Blade element approach for computational modeling of lift driven horizontal axis wind turbine performance

Abraham Ittycheri

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Blade element approach for computational modeling of lift driven horizontal axis wind turbine performance

By

Abraham Ittycheri

Approved by:
Shanti Bhushan (Major Professor)
Omid Askari
Prashant Singh
Yucheng Liu (Graduate Coordinator)
Jason Keith (Dean, Bagley College of Engineering)

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Name: Abraham Ittycheri

Date of Degree: November 25, 2020

Institution: Mississippi State University

Major Field: Mechanical Engineering

Major Professor: Shanti Bhushan

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Candidate for Degree of Master of Science

The United Nations have declared the effects of climate change as the “defining issue of our time” (United Nations, 2019). As a result of increased industrialization in the last century to keep up with the demands of a growing global population, the global output of greenhouse emissions has rocketed, which is linked to the shifting and abnormal weather patterns of the planet. Electricity and heat production alone are attributed to generating 25% of greenhouse gas emissions (Edenhofer, et al.). To alleviate the increasing levels of carbon emission there is an effort to transition in green energy power generation sources like wind energy that is abundantly available in the midwestern United States.

This study aims to implement the Blade Element Method derived modeling methods for predicting the performance of a wind turbine. The experimental results obtained from the MEXICO project is employed as the validation source for the research.
DEDICATION

To my dearest mother, mama jan and my kindest ammachi.
ACKNOWLEDGEMENTS

The sincerest appreciation to Dr. Bhushan whose immense advisement even through holidays has helped this thesis possible. I would also like to thank Delia for guiding me through this complicated process and to Dr. Askari and Dr. Singh whose input was key in development of the paper. Lastly, I would like to thank the members involved with the MEXICO rotor project for conducting the experiment and publishing the experimental data.
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CHAPTER I
INTRODUCTION

1.1 Motivation for research

The effects of climate change have been taking a toll on biodiversity with a significant number of species on the verge of extinction (Harley, 2011). The effects of the phenomenon have also branched out to potentially causing a damaging and lasting impact on the economy and quality of life with major implications on the future of human health and existence (Bosello, Roson, & Tol, 2005). Additionally, there has been a great interest in the diversification of power generation, with the growing industry interest of not being heavily dependent on a single type of energy source, where fossil fuel-based (coal and natural gas) electric power generation sources alone account for more than 60% of electric power in America, while renewable energy sources only account for 17% of electric power generation in the United States, with wind energy only accounting for 6% of total power generation (U.S Energy Information Administration, 2019). Increasing this share can reduce the dependency on extraction-based non-renewable energy generation.

One of the reasons behind the imbalance in the share of the sources for power generation, especially with renewable energy sources like wind power is the current economics of the technology. The investment in wind turbines and analyzing it with the energy it yields is comparably more risk intensive than coal or natural gas-based power-plants (U.S. Department of Energy's Office of Energy Efficiency and Renewable Energy, 2019). With smaller organizations
and individuals lacking tools to model performance predictions of potential investment on wind turbines further worry investors away from the potential expansion of the wind energy infrastructure. Hence, this study aims to develop a low-fidelity calculation model that could predict turbine performance and validate the model against experimental data.

1.2 Background history

The recorded history of the usage of wind energy goes back as far as 5000 BC for the propulsion of boats on the Nile river, while the usage of windmills dates back as early as 200 BC for water pumps in China and grain grinding in Persia (U.S. Energy Information Administration, 2019). For much of recorded history, the usage of wind energy was mostly limited to food production, boat/ship propulsion, or pumping fluids.

The interest in applying the principles of wind energy to generate electricity started gaining interest and moment following the oil shortages the US faced in the 1970s. As a result of the crisis, there was a renewed interest in the development of renewable energy sources for power generation (U.S. Energy Information Administration, 2019).

In the following years, there has been a rise in the installations of wind turbines and the establishment of wind farms, mostly credited to government incentives (U.S. Energy Information Administration, 2019). The share of wind energy for total power generation in the U.S back in 1990 was less than 1%, in 2018 the share grew to 7%.
1.3 Understanding the types of wind-turbines

- Horizontal axis and vertical axis wind turbines:
  
a) Vertical axis wind turbines (VAWT): sometimes described as an “egg-whisk”, the vertical-symmetrical airfoils operates from the force from the fluid flow in the vertical axis (z-axis). The advantages of VAWT is that there are no towers needed and heavy equipment like generators and gear-boxes can be placed on the ground level (Takao, et al., 2009). However, the drawbacks of VAWTs lie in the lack of self-starting capability, difficulty in speed regulations, low revolving speed, and torque fluctuation between each revolution.

b) Horizontal axis wind turbines (HAWT): most wind turbines currently in use are HAWT. HAWT consists of a rotor shaft, gear-box, rotating shafts, generator, and tower-style support structure (Yahyaoui & Cantero, 2018). Most HAWTs also contain a controller that implements control feedback calculation that changes the behavior and angle of the turbine-based on live environmental conditions.
• **Aerodynamic drag and aerodynamic lift wind turbines:**

  a) Aerodynamic drag propelled wind turbines: drag force is generated when the force of wind acts against a surface allowing for work to happen as a result. The application of wind energy using its drag force can be traced back as early as sailboats. Savonius wind turbine (a type of VAWT) operates on the drag-based aerodynamics. However, one of the more critical drawbacks of drag based wind turbines is the low rotational speeds as the speed of the turbine is lower than the speed of the wind, with most drag propelled wind turbines achieving maximum efficiency of 15% (Diaz, Pajaro, & Salas, 2015).

  b) Aerodynamic lift propelled wind turbines: lift force opposes weight (henceforth perpendicular to the surface of the airfoil) and is generated when fluid at any given
velocity passes through the airfoil/blades of a turbine generating a differential pressure between the upper and lower surfaces of the turbine (hence the “thinning out of the blades”), which then generates a lifting force (tangential force in the case of wind turbines) which is greater than the force exerted by gravity, which in turn causes the blade to be lifted up in a rotational axis (Schubel & Crossley, 2015).

