, localization procedures, which are also referred to as unsmearing or recovering techniques [4], address the inverse case of taking effective properties and mapping down to the preceding lower scale. However, despite the recent increase in multiscale modeling research, many of these techniques focus on determining effective material properties while few involve predictions of local failure and strength as shown in Figure 1.1 [3]. Hence, the development of a multiscale modeling strategy which accounts for distinct damage at different relevant length scales is crucial for establishing robust predictive models for the progressive failure behavior of composites.
1.3 Generalized Method of Cells

The generalized method of cells (GMC) is a robust computational micromechanics technique which can be used to model composite materials [5]. Using this methodology, a repeating unit cell (RUC) is discretized into an arbitrary number of rectangular subcells, each of which can be assigned a different material and/or constitutive model. Continuity of displacements and tractions are then imposed on the subcell boundaries in an average sense and all field quantities are evaluated at the subcell centroid. This technique can be used to simulate both doubly-periodic (2D) and triply-periodic (3D) RUCs.

In the original method of cells [6], a doubly-periodic RUC representing a unidirectional fiber-reinforced material was discretized into four subcells prior to...
performing the analysis with one subcell representing the reinforcement and three subcells representing the matrix. This technique was later generalized [7-8] to allow for an arbitrary number of subcells as well as account for both doubly- and triply-periodic RUCs. The GMC was eventually reformulated [9] to increase its’ computational efficiency. The high fidelity GMC (HFGMC) was also developed to increase the accuracy of the local stress-strain fields as well as account for extension-shear coupling, albeit at a computational expense. Comparisons of GMC and HFCMC can be found in Refs. [5,10].

The GMC has also been implemented within the framework of the NASA Micromechanics Analysis Code with the Generalized Method of Cells (MAC/GMC) [11] as a means to simulate the behavior of composite materials. In addition to performing composite property determination, MAC/GMC can be used to simulate subcell interfacial damage and to impose failure criteria both at the subcell and RUC level. This code has also been coupled with ABAQUS Standard/Explicit [12] through the use of user material subroutines resulting in another code, FEAMAC. By implementing the GMC calculations at finite element integration points within ABAQUS through FEAMAC, coupled global-to-local-to-global finite element progressive failure analyses can be performed.

1.4 Motivation for Thesis

This work focuses on the development of a multiscale modeling methodology that accounts for experimentally observed variations in fiber tensile strength. The effect of fiber length on the microscale (RUC) stress-strain and failure behavior was investigated by using a modified Weibull CDF which accounts for length scale. A distribution methodology was developed for assigning an experimentally observed, statistical
distribution of fiber tensile strengths to individual RUCs. These RUCs were then assigned to element integration points within a global FE (macroscale) model of a MMC tensile dogbone specimen, and multiscale progressive failure analyses were performed. A key part of this work is to characterize the influence of statistically varying properties at the microscale on the global macroscale response.
1.5 References


CHAPTER II

A MULTISCALE MODELING METHODOLOGY FOR COMPOSITES THAT INCLUDES FIBER STRENGTH STOCHASTICS

2.1 Abstract

A multiscale modeling methodology was developed for continuous fiber composites that incorporates a statistical distribution of fiber strengths into coupled multiscale micromechanics/ finite element (FE) analyses. A modified two-parameter Weibull cumulative distribution function, which accounts for the effect of fiber length on the probability of failure, was used to characterize the statistical distribution of fiber strengths. A parametric study using the NASA Micromechanics Analysis Code with the Generalized Method of Cells (MAC/GMC) was performed to assess the effect of variable fiber strengths on local composite failure within a repeating unit cell (RUC) and subsequent global failure. The NASA code FEAMAC and the ABAQUS finite element solver were used to analyze the progressive failure of a unidirectional SCS-6/ TIMETAL 21S metal matrix composite tensile dogbone specimen at 650°C. Multiscale progressive failure analyses were performed to quantify the effect of spatially varying fiber strengths on the RUC-averaged and global stress-strain responses and failure. The predicted composite failure behavior suggests that use of models that exploit global geometric symmetries are inappropriate for cases where the actual distribution of local fiber strengths displays no such symmetries. This issue has not received much attention in the
literature. Moreover, the model discretization at a specific length scale can have a profound effect on the computational costs associated with multiscale simulations.

2.2 Introduction

As a result of recent increases in computational capabilities, numerous models have been developed to simulate material behavior across multiple length scales [1]. While most material models are deterministic in character, real materials exhibit statistical variations in properties and features over a range of different length scales. When performing multiscale analyses, a number of challenges arise when accounting for statistically varying material characteristics [2-3]. For instance, how does statistical variability at one length scale affect the predicted material response over a hierarchy of scales including the macroscale [3]? To investigate this question, multiscale modeling strategies have been developed to account for variations in properties and morphologies at different scales (cf., Ref. [2] for a summary of different methods for fiber-reinforced polymer matrix composites). For example, Leggoe et al. [4] used a finite element (FE) approach to study the effect of statistically varying mesoscale reinforcement volume fractions on the global FE response for particulate reinforced metal matrix composites (MMCs). In addition, Xu et al. [5] and Shen and Xu [6] developed the Multiscale Stochastic Finite Element Method (MSFEM) as a means of simulating random heterogeneous materials from the micro- to meso- to macroscales.

