CHARACTERIZATION OF FATIGUE CRACK PROPAGATION

IN AA 7075-T651

By

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To better design structures and machines, understanding of flaws and failures is essential. The body of this work has addressed numerous facets of fatigue crack propagation. The affect of crack closure, testing errors, and data scatter are a few important components of crack growth developed and investigated. It was found that the widely accepted compliance-offset technique for closure measurement may be sensitive to increases in load ratio. Opening load uncertainty was calculated to be on the order of 5%. The application of practical regression techniques and the use of $\Delta K_{eff}$ were used to characterize closure-free crack growth data to develop a single intrinsic $da/dN$ curve. The best form of regression was found to be a multi-linear fit. A strip-yield model requiring the intrinsic curve was used to successfully predict crack growth at other load ratios. Uncertainties with a strong dependence on crack mouth displacement were found for $da/dN$, $\Delta K$, and $a$. 
DEDICATION

This thesis, which is the greatest embodiment of my time and energy, is dedicated to my parents Lloyd and Daphne Blandford, and my brothers Dawson and Jeff. Thanks for everything.
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First and foremost I would like to express a great deal of thanks and appreciation to Dr. Steve Daniewicz. Dr. Daniewicz far exceeded my expectations from an advisor and made my entire graduate experience unforgettable. Moreover, I am very appreciative that Dr. Daniewicz continually encouraged an atmosphere of partnership rather than employment. Secondly, I would like to thank the Department of Mechanical Engineering for the opportunity to further my horizons. Special thanks within the department are given to my committee members Dr. W. G. Steele, Dr. E. W. Jones, Dr. J. T. Berry as well as Vic Latham, Ann Dawson, Dot Jenkins, and Liz Rook. Finally, I must also acknowledge and thank my colleague Jeff Skinner for the valuable finite element results contained in this work.
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CHAPTER I

INTRODUCTION

1.1 Background

Incorporating the existence of cracked geometries into current design methodologies requires an understanding of fatigue crack propagation. Currently, the use of varied experimental means allow for a very thorough understanding of crack propagation. The information gained through experimentation can be implemented in computer models to predict behavior in more advanced geometries that cannot be easily or inexpensively tested. Furthermore, models derived from theoretical means can be validated with the use of crack propagation data.

The best characterization of fatigue crack propagation lies in the crack driving force $\Delta K$. $\Delta K$ is the stress intensity factor range and is expressed as

$$\Delta K \cdot \cdot K_{\text{max}} \cdot \cdot K_{\text{min}}$$ (1.1)

Where $K_{\text{max}}$ and $K_{\text{min}}$ are the stress intensity factors at maximum and minimum load, respectively. A stress intensity factor describes the stress field immediately ahead of the crack tip. There are three types of stress intensity factors for the loadings of tension, in-plane shear, and out-of-plane shear. To expand the use of $\Delta K$ to structures revolves around the theory of similitude, which states that a stress intensity factor is independent of size and geometry as long as the body remains linear elastic. Using $\Delta K$ has allowed practical and informative crack propagation data to be used in service life predictions.
Fatigue crack propagation data obtained from a laboratory can be divided into three regions as seen in Figure 1.1. Region I is typically referred to as the threshold regime and is composed of slow crack growth behavior. Region II is sometimes called the Paris regime in recognition of P.C. Paris as the researcher who first correlated stress intensity factor range with fatigue crack growth rate (Paris 1963). The mentioned correlation is a linear one in log-log coordinates. The final region is one in which the maximum stress intensity factor $K_{\text{max}}$ approaches the fracture toughness $K_{\text{fc}}$ and is composed of rapid crack growth. The axes in Figure 1.1 are $da/dN$ and $\Delta K$ the fatigue crack growth rate and stress intensity factor range respectively. In the Paris regime they are related by the following expression.

$$\frac{da}{dN} = C\Delta K^m$$

(1.2)

Where $C$ and $m$ are material specific constants.