Figure 1.2  The two mechanisms of propulsion compared. (Schubel & Crossley, 2015)

See “Aerodynamic drag and aerodynamic lift wind turbines” section in chapter 1.3 for further details between horizontal axis wind turbine and vertical axis wind turbine

- **Constant-speed and variable-speed wind turbines:**
  
a) **Constant-speed wind turbines** refers to wind turbines rotors that operate at a constant speed regardless of the wind speed. While fixed speed turbines have an advantage of being relatively cheap (compared to variable speed wind turbines) due to the lower cost of the electrical components, they are only optimal at a particular wind speed (Datkhile, Veeresh, & Tapre, 2016).
b) **Variable-speed wind turbines** refers to the latter and more recent family of wind turbines where the rotors have the capability of dynamically responding to the differing speed of the wind. While variable-speed has the capacity of increased energy capture, the operating mechanism, and components to keep it operational is fairly complex and often require control feedback logic (Datkhile, Veeresh, & Tapre, 2016)

### 1.4 Thesis Objectives

The purpose of this thesis is to develop a blade element driven computational model to predict the power performance for wind turbines and validate the model’s eminence against experimental data obtained from the MEXICO rotor project. The thesis will develop three progressive approaches towards developing the model, with the first approach modeling the performance with the traditional BEM equations, while the second and third approach focusing on developing a modified performance parameter based on the Buckingham Pi approach. The second approach to modeling is distinctive by ignoring the influence of induction factor on most dependent variables of power coefficient, while the third approach focusing on the influence of the induction factors on all variables influencing the power coefficient. The validation would use the power performance data from a variety of inflow velocity from the MEXICO rotor project.

### 1.5 Thesis Outline

This thesis has been split into three chapters. The first chapter introduces the research, discusses the background history of a wind turbine as well as the types of wind turbine available in the market. Chapter two will go through the derivation of the equations used for blade element modeling. Chapter three will utilize equations derived in chapter two and apply to the
development of the models. Chapter four will contain the results of the model. Lastly, chapter five would contain the analysis of the results when validated against experimental data and corrections taken in preceding models. Chapter five will also contain suggestions for future research on the topic.
CHAPTER II
DERIVATION OF THE WIND TURBINE MODEL

2.1 Newtonian Derivation-Actuator Disc Model

The following sections below will discuss the equation development of lift driven blade turbine system. The role of a wind turbine as discussed above is to generate power exacting the kinetic energy from the wind. The kinetic energy experienced by the turbine can be described by the Newtonian equation: \( KE_{\text{wind}} = \frac{1}{2} m v^2 \). Since it is complex to calculate and measure the mass of a constantly flowing stream of air particles, it is relevant to associate the changing mass as a measure of density in a given area, i.e: \( \frac{dm}{dt} = \rho A v \), where area \( A \), is the total aerial span of a turbine \( (A = \pi R^2) \). The potential energy experienced by the turbine can be illustrated by the following equation:

\[
P_{\text{wind}} = \frac{1}{2} \rho v^3 \pi R^2
\]

When modeling the stages of the interaction of the turbine to the fluid stream, it can be split into 4 stages: 1: free-stream region, 2: just before the disc, 3: just after the disc, and 4: far wake region (Kulunk, 2011). This structured interaction is described in figure 2.1, which forms the basis of the understanding for the development of the actuator disk model (ADM) equations.
Figure 2.1  Actuator Disk Model. (Kulunk, 2011)

The figure reveals the 4 stages of fluid interaction in a lift driven wind turbine profile, 1: free-stream region, 2: just before the disc, 3: just after the disc, and 4: far wake region.

Assuming continuity of velocity through the disk: $V_2 = V_3$, the continuity equation can be applied as:

$$
\rho A_x V_\infty = \rho A_D V_D = \rho A_W A_W; A_x V_\infty = A_D V_D = A_W A_W
$$

(2.2)

Stages of the ADM predicts and rise in pressure in region 2, with drop-in pressure and velocity in region 3 before normalizing to the free stream pressure in region 4. Velocity is predicted to drop progressively from stages one to three, before renormalizing to the inflow conditions in the far wake region. The characterization is described in figure 2.2.
Figure 2.2  Variation of the velocity and dynamic pressure through the stream-tube (Corke & Matlis, 2016)

The figure reveals the changes in dynamic pressure and velocity in the 4 stages of fluid interaction in a lift driven wind turbine profile

The variation of the velocity between stage 2 and stage can be simplified to the relative velocity experienced at the disk, hence a simplified relationship of the flow is described in figure 2.3.
The figure reveals the changes in static pressure in the 4 stages of fluid interaction in a lift driven wind turbine profile.

Applying Bernoulli’s equation yields the expression for the pressure differential between region 2 and 3 as:

$$P_2 - P_3 = \frac{1}{2} \rho (V_1^2 - V_4^2)$$

(2.3)

Relating force as a function of the differential pressures between region two (p2) and region three (p3) yields:

$$F = (P_2 - P_3)A = \frac{1}{2} (V_1^2 - V_4^2)A$$

(2.4)
Where \( A \), is the area of the disk, which as discussed above can be described as \( A = \pi R^2 \).

It was discussed in the introduction of the ADM method, the inflow velocity experiences a variation as it approaches stage 2, due to the rise in pressure. The expression relating the drop in velocity to the inflow velocity is described as the axial induction factor:

\[
a = \frac{V_1 - V_2}{V_1}
\]  

(2.5)

Using the relation derived from the axial induction factor yields a relationship between velocities at region 1 \((V_1)\) and region 2 \((V_2)\) as \(V_2 = V_1(1-a)\), and region 4 \((V_4)\) as \(V_4 = V_1(1-2a)\).

Substituting the derived relation to solve for force as a function of inflow velocity:

\[
F = \frac{1}{2} \rho V_1^2 4a(1 - a)A
\]  

(2.6)

Figure 2.4  Velocity triangle (without the influence of the induction factors)

The figure reveals the velocity triangle in the elemental airfoil profile without the influence of the induction factors.

Figure 2.4 describes the interaction of inflow velocity \((V_1)\) on the elemental section of the blade rotating with a speed of \(\Omega_r\). The vectors together yield a resultant vector \(V_r\), which the
velocity relative to the blade. However, as discussed before, the inflow velocity experiences a variation in speed as a portion of the speed is induced before the interaction with the elemental blade object (induction factor), hence necessitating an updated velocity triangle in figure 2.5.

The induced effect on the inflow velocity also causes an induction on the rotational speed of the turbine, which is referred to as the angular induction factor \( a' \), the combined vectors of the induced inflow speed and the induced rotational speed yields elemental wake rotational speed \( \omega \).