When performing multiscale analyses involving damage and failure of fibrous composites, the stochastic variation in fiber strengths may be characterized using Weibull cumulative distribution functions (CDFs) [7]. The classic two-parameter Weibull CDF is given by:

$$F(x) = 1 - e^{-(x/\lambda)^k}$$
where $P_f$ represents the cumulative probability of failure at a given stress, $\sigma$. The Weibull scale ($\sigma_0$) and shape ($\beta$) parameters are determined from measured fiber strength data. The preceding CDF, however, does not account for the effect of fiber length on the measured strengths and has been shown to yield inaccurate strength predictions [8]. Accordingly, a modified two-parameter Weibull CDF was proposed by Watson and Smith [9] and Padgett et al. [10] to account for the effect of fiber length on the probability of failure, i.e.,

$$P_f(\sigma) = 1 - \exp \left[ - \left( \frac{\sigma}{\sigma_0} \right)^\beta \right]$$

where $L_0$ represents the reference fiber length (i.e., the length at which $\sigma_0$ and $\beta$ were determined) and $L$ represents the characteristic fiber length of interest. The unitless fiber strength parameter $\alpha$ can be determined from experimental strength data in which the tested fiber lengths are varied (e.g., $0 \leq \alpha \leq 1$). If $\alpha = 0$, then it is easily seen that Eq. 2.2 reduces to the classic two-parameter Weibull CDF (Eq. 2.1). Various researchers have used Eq. 2.2 to characterize the effect of fiber length on failure in the development of analytical models for a wide variety of fibrous composites with silicon carbide monofilaments and carbon, glass, and flax fibers [8, 11-17]. As will be shown in this study, accounting for the effect of fiber lengths on the probability of failure is of critical importance when incorporating fiber strengths into multiscale analyses and can result in drastically different macroscale strength predictions.

To simulate progressive failure at the microscale or RUC level, the modified two-parameter Weibull CDF (Eq. 2.2) was implemented within the framework of the
Micromechanics Analysis Code with the Generalized Method of Cells (MAC/GMC) [18]. MAC/GMC provides a computationally efficient means of modeling composites based on Aboudi’s method of cells micromechanics theories [19-24]. Using this method, a doubly or triply periodic RUC is discretized into an arbitrary number of subcells. Each subcell is then assigned material properties and a constitutive law to describe the local material behavior. Continuity of displacements and tractions are then enforced along the subcell boundaries in an average sense, and all field quantities are evaluated at the subcell centroids. An illustration of this scheme for a unidirectional composite is shown in Figure 2.1. Using this model, a doubly-periodic RUC is defined in the $x_2$-$x_3$ plane and is discretized into an arbitrary number of subcells along the $x_2$ direction (height) and the $x_3$ direction (width), respectively, while the fibers extend in the $x_1$ direction (length). If a triply periodic RUC is selected, the RUC can be discretized along the $x_1$ direction as well. MAC/GMC implementation within the ABAQUS Standard or Explicit [26] FE solver was also achieved by NASA GRC and is known as FEAMAC.
Previous work at NASA Glenn Research Center (GRC) investigated the progressive failure behavior of SiC/Ti MMCs using MAC/GMC and FEAMAC where a statistical distribution of vendor fiber strength data was used in multiscale analyses. Bednarcyk and Arnold [27] developed and incorporated the evolving compliant interface model into MAC/GMC where the interface between fiber subcells in adjacent mating triply periodic RUCs were given a fiber tensile strength consistent with fiber vendor data. This method was used to simulate the longitudinal failure of unidirectional composites and compared with the Curtin fiber breakage model [28-29] that combined a statistical probability of fiber failure based upon a shear lag approach. Bednarcyk and Arnold [30] later used FEAMAC to simulate the progressive failure of a longitudinally reinforced MMC tensile dogbone specimen. A traditional maximum stress failure criterion and the Curtin fiber breakage model were both implemented at the RUC level (microscale) and used to simulate progressive failure at the FE level (macroscale) using
global-to-local-to-global analyses. This work demonstrated that to realistically simulate both strength of and failure location within a tensile specimen, it is necessary to account for the stochastic fiber strength special distribution over the geometry of the specimen and not just within the individual RUC (i.e., at a material point).

When performing global analyses using FEs, it is common to use models that exploit any global geometric symmetries (e.g., half, quarter, eighth symmetry models) to reduce the number of model degrees-of-freedom and decrease the computational time. In traditional deterministic FE modeling of composites, the mean constituent strength value is typically assumed for every fiber, tow, or ply throughout the FE mesh. The use of global geometric symmetries, however, inevitably leads to symmetrical failure behavior, particularly when constant constituent strengths are employed throughout the models. As a consequence, geometric symmetries are inappropriate in the global failure analyses where the actual distribution of strengths display no such symmetries. This issue has not received much attention in the literature. One central goal of this work is to present a method for systematically assigning an experimentally determined spatial distribution of fiber strengths to individual RUC subcells and/or FEs. These RUCs can then be analyzed separately or implemented within a multiscale framework (e.g., FEAMAC).

The first part of this study analyzes the effect that a stochastic distribution of fiber strengths has on the RUC-averaged stress-strain response and failure using MAC/GMC. While this initial study provides insight into local material behavior, it is important to understand how variations in the distribution of local fiber strengths at the RUC level affect the macroscale stress-strain and failure response when combined micromechanical/FE analyses of a MMC tensile dogbone specimen are performed.
2.3 Material System

The effect of a statistical variation in fiber strengths on the failure behavior of a 25% fiber volume fraction SCS-6/ TIMETAL 21S MMC at 650°C was analyzed in this study. The SCS-6 fiber is a high-stiffness, high-strength silicon carbide monofilament with a diameter of approximately 142 μm. In a previous work [30], a two parameter Weibull probability density function (PDF) was fit to the fiber strength data (Weibull, scale parameter $\sigma_0 = 4198.9$ MPa and shape parameter $\beta = 10$). These Weibull parameters were determined based on monofilament tensile tests at a fiber length of $L_0 = 25.4$ mm [30]. Additionally, to account for the fiber strength dependence on temperature, the Weibull scale factor was reduced by 5.3% for simulations performed at 650°C [30-31]. In the current study, the SCS-6 fiber was assumed to be linearly elastic and isotropic.