Figure 1.1 Fatigue Crack Propagation
Various other researchers have correlated $da/dN$ and $\Delta K$. The work of Forman et al. developed a relationship, expressed in equation 1.3, incorporating Regions II and III as well as the effect of stress ratio $R$ (Forman, 1967).

$$\frac{da}{dN} = \frac{C\Delta K^m}{(1 \cdot R)K_{crit} - \Delta K} \quad (1.3)$$

Where $C$ and $m$ are material specific constants different from the Paris coefficients and $K_{crit} \cdot K_{fc}$. Similar to the Forman equation is the correlation developed by Walker, given in equation 1.4 (Walker, 1970).

$$\frac{da}{dN} = \frac{C\Delta K^m}{(1 \cdot R)^{m(1-\gamma)}} \quad (1.4)$$

Once again the constant coefficients $C$ and $m$ are not related to the previous expressions and the $\gamma$ term adjusts for $R$ effects. Of the two previous expressions, the addition of stress ratio $R$ into the equations is of considerable significance. Research has shown that $da/dN$ vs. $\Delta K$ data for a single material varies with stress ratio $R$. Due to this dependence, a plot of $da/dN$ vs. $\Delta K$ cannot truly be considered a material property.

Aside from $da/dN$ versus $\Delta K$ data, other pertinent issues must be considered in fatigue crack growth. Small cracks, variable amplitude loading, environment, data scatter, residual stresses, and crack closure are just a few to consider. Of particular interest is the crack closure phenomenon. Crack closure may be described as crack surface contact while under tensile stress. The effect can be produced by crack surface roughness, corrosive debris between the crack surfaces, and a wake of plastically deformed material behind the crack tip. It is a prevailing thought that plasticity-induced closure is the dominant form in many applications. Conservative design estimates are
typically the result when crack closure is not accounted for in crack growth predictions. In addition, the consideration of plasticity-induced closure greatly facilitates analyses of variable amplitude loading. Figure 1.2 shows the presence of plasticity-induced closure for a growing fatigue crack.

![Diagram of Plastic Wake](image)

Figure 1.2 Plasticity-Induced Crack Closure

The effect of crack closure is measured by a crack opening load, stress, or stress intensity factor. The latter crack opening stress intensity factor $K_o$ has a significant effect on how fatigue crack growth is characterized. The effect of stress ratio as discussed earlier prevents a single $da/dN$ vs. $\Delta K$ plot for a given material. However it is thought that this effect is a consequence of the opening load dependence on $R$. Researchers acquired $da/dN$ vs. $\Delta K_{eff}$ data where $\Delta K_{eff}$ is defined as:

$$\Delta K_{eff} = K_{max} - K_o$$

(1.5)
These curves regardless of stress ratio collapse on top of one other indicating the single curve may be considered a material property. Research continues as to whether $\frac{da}{dN}$ and $\Delta K_{eff}$ are the intrinsic fatigue crack growth rate parameters (Bray, 1999) (Donald, 1998).

Both variable amplitude loading and environmental effects are also of significant interest. An overload during constant load crack growth can cause significant growth retardation. In the extreme case under random loading where numerous overloads and underloads are occurring, crack growth behavior is difficult to quantify. As for the effect of environment, the crack growth rates can be altered considerably in the presence of corrosive media or simple humidity changes. Most of the $\frac{da}{dN}$ vs. $\Delta K$ data generated is under standard lab conditions, which may or may not be what the component sees in service.

The issue of variability should also be considered in the acquisition of fatigue crack growth data. A single specimen test will tend to mislead an investigator as to the true behavior of a material. Multiple tests will allow a measure of how much a particular material varies with regard to crack growth. This measure of variation will change from material to material in almost a defined property sense. Designers may prefer to work with materials that do not exhibit high degrees of variability. Furthermore, in the construction of predictive models variability should be accounted for in some way.

The issues thus far discussed have been applied in varying degrees to predictive models. Unfortunately most of these models are developed around $\frac{da}{dN}$ vs. $\Delta K$ data. Giving a predetermined flaw, most models will result in a cycle count to failure as the output. Designers can determine if this time period will fall under scheduled inspections
or possibly last throughout the components life. The shortcoming of this approach is the lack of \( \frac{da}{dN} \) vs. \( \Delta K_{eff} \) data, which more appropriately characterizes how a material will behave with a fatigue crack present. Unfortunately, as with most new ideas or concepts, acceptance is a slow process.

1.2 Research Objective

The objective and focus of the current research is to characterize the fatigue crack growth behavior in AA 7075-T651. The investigation must include an examination of crack closure and data variability as well as the acquisition of a single intrinsic \( \frac{da}{dN} \) vs. \( \Delta K_{eff} \) curve for AA 7075-T651. The crack closure research should include different experimental data reduction techniques, analytical models, and a comparison between the two. The study of variability should include measures of specimen-to-specimen variation but most importantly a detailed look at single specimen errors and their effect. The acquisition of closure-free crack growth data should consist of testing in all three regions of crack growth and determination of regression coefficients for the AA 7075-T651 plate.