The elemental torque experienced by the elemental object is a function of the rate of change of the angular moment (L, also the lift force) experienced by element \( (I\omega) \), where I is the moment of inertia, see below:

\[
I = mr^2 \quad (2.7)
\]
\[
L = I\omega \quad (2.8)
\]
\[
T = \frac{d\omega}{dt} = \frac{dm}{dt}r^2\omega \quad (2.9)
\]
From the velocity triangle in figure 7, the angular induction factor can be derived to be

\[ a' = \frac{\omega}{2\Omega} \]

therefore deriving the elemental torque as a function of axial and angular induction factor yields:

\[ T = \frac{dm}{dt} r^2 \omega, \text{ where } \frac{dm}{dt} = \rho A V r, \]

substituting the expression for the mass flow rate into the torque equation produces:

\[ T = \rho AV_r (r^2 \omega) \]

(2.10)

\[ dT = \rho (2\pi r dr)V_1 (1 - a)(2r^2a' \omega) = 4\pi \rho V_1 a' \Omega (1 - a)r^3 dr \]  

(2.11)

### 2.1 Blade Element Momentum (BEM) Theory

The differential torque equation described in equation 2.11, describes the torque exerted by the elemental section of the wind turbine. The modeling approach of splitting the blade to elemental sections is called blade element momentum theory or the BEM approach. Compared to the ADM method, the BEM theory approach in predicting the power output of the turbine model on each elemental object could then be integrated to obtain the total sum of force, torque, and power. The approach acknowledges the dissimilarity and the complexity of the geometric variables at different lengths of the blade.

The kinetic force from the inflow velocity is converted to a series of forces on the elemental object. Expanding on the velocity developed in figure 7 and analyzing the forces generated from it, yields figure 8, which reveals the lift force, drag force, tangential force and the normal force to be derived from the initial kinetic force. The coefficient of lift (\(C_L\)) experienced by an elemental cross-section is a function of the ratio of the lift force to the kinetic force generated by the relative velocity and the chord length of the elemental section (see equation
2.12). Similarly, the coefficient of drag ($C_D$) experienced by an elemental cross-section is a function of the ratio of the drag force to the kinetic force generated by the relative velocity and the chord length of the elemental section (see equation 2.13).

![Diagram of forces influencing the cross-section of a blade](image)

Figure 2.6  Differential forces influencing the cross-section of a blade (Corke & Matlis, 2016)

The figure reveals the forces generated on an elemental airfoil profile by the induced velocity triangle

\[ dL = \frac{1}{2} C_L \rho V_r^2 c \, dr \]  
\[ dD = \frac{1}{2} C_D \rho V_r^2 c \, dr \]

Since the forces of lift and drag are offset from the normal and tangential force by the variation of the relative angle ($\phi$) between them, the following proportional relationship can be derived to compute the coefficient of normal and tangential force (Corke and Matlis).
\[ C_N = C_L \cos \phi + C_D \sin \phi \quad (2.14) \]

\[ C_T = C_L \sin \phi - C_D \cos \phi \quad (2.15) \]

\( \phi \), the relative angle is the angle at which the kinetic force vector from the relative velocity interacts with the elemental object. Hence using the velocity triage developed in figures 2.4 and 2.5, the relative angle is tangent of the induced inflow velocity and the rotational speed of the blade element or: local twist experienced at the element and the pitch angle, see equation 2.16.

\[ \phi = \tan^{-1} \left( \frac{V_1(1 - a)}{\Omega r(1 + a')} \right) \quad (2.16) \]

The relative angle can be described as the sum of the angle of attack of the airfoil with respect to the relative flow and the local twist and pitch angles:

\[ \phi = \alpha + [\theta_T + \theta_{CP}] \quad (2.17) \]

The modified equation for elemental force can be further developed as a function of the kinetic force exerted by the induced inflow velocity (see equation 2.11), coupled with positive lift force and negative drag force based on the geometric orientation of the elemental airfoil on each blade element yields a modified (Ingram):

\[ dF = \frac{1}{2} \beta \rho V_r^2 (C_L \cos \phi + C_D \sin \phi) crdr \quad (2.18) \]

\[ dT = \frac{1}{2} \beta \rho V_r^2 (C_L \cos \phi - C_D \sin \phi) crdr \quad (2.19) \]

Relating the relative velocity to the inflow velocity from the velocity triangle in figure 2.6, where \( V_r = \frac{V \cos(1-a)}{\cos \phi} \), yields the following modified equation for elemental torque (Ingram):
\[
dT = \frac{1}{\pi} \frac{B \rho V_\infty^2 (1 - a)^2 (C_L \cos \phi - C_D \sin \phi) crdr}{\cos^2 \theta}
\]

(2.20)

While the equations above succeed in capturing an idealized physical performance of a turbine, two variables need to be added to the elemental torque analysis:

1) **The local solidity**: (sometimes referred to as the blade solidity) is a design parameter at a localized element analyzing the blade chord length to the elemental radius (Yan) and can be calculated using equation 21 (Corke & Matlis, 2016)

\[
\sigma' = \frac{BC}{2 \pi r}
\]

(2.21)

2) **Tip loss correction**: Because BEM approaches the modeling of a turbine based on the assumption of each element independent of each other, the vortices formed at the tip is not accounted for. Hence a tip loss correction factor is implemented through equations 2.22 and 2.23 (Corke & Matlis, 2016).

\[
f = \frac{R - r B}{r \sin \phi \, \frac{2}{\pi}}
\]

(2.22)

\[
F = \frac{2}{\pi} \cos^{-1}(e^{-f})
\]

(2.23)

Inputting in the geometric influence from the local solidity and the tip loss correction on the elemental torque equation yields (Ingram):

\[
dT = \frac{\sigma' F B \rho V_\infty^2 (1 - a)^2 (C_L \cos \phi - C_D \sin \phi) r^2 dr}{2 \cos^2 \theta}
\]

(2.24)

The elemental power is generated from the rotational movement from the torque experienced at each element and the overall angular velocity of the turbine.

\[
dP = dT \Omega
\]

(2.25)
Equating the expression derived from the ADM based equations and the BEM based equation an updated expression to calculate the axial and angular induction factors is expressed in equations 2.26 and 2.27. To go through the derivation of the above two expressions of the axial and angular induction factors see Appendix D.

\[
a = \frac{B(C_N)c}{8\pi rsin^2\phi} \left( \frac{B(C_N)c}{8\pi rsin^2\phi} + 1 \right) \quad (2.26)
\]

\[
a' = \frac{BR(1 - a)(C_T)c}{8\lambda\pi sin^2\phi r^2} \quad (2.27)
\]
CHAPTER III
MEXICO ROTOR PROJECT AND MODELING APPROACHES FOR VALIDATION

3.1 About the MEXICO Rotor Project

The Model Rotor Experiments under Controlled Conditions (MEXICO) project collected controlled experimental data for wind turbine performance when subjected to a variety of input variables. The Blade profile used in the MEXICO project consists of 3 blade profiles: 1) DU 91-W2-250 at the root, 2) Risø A1-21 at the midspan region, and 3) NACA 64-418 airfoil at the tip (see figure 3.1). In addition to the variable airfoil profiles throughout the blade, the turbine blades of the MEXICO project also have varying twist angles at different elemental sections of the blade (see figure 3.2). For the experimental power output from this project see figure 3.3.