TIMETAL 21S is a metastable beta strip titanium alloy possessing a high strength and good creep and oxidation resistance. The titanium matrix is considered to be viscoplastic and was simulated using the Generalized Viscoplasticity with Potential Structures (GVIPS) constitutive model [32]. Table 2.1 contains a summary of the thermoelastic material properties for both the SCS-6 fiber and the TIMETAL 21S matrix [30]. The viscoplastic material properties employed in this study can be found in Ref. 30.
Table 2.1 Material properties for SCS-6 and TIMETAL 21S

<table>
<thead>
<tr>
<th>Material</th>
<th>Temperature (°C)</th>
<th>Young's Modulus (GPa)</th>
<th>Poisson's Ratio</th>
<th>Coefficient Of Thermal Expansion (1x10^-6/°C)</th>
</tr>
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<tr>
<td>SCS-6</td>
<td>21</td>
<td>393</td>
<td>0.25</td>
<td>3.56</td>
</tr>
<tr>
<td></td>
<td>316</td>
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<td></td>
<td>860</td>
<td>358</td>
<td>0.25</td>
<td>4.57</td>
</tr>
<tr>
<td>TIMETAL 21S</td>
<td>23</td>
<td>114.1</td>
<td>0.365</td>
<td>7.717</td>
</tr>
<tr>
<td></td>
<td>300</td>
<td>107.9</td>
<td>0.365</td>
<td>9.209</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>95.1</td>
<td>0.365</td>
<td>10.700</td>
</tr>
<tr>
<td></td>
<td>650</td>
<td>80.7</td>
<td>0.365</td>
<td>12.130</td>
</tr>
<tr>
<td></td>
<td>704</td>
<td>59.7</td>
<td>0.365</td>
<td>14.090</td>
</tr>
</tbody>
</table>

Note: Table adapted from [30].

2.4 RUC Analyses

A parametric study using MAC/GMC was performed to assess the effect of variable fiber strength distributions and simulated RUC architectures on global composite failure at an elevated temperature of 650°C. Recognizing that individual RUCs will be assigned to FE integration points in multiscale analyses using ABAQUS, understanding the influence of variable fiber strength distributions and RUC architectures on local failure is crucial for predicting the progressive composite failure at the global structural level. Five different doubly-periodic RUCs were analyzed in this study: 2x2, 4x4, 6x6, 10x10, and 14x14 subcell RUCs. These RUCs have 1, 4, 9, 25, and 49 fiber subcells, respectively, while maintaining a constant fiber volume fraction of 25%. Figure 2.2 shows the single-fiber (2x2) and 25-fiber (10x10) doubly periodic RUCs considered in this study. Note that a 10x10 RUC can be subdivided into 25 individual 2x2 RUCs. The MMC material system considered in this work was specifically fabricated to provide
uniform fiber volume fractions throughout the composite [34] and hence, these RUC architectures are representative of the as-fabricated material. For each individual fiber subcell in the RUC, a fiber tensile strength value obtained from an experimentally determined Weibull CDF was assigned. Essentially, a random number was generated (i.e., [0,1]) and used to solve Eq. 2.2 for the fiber strength, $\sigma$. These strength values were then assigned to the individual subcells in the microscale analyses. To assess the effect of fiber length on the predicted RUC response, MAC/GMC analyses were performed where the fiber length dependent strength was obtained from Eq. 2.2. For example, since the 2x2 RUC has one fiber subcell, an individual fiber tensile strength was selected using the modified Weibull CDF (Eq. 2.2) and assigned to the fiber subcell. Similarly, for the 4x4 RUC, four fiber tensile strengths were selected using the CDF and assigned to the RUC.

![Microstructural representation of a unidirectional SiC/Ti composite](image)

**Figure 2.2** Microstructural representation of a unidirectional SiC/Ti composite

Notes: Representation of a) a single-fiber RUC and b) a 25-fiber RUC.

In these MAC/GMC analyses, the modified Weibull parameters $\alpha = 1$, $L = 1.27$ mm, and $L_0 = 25.4$ mm ($L/L_0 = 0.050$) were chosen. This value of $L$ corresponds to the average FE dimension measured in the fiber direction as will be discussed later.
Simulations were also performed where $\alpha = 0$ in order to bound the effect of fiber length on the fiber strength distribution. Figure 2.3 compares the CDFs and associated PDFs for Eq. 2.2 for the case where $0 \leq \alpha \leq 1$. Since the characteristic fiber length is much smaller than the reference length (i.e., $L/L_0 = 0.050$), the length dependent strength distribution is shifted to higher stresses as $\alpha$ increases. Since the modified Weibull CDF (Eq. 2.2) is based on “weakest link” theory, a distribution of shorter fibers will typically have higher strengths than an analogous distribution of longer fibers due to the decreased likelihood of severe flaws. Additionally, for $L/L_0 \leq 1$, the mean fiber strength increases and there is more scatter in the strength distribution as $\alpha$ is increased. Since a longer fiber will have a higher probability of having more flaws, there is similarly a higher probability that a severe flaw would be present thus resulting in a lower mean strength and less scatter in the fiber strength distribution (cf., Fig. 2.3). When the extreme cases are examined (i.e., $\alpha = 0$, the fiber strength distribution is based upon measured data for 25.4 mm long fibers; $\alpha = 1$, the distribution of fiber lengths is consistent with typical FE dimensions), the resulting fiber strength distribution is noticeably different. This underscores the importance of accounting for fiber lengths in an appropriate manner.
Figure 2.3  Modified Weibull probability functions

Notes: Representation of the a) CDF and b) PDF as a function of the fiber strength parameter $\alpha$.

Similar to the work in Bednarcyk and Arnold [30], residual stresses in the fiber and matrix subcells were accounted for by simulating a 16 hour stress-free cooldown.
from the heat treatment temperature to room temperature, followed by a stress-free
temperature rise to 650°C over five minutes. A uniform axial strain in the $x_1$ direction
(*c.f.*, Fig. 2.1) was then applied at a rate of $1 \times 10^{-4}$/s. While the matrix was permitted to
yield in accordance with the GVIPS model, ultimate failure (fracture) of the matrix was
not considered in the analyses. 100 MAC/GMC simulations were performed for each
individual RUC at 650°C to estimate the mean and range in the RUC-averaged tensile
strengths associated with the stochastic distribution of fiber strengths. This resulted in a
total of 1000 distinct MAC/GMC simulations performed in this study.

Figure 2.4 shows a plot of the RUC-averaged composite tensile strengths resulting
from a statistical distribution of individual fiber strengths in RUCs with 1, 4, 9, 25, and
49 fiber subcells, respectively, where two different fiber length dependent strength
distributions were employed in MAC/GMC (*i.e.*, $\alpha = 0$; $\alpha = 1$, $L/L_0 = 0.050$). As would be
expected, the simulations that account for the effect of fiber length on strength ($\alpha = 1$)
yield a higher mean RUC-averaged strength and more variation in calculated strengths
than simulations where $\alpha = 0$ (*i.e.*, no fiber length dependence). For both cases, when one
fiber strength is used (*i.e.*, a 2x2 RUC), a higher mean RUC-averaged strength results. As
the number of fiber subcells is increased, however, the strength decreases. Similarly, the
range in RUC-averaged strength decreases as the number of fiber subcells is increased.
When only one strength value is employed in the model (*i.e.*, using a 2x2 RUC), the
RUC-averaged strength becomes highly dependent on the selected strength resulting in a
higher mean RUC-averaged strength and more variation in calculated strengths. This
effect is not as pronounced as the number of fiber strength values used in the model is
increased since the load carried by one fiber at failure gets primarily shed to the
remaining fibers. By increasing the number of strength values employed in the model, a higher probability of selecting a weaker fiber occurs thus leading to a decrease in mean RUC-averaged strength. Additionally, as the number of fiber subcells is increased, the distribution of fiber strengths within individual RUCs of the same architecture become more similar resulting in less variation in predicted ultimate strength. For strength distributions where $\alpha = 0$ and $\alpha = 1$, as the number of fiber subcells approaches 25 (10x10 RUC), only a slight difference is observed in the mean value and variation in the RUC-averaged strength. While the use of a 25-fiber subcell RUC led to approximately the same RUC-averaged strength as for a 49-fiber subcell RUC, roughly one-half the computational time was needed. This savings in computational time becomes crucial when performing global-to-local-to-global simulations.
Figure 2.4  Mean RUC-averaged ultimate strength

Notes: Strength is compared against the number of fiber subcells (at a constant fiber volume fraction) after 100 simulation runs per RUC with the error bars corresponding to the maximum and minimum ultimate strength values out of the 100 simulations. Eq. 2.2 was used to generate the fiber strength values for $\alpha = 0$ and $\alpha = 1$ ($L/L_0 = 0.050$).

Figures 2.5a-e contain ten typical RUC-averaged uniaxial stress-strain curves in the $x_1$ (fiber) direction for the single-fiber, four-fiber, nine-fiber, ten-fiber, and 25-fiber RUCs, respectively. Similar results were obtained for simulations that accounted for the effect of fiber length on strength ($\alpha = 1$). For all of the RUC architectures, the RUC-averaged stress-strain response increased monotonically up until the onset of fiber failure. For RUCs containing only one fiber subcell (i.e., a $2\times2$ RUC), once the fiber failed, the RUC-averaged axial stress-strain response displayed a discrete sudden load drop and the remaining stress was carried by the matrix (Fig. 2.5a). The strain at failure, of course, was a strong function of the assumed fiber strength. A comparison of Fig. 2.5a-e suggests that as the number of fiber subcells is increased, a gradual softening behavior is observed. The
total strain range over which failure occurred was nearly constant irrespective of the number of fiber subcells. Additionally, the variation in the RUC-averaged stress-strain response diminishes as the number of fiber subcells increases since the local RUC fiber strength distribution becomes more similar to other RUCs. By adding more fiber subcells, a more gradual continuum-like local stress-strain response is observed, as each fiber failure represents a smaller fraction of the RUC volume. Clearly, understanding the progressive failure behavior associated with a statistical fiber strength distribution at the RUC level is crucial to establishing and accurately capturing length scale effects in a robust computationally efficient multiscale modeling methodology.
Figure 2.5  RUC stress-strain curves

Notes: A random sampling of ten stress-strain curves from a batch of 100 simulations for a) single-fiber RUC b) four-fiber RUC c) nine-fiber RUC d) 25-fiber RUC e) 49-fiber RUC using Eq. 2.2 for the $\alpha = 0$ case.
2.5 Coupled FE/Micromechanics Analyses

While the preceding parametric study investigated the effect of RUC architecture and fiber strength distributions on the RUC-averaged local failure, the ultimate goal of this work is to use RUC deformation and damage evolution at a given integration point within an FE analysis (i.e., FEAMAC/ABAQUS) to perform *global* progressive failure analyses of a 25% fiber volume fraction SCS-6/TIMETAL 21S dogbone specimen under a monotonic tensile load at 650°C. Figure 2.6 contains a schematic of the NASA GRC dogbone specimen [33]. Such specimens were specifically designed to reduce the magnitude of the stress concentration associated with a reduction in cross-sectional area commonly observed in dogbone tensile test specimens. The two FE models were constructed using relatively coarse (2400 elements) or fine (19,200 elements) meshes comprised of eight-noded linear isoparametric brick elements with eight integration points per element. Initial analyses showed that the use of higher-order quadratic elements had a negligible impact on the predicted stress-strain and failure response for this problem. Since FEAMAC assigns an RUC to each element integration point, MAC/GMC is called over 150,000 times *per time step* for the fine FE mesh. Hence, the computational efficiency of MAC/GMC [24] is a crucial element in the multiscale FE analyses performed here and is essential to the analysis of more complex structures.