Figure 3.1 Elemental Blade Profile: Cross-sectional airfoil profile (Snel & Schepers, 2009)
The figure reveals the airfoil profiles used for the blade used in the MEXICO rotor project
Figure 3.2  Elemental Blade Profile: Chord Length, Local Twist (Micallef, Kloosterman, Ferreira, & Bussel, 2009)

The figure depicts the chord length and local twist variation (/10°) for the blade used in the MEXICO rotor project.

Figure 3.3  MEXICO Rotor Project Power Plot, Pitch angle of -2.3° (Micallef, Kloosterman, Ferreira, & Bussel, 2009)

The figure depicts the experimental power output based on variable inflow velocity conditions.
3.2 Modeling Approach 1: Traditional BEM Approach with Idealized Angle of Attack

Pitch control in a wind turbine system allows for controlling the angle of the blade for the optimal lift force and minimize the impact of drag, which in turn allows for greater torque and power output. In the described method, the idealized angle of attack is chosen based on the ideal lift coefficient, often a value right before the stall point. For this prescribed method, the ideal angle of attack is chosen from the blade profile element DU91-W2-250 at 23°, due to the profile’s greater prevalence throughout the blade compared to the other profiles. The angle of 23° was chosen as it was the value right before the stall point for the $C_L$ plot for the blade profile (see appendix B). Assumption of an idealized angle of attack is made due to the relatively low variation in the local twist (see figure 3.2), hence, variation in the lift and drag between the blade elements is assumed to be minimal. Once the idealized angle of attack is selected, the relative inflow angle could be calculated by taking the sum of the selected angle of attack, pitch control angle, and the local elemental twist. Following the choice of the specified angle of attack, the power output could be calculated through the calculation process described in figure 3.4.
Figure 3.4  Modeling Approach 1 Flow Chart

The figure depicts the equation flow steps for modeling approach 1 based on the assumption on an idealized angle of attack.

### 3.3 Buckingham π Approach for BEM Model for Power Output

Non-dimensional analysis methods like the Buckingham π theorem approach to wind turbine aerodynamic equations focus on the approach of developing functional relationships to vital non-dimensional parameters in the analysis of turbine performance. The procedure is conducted by analyzing an equation that results in the output of interest (power) and solves for the non-dimensional parameters that influence the variable. The steps for the Buckingham π analysis are detailed in Appendix C.
3.3.1 Derivation of the BEM Model Using Non-Dimensional Approach

The two π terms derived in Appendix C represent two non-dimensional variables relevant to the performance of wind turbine aerodynamic analysis. π2 relates to the expression that determines the coefficient of power, while π1 is the expression for the tip speed ratio (briefly discussed in the development of the angular induction factor in equation 27, see appendix D).

Using the relationship developed from π1 and the relationship from the velocity triangles of figures 2.4 and 2.5, the expression for the local tip speed ratio is developed in equation 3.1, which a ratio of the angular speed of the blade element relative to the tip and the inflow velocity.

\[ \Omega = \frac{V_x \lambda r}{r} \]  

(3.1)

Rewriting the expression above in relation to the local tip speed ration yields:

\[ \lambda r = \frac{\Omega r}{V_x} \]  

(3.2)

The coefficient of power is an expression of the percentage of wind power converted to potentially useful power from the turbine. Dividing the expression of power in equation 2.25, to the wind power equation developed in equation 2.1 yields the following expression for coefficient of power:

\[ dC_p = \pi^2 = \frac{\text{Power of the turbine}}{\text{Theoretical Wind Power}} = \frac{\Omega F B \sigma \pi p V_x^2 (1-a)^2 (C_l \cos \phi - C_D \sin \phi) r^2 dr}{\cos^2 \theta} \]

\[ = \frac{1}{2} \rho \pi V_x^3 R^2 \]  

(3.3)
Simplifying and substituting the relationship between inflow velocity and the angular velocity of the blade (as described in equation 3.2), allows from the above expression of the coefficient of power to further simplified to equation 3.4.

\[
dC_p = \frac{2\sigma'FB(1 - a)^2(C_L \cos \phi - C_D \sin \phi)r^2dr \lambda rV_\infty}{\cos^2 \theta R^2V_\infty} = \frac{2\lambda r\sigma'FB(1 - a)^2(C_L \cos \phi - C_D \sin \phi)rdr}{\cos^2 \theta R^2} (3.4)
\]

Hence, as derived from the Buckingham π theorem, the performance of power is a function of the local tip speed ratio. The relationship would form the basis for the progressive methods for modeling the power coefficient. The power output from these modeling approaches can be calculated by multiplying the theoretical power to the coefficient of power calculated in equation 3.4.

\[
dP = dC_p \frac{1}{2} \rho v^3 \pi R^2 (3.5)
\]

### 3.4 Modeling Approach 2: Non-Dimensional Approach to BEM

Using equation 3.4 as the equation of focus, modeling approach two will not assume a homogeneous angle of attack throughout the blade profile. However, to obtain the angle of attack and the relative inflow velocity angle, modeling approach 2 will ignore the influence of the induction factors in the computation of the relative inflow velocity angle. Using the relative inflow angle and force coefficient variables calculated from the elemental angle of attack, induction factors are computed. Using the calculated induction factor, an updated angular velocity is calculated. The updated angular velocity is then used to calculate the coefficient of power. It is important to note that while induction factor in introduced in the calculation of the angular velocity, like indicated in figure 2.5 the induction factor also influences the magnitude of
the relative angle and the subsequent variables tied to it, however, modeling approach two approaches the calculation by ignoring the influence of the induction factor. See figure 3.5.

![Modeling Approach 2 Flow Chart](image)

Figure 3.5  Modeling Approach 2 Flow Chart

The figure depicts the equation flow steps for modeling approach 2 based on the assumption on 0 induction factor for the calculation of the relative angle.

### 3.5 Modeling Approach 3: “Corrected” Non-Dimensional Approach to BEM

The procedure in modeling approach 2, succeeds in utilizing the induction factor to calculate a “corrected” angular velocity. Modeling approach three aims in applying the effects of the induction factor to calculate “corrected” inflow velocity angle, angle of attack and lift and drag coefficients. The above variables were chosen to utilize a “corrected” values as the coefficient of power is a function of the above chosen variables. The justification for the greater incorporation of the induction factor is because of it’s ability to effect the overall magnitude of
the output and because it is relevant aerodynamic influence on the performance of the turbine. See figure 3.6.