Similar to the previous local RUC analyses using MAC/GMC, thermal residual stresses were determined from FEAMAC/ABAQUS analyses involving a 16 hour assumed stress-free cooldown from the heat treatment temperature to room temperature. This was followed by a stress-free temperature rise to 650°C over five minutes. Multiscale progressive failure analyses were then performed with a constant temperature
distribution at 650°C. The surface nodes corresponding to the machine grips were fixed at one end of the specimen while the surface nodes in the grip region at the opposite end of the specimen were given a longitudinal tensile displacement consistent with an initial elastic strain rate of 1x10^{-4}/s in the gage section.

![NASA GRC MMC tensile dogbone specimen](image)

**Figure 2.6** NASA GRC MMC tensile dogbone specimen

Notes: a) Specimen geometry with top and side views of the b) coarse 2400 FE mesh and c) fine 19,200 FE mesh.

The coarse FE mesh was initially used to study the effect of RUC architecture and fiber strength parameter ($\alpha$) on the global composite stress-strain response failure behavior. In each simulation, RUCs containing single-fiber (2x2 subcells) or four-fiber (4x4 subcells) RUCs were generated (*i.e.*, only one RUC geometry per simulation). As an aside, special care should be taken to ensure that the actual material volume associated with an RUC does not exceed that of the FEs used in the analysis, *i.e.*, the RUC-averaged
continuum response would occur over a domain larger than the typical element size. Recall in the previous local MAC/GMC analyses, large amounts of variation in the RUC-averaged composite strength were observed for both the 2x2 and 4x4 RUCs. Such variability in local properties will manifest itself at the macroscale in multiscale progressive failure analyses. One crucial consideration is to retain a sufficient level of model discretization at each simulation scale to provide accurate results without excessive computational costs. 96 RUCs, each with fiber strength values based upon the modified Weibull CDF given by Eq. 2.2, were randomly assigned in equal numbers to individual FEs throughout the mesh. The fiber strength parameter was varied between $0 \leq \alpha \leq 1$ in increments of 0.25. The characteristic length $L = 1.27$ mm ($L/L_0 = 0.05$) corresponded to the typical element length in the coarse mesh. The element length ($L$) was chosen as a length scaling parameter in order to validate the methodology for different FE meshes. Figure 2.7 contains an overview schematic of the methodology used in assigning local fiber properties to individual RUCs and then distributing RUCs throughout the FE mesh. This process was performed ten times for each RUC architecture (single-fiber, four-fiber) for $\alpha = 0, 0.25, 0.5, 0.75$, and 1.0 resulting in a total of 100 multiscale FE simulations using the coarse mesh. The results from each of the ten analyses were averaged together to obtain the macroscale composite strength.
Figure 2.7  Fiber strength distribution scheme for multiscale analyses

Notes: First, the input parameters for the strength distribution and the number/architecture of the RUC are determined. Then the fiber strengths are generated by using a random number generator and solving Eq. 2.2 for the stress, $\sigma$. These strengths are then assigned to individual fiber subcells within an RUC, and the process is repeated until all RUCs have been defined. Finally, the RUCs are randomly assigned in equal numbers to FEs within the ABAQUS model.

Figure 2.8 shows the average macroscale composite ultimate strength and associated standard deviation for multiscale analyses performed using single-fiber and four-fiber RUCs as a function of the fiber strength parameter $\alpha$. As $\alpha$ is increased, the predicted ultimate strength increases proportionally (Fig. 2.8a). This is due to the increasingly pronounced effect of fiber length on strength as $\alpha \rightarrow 1$ (cf., Fig. 2.3b). In contrast to the \textit{local} MAC/GMC calculations (Fig. 2.4), the use of a four-fiber RUC...
within a global FE analysis leads to higher ultimate strengths than for a single-fiber RUC for all values of $\alpha$. In the multiscale failure analyses, as the far-field global strain is increased, local fiber failures initiate in a distributed fashion throughout the specimen in lower length scale RUCs surrounding FE integration points. This process leads to failure localization within individual RUCs as well as throughout the global FE mesh, culminating in ultimate specimen failure. When single-fiber RUCs are employed in multiscale analyses, some load shedding within an element and between elements is possible once initial fiber failure occurs. However, after the single fiber fails, the load is rapidly shed to neighboring elements, increasing the likelihood of damage localization which leads to a reduction in the predicted composite ultimate strength. While the calculated variation in strength values is less than 5% regardless of $\alpha$ (Fig. 2.8b), the variation in the composite strengths is less for calculations performed using a four-fiber RUC than those for the single-fiber RUC. Additionally, the experimental ultimate strength for this specimen is 973 MPa [34]; this suggests that the fiber strength parameter is likely less than unity. Of course, further testing is required to fully establish an appropriate $\alpha$ value.
Figure 2.8  Strength results from multiscale progressive failure analyses

Notes: a) Average macroscale composite ultimate strength and b) standard deviation as a function of fiber strength parameter \(0 \leq \alpha \leq 1\) over ten ABAQUS/FEAMAC simulations using single-fiber and four-fiber RUCs with the coarse FE mesh. Note that the experimental ultimate strength was 973 MPa [34].

This variability in predicted macroscale composite strength can also be seen in the global FE stress-strain responses. For example, Fig. 2.9a and 2.9b show the predicted gage section stress-strain curves for \(\alpha = 0\) from multiscale analyses performed using both single-fiber and four-fiber RUCs, respectively. Here, the macroscale (continuum averaged) response was determined from the family of elements comprising the gage
section of the specimen. Included in the figure is the measured response from Ref. 34. Both sets of calculations reasonably matched the observed specimen behavior, but the variability in predicted strengths was lower for simulations performed using a four-fiber RUC (Fig. 2.9b) than for a single-fiber RUC (Fig. 2.9a). While an increase in the fiber strength parameter, $\alpha$, can be used to increase the mean composite strength, use of RUCs containing increasing numbers of fibers will reduce the variability in predicted strengths.
Figure 2.9  Gage section stress-strain response of multiscale progressive failure simulations with variable fiber strengths

Notes: Represents a longitudinally reinforced SCS-6/ TIMETAL 21S MMC for simulations using a) single-fiber RUC and b) four-fiber RUC. The coarse FE mesh was used and RUC fiber subcell strengths were assigned using $\alpha = 0$. Results are compared against gage section experimental data obtained from [34].