Figure 3.6 Modeling Approach 3 Flow Chart

The figure depicts the equation flow steps for modeling approach 3 based on a two-step approach to calculating the relative angle.
CHAPTER IV
RESULTS

4.1 Results for Modeling Approach #1

Figure 4.1  Calculated Lift Coefficient at 7.884 m/s

The figure depicts the calculated lift coefficient for modeling approach 1 for the inflow velocity of 7.884 m/s.
Figure 4.2  Calculated Drag Coefficient at 7.884 m/s

The figure depicts the calculated drag coefficient for modeling approach 1 for the inflow velocity of 7.884 m/s.

Figure 4.3  Calculated Lift Coefficient at 21.902 m/s

The figure depicts the calculated lift coefficient for modeling approach 1 for the inflow velocity of 21.902 m/s.
Figure 4.4  Calculated Drag Coefficient at 21.902 m/s

The figure depicts the calculated drag coefficient for modeling approach 1 for the inflow velocity of 21.902 m/s.

Figure 4.5  Modeling Approach 1: Experimental Power Output Values Compared to Computationally Modeled Power Output

The figure compares the output from modeling approach 1 to the experimental data.
4.2 Results for Modeling Approach #2

Figure 4.6  Calculated Lift Coefficient at 7.884 m/s

The figure depicts the calculated lift coefficient for modeling approach 2 for the inflow velocity of 7.884 m/s

Figure 4.7  Calculated Drag Coefficient at 7.884 m/s

The figure depicts the calculated drag coefficient for modeling approach 2 for the inflow velocity of 7.884 m/s
Figure 4.8  Calculated Lift Coefficient at 21.902 m/s

The figure depicts the calculated lift coefficient for modeling approach 2 for the inflow velocity of 21.902 m/s

Figure 4.9  Calculated Drag Coefficient at 21.902 m/s

The figure depicts the calculated drag coefficient for modeling approach 2 for the inflow velocity of 21.902 m/s
Figure 4.10  Modeling Approach 2: Experimental Power Output Values Compared to Computationally Modeled Power Output

The figure compares the output from modeling approach 2 to the experimental data.

4.3 Results for Modeling Approach #3

Figure 4.11  Calculated, “Corrected” Lift Coefficient at 7.884 m/s

The figure depicts the calculated lift coefficient for modeling approach 3 for the inflow velocity of 7.884 m/s.
Figure 4.12  Calculated, “Corrected” Drag Coefficient at 7.884 m/s

The figure depicts the calculated drag coefficient for modeling approach 3 for the inflow velocity of 7.884 m/s.

Figure 4.13  Calculated, “Corrected” Lift Coefficient at 21.902 m/s

The figure depicts the calculated lift coefficient for modeling approach 3 for the inflow velocity of 21.902 m/s.
Figure 4.14  Calculated, “Corrected” Drag Coefficient at 21.902 m/s

The figure depicts the calculated drag coefficient for modeling approach 3 for the inflow velocity of 21.902 m/s.

Figure 4.15  Modeling Approach 3: Experimental Coefficient of Power Output Values Compared to Computationally Modeled Output

The figure compares the output from modeling approach 3 to the experimental data.
CHAPTER V
RESULT ANALYSIS, CONCLUSION AND FUTURE WORK

5.1 Result Analysis

The figure compares the output from the three modeling approaches to the experimental data

In order to conclude the quality of the modeling approaches validation of experimental data, it requires evaluation criteria. Based on the results displayed on figure 5.1, the development of the follow two evaluation criteria becomes evident:

- Criteria 1-percentage error: the criteria is focused on the magnitude of variation between experimental and modeled data.
• Criteria 2-trend accuracy: the criteria is focused on the modeled output’s ability to match slope and local maximum of the experimental data.

5.1.1 Criteria 1-Percentage Error Analysis

![Graph of Percentage Error Analysis](image)

Figure 5.2 Comparative results analysis of the BEM models: Percent Error

The figure compares the percent error of the three modeling approaches when evaluated against experimental data.

Examine the quality of the three modeled approaches for criteria 1, figure 5.2 reveals that modeling approach three has lowest percentage error of the three modeled approaches, while modeling approach two has on average a higher percentage error of the three modeled approaches. In order to evaluate the causation for the percent error behavior observed in figure 5.2, a comparative analysis of the variables influencing the power output will be analyzed at individual elements of the blade at a specific velocity. For the analysis, the inflow velocity of 9.073 m/s is chosen due to it being a point of increased variation between modeling approaches two and three.
While the coefficient of drag does have a negative consequence on the torque and the power generated in the airfoil element, the analysis will be ignoring the influence of the $C_D$ variable as the order of the magnitude of the variable is significantly lower than that of $C_L$, therefore, the effects of drag can be ignored for this analysis. The snapshot analysis at 9.073 m/s will focus on the variables influencing the elemental lift coefficient and the effect of the elemental lift coefficient to the elemental power extracted from the turbine.

Starting, with comparative plot for the relative angle on figure 5.3, the plot reveals the trend for model one with very minimal change. This is due to the assumption of an idealized angle of attack on modeling approach one, when the assumed angle of attack of the $23^\circ$ influencing the relative angle. Note that the minimal variation observed in modeling approach one is due to the twist variation in the blade profile. However, modeling approaches two and three is noted with much more dynamic variation in the relative angle throughout the blade profile.
Unlike modeling approach one, modeling approach two and three does not assume an idealized angle of attack, which then is utilized for the calculation for the relative angle. In the latter two approaches, the relative angle is evaluated based on the tangential relationship between the inflow velocity and the angular velocity of the blade. The variation between the two models is the incorporation of induction factor in approach three.

The effect of the incorporation of the induction factor in approach three is reducing the overshooting observed in modeling approach two. The effects of induction according to equation 2.16 has an effect that reduces the magnitude of the relative angle. The reduced relative angle observed in modeling approach will have a domino effect on the angle of attack, see figure 5.4.
Figure 5.4  Comparative results analysis of the BEM models: Angle of attack ($\alpha$) at 9.073 m/s

The figure compares the calculated angle of attack ($\alpha$) of the three modeling approaches at 9.073 m/s.