This also explains the slightly higher average strength obtained using a four-fiber RUC as shown in Figs. 2.8a and 2.9a. For example, Fig. 2.10 shows the distribution of failed elements after damage localization has occurred for a series of FE meshes with...
different distribution of fiber strengths as well as images of three fractured test specimens (Fig. 2.10d). For instance, progressive failure analyses were performed where a constant strength value was used throughout the mesh (Fig. 2.10a). For this case, although no a priori geometric symmetry has been imposed, a symmetric distribution of local failures occur, consistent with results obtained when using one-quarter or one-eighth symmetry models as in Ref. 30. In contrast, Figs. 2.10b and 2.10c show the distribution of failed elements for each of five multiscale analyses employing single-fiber and four-fiber RUCs, respectively, where a statistical distribution of fiber strengths was employed. Here, the localization of fiber failure occurs throughout the gage section as shown in Figs. 2.10b and 2.10c consistent with the experimentally observed fracture behavior (Fig. 2.10d). Note that much more widespread damage/failure is observed for the four-fiber RUC simulations (Fig. 2.10c) than the single-fiber RUC simulations (Fig. 2.10b). Using a single-fiber RUC, the rapid onset of localized failure also leads to fewer simulated fiber failures across the model when compared against the simulations using a four-fiber RUC.
Figure 2.10 Distribution of fiber failures within the coarse FE mesh after the onset of localization

Notes: Failure Behavior of a) single-fiber RUC with a constant fiber strength value for all elements and b) five single-fiber RUC and c) five four-fiber RUC distributed fiber strength simulations with $\alpha = 0$ where blue represents no failure and red indicates complete fiber subcell failure for a given element. d) experimentally observed failure behavior.

For illustration purposes, ten additional multiscale progressive failure analyses using a coarse FE mesh were performed in which a distribution of fiber strengths ($\alpha = 0$) were employed within 25-fiber RUCs. This RUC was used since it led to local RUC ultimate strengths that were somewhat independent of the number of fiber subcells (cf., Fig. 2.4). Predictions of the macroscale composite stress-strain response and ultimate strength from these analyses were compared to a similar analysis where the local fiber strength was held constant throughout the mesh (cf., Fig. 2.10a). In the constant strength analysis, a fiber strength corresponding to the mean value from the Weibull fiber strength distribution was used. The predicted macroscale stress-strain response in the vicinity of the ultimate composite failure obtained using a constant local fiber strength was markedly different from the responses from analyses where a spatial distribution of local fiber
strengths was simulated. Figure 2.11 contains the measured uniaxial stress-strain response for a NASA SCS-6/TIMETAL 21S dogbone specimen, as well as the predicted results obtained using a constant local fiber strength and variable local fiber strengths. As can be seen from the figure, use of a constant local fiber strength led to a prediction of a macroscale composite strength that over-predicted the observed strength by approximately 12%. Moreover, the location of failed elements occurs in regions with mild stress concentrations (cf., Fig. 2.10a). These results are inconsistent with experimental observations where the actual specimens predominately fail within the gage section (Fig. 2.10d). In contrast, the ultimate composite strengths obtained using spatially varying local fiber strengths better matched the measured strength, and the locations of the predicted failures predominately occurred within the gage section, consistent with experimental observations (Fig. 2.10d). In general, use of a constant local fiber strength led to a predicted macroscale strength that was roughly 25% greater than that for the variable fiber strength simulations. These results underscore the importance of accounting for spatial variations in the distributions of fiber strengths when performing multiscale analyses.
Figure 2.11  Gage section stress-strain response of multiscale progressive failure simulations with variable fiber strengths (25-fiber RUC)

Notes: Represents a longitudinally reinforced SCS-6/ TIMETAL 21S MMC for simulations using 25-fiber RUCs with the coarse FE mesh and $\alpha = 0$. These are compared against the case where a constant fiber strength was used throughout the model and experimental data obtained from [34].

The preceding multiscale calculations investigated the effect of variations in the local distribution of fiber strengths on global MMC composite failure based upon a fixed level of macroscale discretization, *i.e.*, a relatively coarse FE mesh was used to simulate the global response. One key element in the development of robust, computationally-efficient multiscale materials models is to assess the appropriate level of model discretization at each relevant length scale that leads to tractable yet accurate macroscale solutions. To illustrate this point, consider an arbitrary SCS-6/ TIMETAL 21S MMC subvolume containing 16 uniformly distributed aligned fibers. The continuum-level material response of this subvolume may be determined from multiscale models that employ different levels of discretization at the macroscale and microscale, respectively.
For example, using eight-noded linear isoperimetric elements (each with eight integration points), the 16 fiber subvolume could be idealized using one FE to simulate the global response and a *four-fiber RUC* at each integration point to simulate the local axial response using MAC/GMC (*cf.*, Fig. 2.12). Alternatively, the material subvolume could be idealized using *four* FE s at the macroscale and a *single-fiber RUC* surrounding each FE integration point (*cf.*, Fig. 2.12). Of course, the latter multiscale discretization results in an eight-fold increase in the number of FE degrees-of-freedom at the macroscale. Both multiscale discretizations were used to perform multiscale progressive failure analyses of the NASA SCS-6/ TIMETAL 21S dogbone specimen, where relatively coarse (2400 element) and fine (19,200 element) global finite element models (Fig. 2.6) were used in conjunction with four- and single-fiber RUCs, respectively.
Figure 2.12  Comparison of mesh discretization at different scales

Notes: A constant simulation volume is maintained. The first case combines a coarse (2400 FE) mesh with a four-fiber RUC while the second case combines a fine (19,200 FE) mesh (*i.e.*, double the mesh density) with a single-fiber RUC.