Note that the analysis of the angle of attack in figure 5.4, modeling approach one has a maintained value of 23° throughout the blade profile due to the assumption made for the profile in model one for all inflow conditions (refer to section 3.2). Modeling approach two and three experience a similar behavior as the relative angle in figure 5.3, where the angle of attack for modeling approach two experiences overshooting when compared to the value from data from approach three. The similarity maintained between the two figures for approaches two and three, because of the direct relationship between the relative angle and the angle of attack derived in equation 2.17. The equation highlights that a higher relative angle calculated in approach two should yield a higher relative angle compared to approach three. The value of the lift coefficient is directly influenced by the value of the calculated angle of attack.
Figure 5.5  Comparative results analysis of the BEM models: Lift Coefficient ($C_L$) at 9.073 m/s

The figure compares the calculated lift coefficient ($C_L$) of the modeling approaches two and three at 9.073 m/s

Due to the low dynamic behavior observed in the elemental profile for the angle of attack and the relative angle for modeling approach one, the influence of the model on the lift coefficient is ignored in figure 5.5 due to the figure’s focus on comparing the influence on the induction factor on the lift coefficient’s between approaches two and three. As observed in figure 5.5, the higher angle of attack observed on modeling approach two when compared to approach three yields a higher coefficient of lift. However, if the angle of approach goes past the stall point at any given elemental section, the coefficient of lift could experience a drop or a flat line depending on the airfoil profile in question (see Appendix B). However, for the angles of attack observed for modeling approaches two and three in figure 5.4, none of the values approach the stall point for their respective profiles, therefore, the higher angle of attack observed in modeling
approach two yields a high lift coefficient. The high lift coefficient has a positive influence in the power extracted from the radial profile.

Figure 5.6 Comparative results analysis of the BEM models: Differential Power Output (kW) at 9.073 m/s

The figure compares the calculated differential power coefficient (dP) of the modeling approaches two and three at 9.073 m/s

Figure 5.6 reveals an extension of what is observed at figure 5.5, where modeling approach two models a higher power output at each elemental section of the blade, however, converges towards the tip of the blade as the section of the blade does not generate much power due to its limited elemental section. The summation of the power generated at each elemental section adds up to generate power observed at 9.073 m/s at figure 5.1, hence, the variation observed at each elemental section between the two profile compounds to the end results at figure 5.1 explaining the noticeable variation between the two modeling approaches.
5.1.2 Criteria 2-Trend Accuracy

Trend accuracy is considered for the evaluation of the quality of the modeling approaches due to the peak power performance observed in the experimental data at 18.96 m/s. In order to evaluate the quality of the three modeled approach when compared for trend accuracy the following details will be analyzed:

1) Slope (positive and negative slope)
2) Local maximum

For the three modeling approaches, all experience a positive slope like the experimental data until the local maximum is reached. Modeling approach one, however, continues to rise at a steady rate past the experimental maximum. This is because modeling approach one assumed an idealized angle of attack for all inflow conditions, and therefore forces variables like the lift and drag coefficient along with the relative angle to act as a constant to the variable inflow conditions and therefore generate a linear behavior observed in figure 5.1

Modeling approaches two and three experience identical behavior to experimental data, however, experiences variation in the rate of change for the slope, especially with modeling approach three, which experiences a greater slope compared to experimental data between inflow velocities 9 m/s to 11 m/s. This noticed initial variation approach three, set a trajectory for the noted variation between the modeled and experimental data. Note that modeling approach two experiences a similarly high rate of change between 9 m/s to 11 m/s, that similar to modeling approach sets the trajectory for the greater percent variation between experimental and modeled data.

Modeling approach two and three, unlike approach one does not exhibit a linear behavior, and instead like the experimental data experiences a peak power output. Modeling approach
three and two experiences a peak power output at 19.738 m/s, like the experimental data, and like the experimental data experiences subsequent negative slope for the power output. However, modeling approach three fails to recognize the “v-shaped” behavior observed in the experimental data between inflow velocities 18.282 m/s and 19.738 m/s. Modeling approach two observes a behavior similar to the “v-shaped” behavior between the listed range. The potential explanation for the lack of this behavior in modeling approach three is the corrupting influence of the induction factor, which is calculated based on an initial guess, and not necessarily reflective of the experimental or aerodynamic conditions.

### 5.1.3 Summary of The Quality of The Modeling Approaches and Error Sources

For the quality of models based on the two evaluation criteria, the following summary could be developed:

- **Percent error (analysis between modeling approach two and three):** modeling approach three compared to modeling approach two consistently maintains a low percent error. The cause of this is the lack of incorporating in the influence of the induction factor in approach two. Figures 5.3 through 5.6, reveals how the incorporation of the induction factor in modeling approach three reduces the magnitude of elemental relative angle, angle of attack, lift and drag coefficients and the elemental power output when compared to approach two.

- **Percent error (“U-shaped” curve for modeling approach one):** figure 5.2 reveals a “U-shaped” behavior for modeling approach one as lower percentage errors are observed in the middle range of inflow velocities (13 m/s to 17 m/s), while a higher percent errors are observed at lower and higher inflow velocities. The reasoning for this behavior is the assumption of a constant angle of attack for all inflow velocities at an idealized lift
coefficient condition. The ideal range of inflow velocities follows the range where the lowest percent error is observed, where the assumption of an idealized angle of attack holds aerodynamically accurate for the specific range (New York State Energy Research and Development Authority).

- Trend Accuracy (analysis between modeling approaches two and three): Modeling approaches two and three succeed in mimicking the trend of the experimental data as it successfully follows the positive rate of change for the slope, recognizes the local maximum, and follows the negative rate of change observed in the experimental data. The is a visible variation in the magnitude for the slope between the models and the experimental data, especially at higher inflow conditions. The likely explanation for variation between the experimental and modeled slopes is potentially is sourced in the unaccounted factors in the modeling approach (see unaccounted factors bullet).

- Trend Accuracy (modeling approach one): while modeling approach one follows a trend similar to the experimental data, however, the model fails to recognize the presence of a local and continues to experience a steady positive slope for all inflow conditions. The explanation for this behavior lies in the assumption made in modeling approach one for an idealized angle of attack in all inflow conditions forcing the modeled output to behave similar to a constant linear slope, where the variation of between the inflow conditions is driven from the inflow velocities multiplied by the constantly assumed variables for angle of attack, relative angle and lift and drag coefficients.

- Unaccounted factors:
  a) Blade design: figure 3.1 revels the three different airfoil profile for the MEXICO rotor blade. While the calculation of the $C_L$ and $C_D$ at elemental section is done
dynamically based on the airfoil profile the element is part of, it is important to recognize for the regions in between that are not a part of the three listed profiles, an averaged values of $C_L$ and $C_D$ is taken based on the values of the adjacent section. This average values for section in between airfoil profiles might not be accurate.

b) Vibration effects at high inflow conditions: a higher percent error is noted at higher inflow velocities. The BEM models did not account for effects of vibration to the structure of the turbine at higher inflow condition. Vibrations can potentially have negative impact to electrical equipment.

c) Equipment rating: the model does not address the ideal operational rating for the electrical components of the turbine. If an equipment is pressed past the suggested rating, the equipment might fail to perform optimally.
Table 5.1    Quality Evaluation of Modeling Approaches

<table>
<thead>
<tr>
<th>Evaluation Description</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality of Criteria 1:</td>
<td>On average has a lower percentage error than model 2</td>
<td>On average the highest percentage error of the three models</td>
<td>Lowest percentage error of the three models</td>
</tr>
<tr>
<td>Percentage Error</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Source of Percentage</td>
<td>Error sourced in statically maintained angle of attack</td>
<td>Higher lift coefficient compared to model 3 coupled with high local tip</td>
<td>Noted percent error due to expected percent error due compartmentalized of</td>
</tr>
<tr>
<td>Error</td>
<td></td>
<td>speed ratio generates an overshoot C_p</td>
<td>the blade profile</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>High percent error observed in high wind velocity due to exceeding the</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>ideal operational velocity of a turbine</td>
</tr>
<tr>
<td>Quality of Criteria 2:</td>
<td>Fails to mimic experimental data trend</td>
<td>Closely mimics experimental data trend</td>
<td>Closely mimics experimental data trend</td>
</tr>
<tr>
<td>Trend Accuracy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Source of Trend Accuracy</td>
<td>Error sourced in statically maintained angle of attack and lack</td>
<td>Closer trend to experimental data due to ignoring the influence of</td>
<td>Varies slightly from experimental due to the angle of attack being</td>
</tr>
<tr>
<td>Error</td>
<td>incorporation of non-dimensional variable</td>
<td>induction factor in angle of attack</td>
<td>influenced by the induction factor</td>
</tr>
</tbody>
</table>

The table reveals a snapshot summary of the evaluation of the three modeling approaches
5.2 Conclusion

The results from the three modeling approaches highlight the significance of applying a dynamic approach to the calculation of the angle of attack based on the calculation of the relative angle as a function of the inflow velocity and angular velocity. The lacking this derived relationship in modeling approach one, where an assumed angle of attack drives the value for the relative angle and subsequent variables in the modeling forces an inaccurate linear behavior of the method. Modeling approaches two and three correct this mistake in approach and is able to model a trend identical to experimental data. The lack of incorporation of the induction factor in modeling approach two yields the model with overshoot values when compared to experimental data. The issue is addressed in approach two’s two step approach, where the calculation values of induction in step one is used in the calculation of the inducted relative angle in step two, dropping the magnitude observed in approach two to be closer to experimental data.

The progressive approaches highlight the significance of a dynamic evaluation of the blade element approach needing to be independent of overinfluencing guesses of approach one, combined the incorporation of realistic aerodynamic influence of induction.

The three models suggest a progressive capacity of the ability of the three modeling approaches to validate experimental data, with the framework of modeling approach suggesting promising results from the research.
Figure 5.7 Summary of the correctional procedures of the development of the three modeling approaches

The flow chart provides a summary of the progressive flaws and corrections made between the modeling approaches.
5.3 Future Work

Modeled data obtained from the BEM based modeling approach two and three reveals promising results that succeeds in mimicking the trend of the experimental data. The feasibility of deploying useful applications of the modeling approaches derived in the research is contingent upon validating the model against data from multiple experiments and viewing consistent outcomes. The approached derived from the research has the potential capacity of being a useful resource for small scale organizations and communities interested in investing on wind turbine to analyze the potential power output from the infrastructure.

The modeling approaches also highlights the significance of the incorporation of the induction factor into the calculations. The expression for the induction factors in equations 2.26 and 2.27 is derived equation the relationship for aerodynamic performances between the wind turbine modeling methods. In addition, the calculation of the induction factor in the modeling approaches is based on an initial guess, and therefore might not be reflective of the real-world aerodynamic conditions. Henceforth, future research warrants more exploration in the accurate modeling of the induction factor based on design parameters and inflow conditions.
REFERENCES


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*NACA 64(3)-418 (naca643418-il).* n.A. 

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APPENDIX A

NOMENCLATURE
\[B\] \hspace{1em} B \text{ Number of Blades}

\[a\] \hspace{1em} Axial Induction Factor

\[a'\] \hspace{1em} Angular Induction Factor

\[c\] \hspace{1em} Chord length

\[CL\] \hspace{1em} Lift Coefficient

\[CD\] \hspace{1em} Drag Coefficient

\[CN\] \hspace{1em} Normal Force Coefficient

\[CT\] \hspace{1em} Tangential Force Coefficient

\[dL\] \hspace{1em} Differential Lift

\[dD\] \hspace{1em} Differential Drag

\[dQ\] \hspace{1em} Differential torque

\[dP\] \hspace{1em} Differential power

\[f\] \hspace{1em} Tip loss coefficient

\[F\] \hspace{1em} Tip loss factor

\[R\] \hspace{1em} Total radius of the blade from hub to the tip

\[T\] \hspace{1em} Thrust

\[V_\infty\] \hspace{1em} Free stream velocity

\[r\] \hspace{1em} Elemental Radius

\[\alpha\] \hspace{1em} Angle of attack

\[\rho\] \hspace{1em} Density

\[\sigma'\] \hspace{1em} Local Solidity

\[\theta_T\] \hspace{1em} Local Twist

\[\theta_{CP}\] \hspace{1em} Pitch Angle

\[\phi\] \hspace{1em} Relative Flow Angle
angular velocity of the rotor disk
APPENDIX B

AIRFOIL DATA
DU91-W2-250 Lift and Drag Coefficient Plots

Figure B.1  $C_L$ Curve for DU91-W2-250 (Micallef, Kloosterman, Ferreira, & Bussel, 2009)

The image provides the $C_L$ curve for the DU91-W2-250 profile used in the modeling approach calculations.

Figure B.2  $C_D$ Curve for DU91-W2-250 (Micallef, Kloosterman, Ferreira, & Bussel, 2009)

The image provides the $C_D$ curve for the DU91-W2-250 profile used in the modeling approach calculations.
Ris$\varnothing$ A1-21 Lift and Drag Coefficient Plots

Figure B.3  $C_L$ Curve for Ris$\varnothing$ A1-21 (Fuglsang, Dahl, & Antoniou, 1999)

The image provides the $C_L$ curve for the Ris$\varnothing$ A1-21 profile used in the modeling approach calculations.

Figure B.4  $C_D$ Curve for Ris$\varnothing$ A1-21 (Fuglsang, Dahl, & Antoniou, 1999)

The image provides the $C_D$ curve for the Ris$\varnothing$ A1-21 profile used in the modeling approach calculations.
NACA 64-418 Lift and Drag Coefficient Plots

Figure B.5  $C_L$ Curve for NACA 64-418 (NACA 64(3)-418 (naca643418-il), n.d.)