Analyses were performed using each discretization where the local fiber strengths were assigned to individual subcells in the same manner as before using the modified Weibull CDF (Eq. 2.2) with the fiber strength parameter $\alpha = 0$ (no effect of fiber length on strength) or $\alpha = 1$ (fiber strength distribution was a function of fiber length); in the latter analyses the characteristic fiber length ($L$) in Eq. 2.2 corresponded to the average FE dimension in the axial direction for the coarse ($L = 1.27$ mm) and fine ($L = 0.64$ mm) meshes, respectively. Note that as the characteristic length ($L$) is reduced for the fine FE mesh, the mean fiber strength and variation in the associated distribution of fiber strengths increase slightly (Fig. 13). Ten multiscale simulations were performed for each
multiscale discretization/ local fiber strength parameter combination, and the average macroscale ultimate composite strength was determined for each case.

Figure 2.13 Modified Weibull PDFs employed in the multiscale discretization study for both the coarse and fine FE meshes.

Notes: For the $\alpha = 0$ case, the fiber strength distribution no longer depends on length while when $\alpha = 1$, the finite element length effects the strength distribution.

Figure 2.14 contains a plot of the macroscale stress-strain responses from each of five representative multiscale analyses performed using the coarse mesh/ four-fiber subcell and fine mesh/ single-fiber subcell discretizations for $\alpha = 0$. Note that the essential character of the predicted stress-strain responses is the same for each discretization. Similar results were obtained using both discretizations for $\alpha = 1$, where the predicted composite ultimate strengths increased somewhat relative to case where $\alpha = 0$. Table 2.2 contains a summary of the average macroscale composite strength, standard deviation, and range in predicted strength values for each multiscale
discretization/ local fiber strength parameter combination. For a given value of $\alpha$, the average composite strength and the range in calculated strengths were nearly identical for each multiscale discretization. As an aside, the predicted strengths obtained using $\alpha = 1$ substantially over-predicted the observed strength for the MMC specimen. This suggests that use of a lower value for the fiber strength parameter, based upon additional experimental testing, is likely warranted for this material system. Both discretizations resulted in widespread fiber failures that were distributed throughout the gage section. For example, Figure 2.15 shows the distribution of FEs with failed fiber subcells from five separate analyses obtained using the fine mesh/ single-fiber subcell discretization for $\alpha = 0$. The distribution of failure locations is similar to that found using a coarse mesh/ four-fiber subcell discretization for $\alpha = 0$ (Fig. 2.10c). These results suggest that both multiscale discretizations lead to similar estimates of the macroscale composite material behavior. The solution time for the fine mesh/ single-fiber subcell analyses, however, was roughly three times greater than for the coarse mesh/ four-fiber subcell analyses without a significant difference in calculated results. Clearly, the model discretization at a specific length scale can have a profound effect on the computational costs associated with multiscale simulations. Since the solution algorithm implemented in MAC/GMC is substantially more efficient than that for traditional FE analyses [24], use of a more highly refined microscale model in combination with a coarser global FE mesh led to more computationally effective solutions for this problem. In general, the optimal discretization at each relevant length scale is likely to be problem dependent. For instance, problems with a global stress gradient (e.g., open-hole composites or full scale
structures), may require a different discretization than that used here. These sorts of issues must be addressed in order to fully exploit the benefits of multiscale analyses.

Figure 2.14  Gage section stress-strain results from the mesh discretization study

Notes: Represents a longitudinally reinforced SCS-6/ TIMETAL 21S MMC for multiscale simulations ($\alpha = 0$) using the coarse FE mesh with a four-fiber RUC and the fine FE mesh with a single-fiber RUC.

Table 2.2  Macroscale strengths from multiscale analyses involving coarse FE mesh/four-fiber RUCs, and fine FE mesh/single-fiber RUCs

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<th>Modified Weibull CDF ($\alpha=0$)</th>
<th>Modified Weibull CDF ($\alpha=1$)</th>
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<td>Fine FE + Single-Fiber RUC</td>
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<td>Mean</td>
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<td>Maximum</td>
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</tr>
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Figure 2.15  Distribution of fiber failures within the fine mesh after the onset of localization

Notes: Represents five single-fiber RUC distributed fiber strength simulations where blue represents no failure and red indicates complete fiber subcell failure for a given element. Each illustration denotes the outer layer of elements in a separate simulation.

2.6 Summary and Conclusions

A parametric study investigating the effect of a statistical fiber strength distribution and repeating unit cell (RUC) architecture was performed using the computationally efficient code, MAC/GMC, for an SCS-6/ TIMETAL 21S material system. Progressive failure at the microscale was simulated at an elevated temperature of 650°C by randomly distributing fiber strengths to individual RUC subcells based upon a modified Weibull cumulative distribution function (CDF), which accounts for the effect of fiber length on the probability of failure. By increasing the number of fibers in the RUC, a more gradual, continuum-like stress-strain behavior was observed.

Global multiscale progressive failure analyses of a 25% fiber volume fraction SCS-6/ TIMETAL 21S tensile dogbone specimen were performed at 650°C by
implementing the modified Weibull CDF of fiber strengths within FEAMAC/ABAQUS. Fiber strengths were appropriately assigned to individual fiber subcells within RUCs surrounding FE integration points in order to assess the effect of a spatial distribution of local fiber strengths on the macroscale predicted composite stress-strain response and failure. The ultimate composite strengths and distribution of failure locations (predominately within the gage section) reasonably matched the experimentally observed failure behavior. Moreover, these analyses suggest that the use of models that exploit global geometric symmetries biases the characteristics of failure and are thus inappropriate for cases where the actual distribution of local fiber strengths displays no such symmetries. This issue has not received much attention in the literature. Additionally, the discretization at a specific length scale can have a profound effect on the computational costs associated with multiscale simulations. Multiscale analyses were performed using coarse FE mesh/four-fiber subcell and fine FE mesh/single-fiber subcell discretizations. Both multiscale discretizations led to similar estimates of the macroscale composite material behavior and failure. The solution time for the fine mesh/single-fiber subcell analyses, however, was roughly three times greater than for the coarse mesh/four-fiber subcell analyses. Clearly, the model discretization at a specific length scale can have a profound effect on the computational costs associated with multiscale simulations. Understanding these issues is crucial to the development of robust multiscale material models that yield accurate yet tractable results.
2.7 References