The image provides the $C_L$ curve for the NACA 64-418 profile used in the modeling approach calculations.

Figure B.6  $C_D$ Curve for NACA 64-418 (NACA 64(3)-418 (naca643418-il), n.d.)

The image provides the $C_D$ curves for the NACA 64-418 profile used in the modeling approach calculations.
APPENDIX C

BUCKINGHAM PI EVALUATION OF BEM BASED POWER EQUATION
Writing and expanding out the equation of power from equation 2.25:

\[
\frac{dP}{d\sigma} = \frac{\Omega' \pi \rho V_c^2 (1 - a)^2 (C_L \cos \phi - C_D \sin \phi) r^2 dr}{\cos^2 \theta}
\]  

(C.1)

Reveals power is a function of the following dimensional variables: angular velocity, density, inflow velocity and radial element:

\[dP = f(\Omega, \rho, V, r)\]  

(C.2)

Hence, writing out the base dimensional units (L=unit of length, M=unit of mass, T=unit of time) for the dimensional variable above yields:

\[dP = f(\Omega, \rho, V, r)\]  

(C.3)

\[P = L^2 MT^{-3}\]  

(C.4)

\[\Omega = T^{-1}\]  

(C.5)

\[\rho = ML^{-3}\]  

(C.6)

\[V = LT^{-1}\]  

(C.7)

\[r = L\]  

(C.8)

In order to solve for the \( \pi \) variables, the number of total dimensional variable in the power equations (\( P, \Omega, \rho, V \) and \( r \)) is subtracted by the number of dimensions in the power equations (\( L, M, T \)); 5-3=2. Hence, the Buckingham \( \pi \) method is evaluated for two \( \pi \) variable, while choosing the dependent variables (\( \Omega, \rho, V \) and \( r \)) as the repeating variables.

Solving for \( \pi_1 \):

\[\pi_1 = \Omega \rho^a V^b r^c = T^{-1} (ML^{-3})^a (LT^{-1})^b (L)^c\]  

(C.9)

\[M: a = 0\]  

(C.10)
\[ L: -3a + b + c = 0 ; c = 1 \]  \hspace{1cm} (C.11)

\[ T: -1 - b = 0 ; b = -1 \]  \hspace{1cm} (C.12)

\[ \pi_1 = \Omega \rho^a V^b r^c = \Omega \frac{\rho^0 r}{V} = \frac{T^{-1}(1)(L)}{(LT^{-1})} \]  \hspace{1cm} (C.13)

Solving for \( \pi_2 \):

\[ \pi_2 = P \rho^a V^b r^c = L^2 M T^{-3} (ML^{-3})^a (LT^{-1})^b (L)^c \]  \hspace{1cm} (C.14)

\[ M: 1 + a = 0 ; a = -1 \]  \hspace{1cm} (C.15)

\[ L: 2 - 3a + b + c = 0 ; c = -2 \]  \hspace{1cm} (C.16)

\[ T: -3 - b = 0 ; b = -3 \]  \hspace{1cm} (C.17)

\[ \pi_2 = P \rho^a V^b r^c = \frac{P}{\rho V^3 r^2} = \frac{L^2 M T^{-3}}{(ML^{-3})(LT^{-1})^3(L)^2} \]  \hspace{1cm} (C.18)

The two \( \pi \) terms derived above represent two non-dimensional variables relevant to the performance of wind turbine aerodynamic analysis. \( \pi_2 \) relates to the expression that determines the coefficient of power, while \( \pi_1 \) is the expression for the tip speed ratio.
APPENDIX D

DERIVATION OF THE UPDATED EXPRESSION OF INDUCTION FACTORS
Derivation of Updated Expression of Axial Induction Factors

Relating the differential forces developed from the ADM based model and the BEM allows for the following expression, see equations 2.6 and 2.18:

$$\frac{1}{2} \rho V_x^2 4a(1-a)2\pi r dr = \frac{1}{2} B \rho V_r^2 (C_L \cos \phi + C_D \sin \phi) cd r$$  \hspace{1cm} (D.19)

Recalling the relationship of the normal force coefficient and the lift and drag coefficients in equation 2.14, the expression above could be modified to:

$$\frac{1}{2} \rho V_x^2 4a(1-a)2\pi r dr = \frac{1}{2} B \rho V_r^2 (C_N) cd r$$ \hspace{1cm} (D.20)

To eliminate all dimensional variables in the expression above, the elimination of the velocity components could be the top priority, where the relative velocity can be substituted with the previously derived relationship $V_r = \frac{V_x (1-a)}{\sin \phi}$

$$V_x^2 4a(1-a)2\pi r dr = B \frac{V_x^2 (1-a)^2}{\sin^2 \phi} (C_N) cd r$$ \hspace{1cm} (D.21)

$$8\pi ar = B \frac{(1-a)}{\cos^2 \phi} (C_N)c$$ \hspace{1cm} (D.22)

$$\frac{a}{1-a} = \frac{B(C_N)c}{8\pi r \sin^2 \phi}$$ \hspace{1cm} (D.23)

$$a = \frac{B(C_N)c}{8\pi r \sin^2 \phi} \frac{1}{\frac{B(C_N)c}{8\pi r \sin^2 \phi} + 1}$$ \hspace{1cm} (D.24)
Derivation of Updated Expression of Axial Induction Factors

Relating the differential forces developed from the ADM based model and the BEM allows for the following expression, see equations 2.10 and 2.19:

\[ 4\pi \rho V_x a' \Omega (1 - a) r^3 dr = \frac{1}{2} B \rho V_r^2 (C_L \cos \phi - C_D \sin \phi) crdr \]  
(D.25)

Recalling the relationship of the tangential force coefficient and the lift and drag coefficients in equation 2.15, the expression above could be modified to:

\[ 4\pi \rho V_x a' \Omega (1 - a) r^3 dr = \frac{1}{2} B \rho V_r^2 (C_T) crdr \]  
(D.26)

To eliminate all dimensional variables in the expression above, the elimination of the velocity components could be the top priority, where the relative velocity can be substituted with the previously derived relationship \( V_r = \frac{V_x(1-a)}{\sin \phi} \) and \( \Omega = \frac{V_x}{R} \)

\[ 8\pi V_x a' \frac{V_x \lambda}{R} (a - 1) r^3 dr = B \frac{V_x^2 (1-a)^2}{\sin^2 \phi} (C_T) crdr \]  
(D.27)

\[ 8\pi a' \frac{\lambda}{R} r^2 = B \frac{(1-a)}{\sin^2 \phi} (C_T) c \]  
(D.28)

\[ a' = \frac{BR (1-a) (C_T) c}{8\lambda \pi \sin^2 \phi r^2} \]  
(D.29)