CHAPTER III
CONCLUSIONS AND RECOMMENDATIONS

3.1 Conclusions

While the use of multiscale modeling techniques to determine effective properties has received much attention in the literature, less attention has been focused on using these techniques to predict failure and/or account for damage. This study presents a multiscale modeling methodology for composite materials in which stochastic variations in fiber strength were simulated at the microscale. A modified Weibull cumulative distribution function (CDF), which accounts for the effect of fiber length on the probability of failure, was used to characterize the fiber strength distribution. Progressive failure of a 25% fiber volume fraction unidirectional SCS-6/ TIMETAL 21S metal matrix composite (MMC) at 650°C was simulated by implementing the modified Weibull CDF of fiber strengths at the microscale within the framework of the Micromechanics Analysis Code with the Generalized Method of Cells (MAC/GMC).

A parametric study using MAC/GMC was first performed to assess the effect of variable fiber strength distributions and simulated repeating unit cell (RUC) architectures on the RUC-averaged ultimate strength and failure at an elevated temperature of 650°C. This resulted in a total of 1000 distinct simulations. By increasing the number of fiber subcells (i.e., at a constant fiber volume fraction), a more gradual continuum-like local stress-strain response was observed. A decrease in the RUC-averaged ultimate strength
and variation in ultimate strengths was also found as the number of fiber subcells was increased. This was attributed to an increased probability of simulating a weak fiber.

By implementing these RUCs within FEAMAC/ABAQUS, global multiscale progressive failure analyses of a 25% fiber volume fraction SCS-6/TIMETAL 21S dogbone specimen under a monotonic tensile load at 650°C were performed. A local distribution of fiber strengths was assigned to individual RUCs based upon the modified Weibull CDF prior to distributing the RUCs throughout a relatively coarse or fine finite element (FE) mesh.

In the first set of analyses, the coarse finite element (FE) mesh was used to capture the effect of the fiber strength parameter (a) on the macroscale (FE) response resulting in a total of 100 multiscale progressive failure analyses. The predicted composite ultimate strength was found to increase proportionally with a due to a more pronounced effect of fiber length on strength as a→1. In contrast to the local MAC/GMC calculations, use of a four-fiber RUC within a global FE analysis led to higher ultimate strengths and less variation in strengths than for a single-fiber RUC for all values of a. However, when using single-fiber RUCs, a rapid onset of localized failure occurred which led to fewer simulated fiber failures across the model than simulations using a four-fiber RUC. It is believed that if simulations were performed at room temperature as opposed to 650°C, the failure behavior would be similar although a higher effective stiffness, higher composite ultimate strength, and lower strain to failure would be expected.

Additional multiscale progressive failure analyses were performed in which a distribution of fiber strengths (a = 0) were employed within 25-fiber RUCs and compared
against the case where a constant local fiber strength was used throughout the model. The use of a constant local fiber strength led to a macroscale composite strength prediction somewhat larger than the observed strength, as well as a symmetric distribution of failed elements inconsistent with experimental observations. Such results are identical to those obtained using one-eighth symmetry models with a constant fiber strength. In contrast, the ultimate composite strengths obtained using spatially varying local fiber strengths better matched the measured strength and the experimentally observed failure behavior. Therefore, the use of global geometric symmetries is inappropriate in failure analyses where the actual distribution of strengths in the specimen display no such symmetries and can lead to inaccurate results.

In order to characterize the effect of model discretization at different length scales within multiscale analyses, both the macroscale (FE) and microscale (RUC) discretizations were varied for a fixed material subvolume. In these simulations, the relatively coarse and fine global FE models were paired with four- and single-fiber RUCs, respectively. While both multiscale discretizations lead to similar estimates of the macroscale composite material behavior, the solution time for the fine mesh/ single-fiber subcell analyses was roughly three times greater than for the coarse mesh/ four-fiber subcell analyses. Since the solution algorithm implemented in MAC/GMC is substantially more efficient than that for traditional FE analyses, use of a more highly refined microscale model in combination with a coarser global FE mesh led to more computationally effective solutions for this problem while providing similar results. Understanding these issues are key in the development of robust, computationally-efficient multiscale materials models that retain an appropriate level of model
discretization at each relevant length scale yet yield accurate tractable macroscale solutions.

3.2 Recommendations

This study highlighted some of the issues and challenges associated with multiscale modeling of composite materials. As a result, several recommendations can be made for future studies. For any simulation involving tensile dominated failure of composites, it is crucial to simulate a statistical distribution of fiber strengths at the microscale in order to accurately capture the failure behavior at the macroscale. Additional studies could further investigate the competing influence of discretization at different scales on the macroscale response for problems including other composite materials (ceramic and polymer matrix composites), fiber architectures (laminated or woven fabric composites) and damage (fiber matrix debonds, delaminations, etc.). In addition, while the dogbone specimen considered in this study led to uniform stress at the macroscale, would a gradient in the global stress field (e.g., open-hole composites) affect the optimal global element type and lower scale discretization? While this study involved calculations at two length scales, inclusion of additional length scales in the simulations (e.g., molecular dynamics) will require a rigorous assessment of the handshake protocol between calculations performed at different scales as well as the computational efficiency at each scale. Similarly, if multiscale modeling approaches are extended to simulate full scale structures, it becomes desirable to selectively employ multiscale calculations in regions with large stress or strain concentrations. Subdomains with no evolving microstructure unnecessarily increase computational costs. Factors such as these are crucial to developing a robust multiscale analysis methodology.