1.3 Thesis Overview

The author’s thesis contains a variety of work revolving around the investigation of an aluminum plate. Chapter II gives a thorough review of all pertinent research previously completed that relates to the current investigation. The reviewed topics include fatigue crack closure, experimental crack closure results, experimental noise, analytical closure modeling, fatigue crack growth rates, experimental crack growth variability, and variability modeling. Chapter III develops the experimental procedures,
equipment used, and support testing results. The support testing includes fracture toughness, tension, and preliminary crack propagation tests. The procedures within Chapter III include specific use of Instron’s fatigue testing software. The remaining chapters embody the bulk of the research. Chapter IV discusses the crack closure measurement results. These results include various data reduction techniques and the use of analytical models to predict crack opening loads. Chapters V and VII develop and apply the ideas of uncertainty analyzes to both crack opening loads and fatigue crack growth rates. Chapter VI contains the fatigue crack propagation data. The use of regression routines and $\Delta K_{eff}$ are within this chapter to aid in predictions and to find a single intrinsic fatigue crack growth curve.
CHAPTER II

LITERATURE REVIEW

2.1 Introduction

In the current investigation several aspects of fatigue crack growth are of interest. They include crack closure measurement and modeling, scatter and uncertainty in crack growth rates, threshold testing techniques, experimental validation, and the impact of experimental noise. Numerous researchers have dealt with these issues over the years and a review of this research is needed. Once past research has been covered then proper analysis of current issues can be achieved.

2.2 Fatigue Crack Closure

2.2.1 Concepts

The first significant work completed in the area of fatigue crack closure was accomplished by W. Elber (Elber, 1970) (Elber, 1971). His research focused on the premise that any growing fatigue crack under zero to tension loading exhibits crack closure. Through the analysis of force systems within a cracked geometry, Elber was able to develop a crack opening force. This analysis then proved that sections of a crack are partially closed even as tensile loads increase. Once the crack opening load is
reached then the crack is presumed fully open. The development of a crack opening load then gave rise to a new effective range of loading contributing to crack growth.

These ideas have been proven over the years and especially reemphasized by J. Schijve (Schijve, 1988). Schijve elaborated on the concepts of Elber and demonstrated the value of fatigue crack closure as it applies to experimentation and design. Model predictions of fatigue crack growth rates are much improved by the incorporation of crack closure. Furthermore, correct anticipation of service life can be obtained by the incorporation of closure principals into design phase analyzes.

2.2.2 Crack Opening Load Measurement

The understanding of fatigue crack closure has been greatly enhanced by the ability to measure its effect in controlled laboratory tests. The effect of closure is easily measured by the acquisition of a crack opening load. The current U.S. standard for measuring a crack opening load is contained within the American Society of Testing and Materials E647 specification (ASTM, 1999a). The compliance-offset technique, discussed in the ASTM specification, reduces load-displacement data to arrive at a crack opening load. Other techniques and methods of applying compliance offset have been developed.

Applying the compliance-offset technique requires the acquisition of load-displacement data from a cracked geometry. The displacement data can be obtained either near the crack tip or from a remote location on the specimen. Remote measurement can be accomplished with a crack mouth opening displacement gage or clip gage, a backface strain gage, or a potential drop apparatus. Near crack tip measurement
can be accomplished with two-stage replication (Allison, 1988), crack tip strain gages (Chen, 1991), or by a digital image displacement system (Sutton, 1999).

Other data reduction techniques include the reduced displacement technique (Elber, 1975). This technique develops displacement deviations from load-displacement data from which a crack opening load can be identified. A similar technique involving stress intensity factors with displacement deviations has been used to successfully identify the crack opening load in a specimen (Ray, 1988). Various other data reduction techniques have been developed that account for crack tip strains below the crack opening load (Donald, 1998) or use of direct load vs. strain traces (Booth, 1988).

2.2.3 Experimental Crack Closure Results

Crack closure, with its ease of measurement in through cracks, has been vigorously researched throughout the last three decades. The more recent thrust responsible for the ASTM compliance-offset technique revolves around a testing series commonly called a round robin (Phillips, 1993). A large number of laboratories tested compact tension C(T) specimens with either clip gages or backface strain gages. Data quality analysis was also a priority, and it was demonstrated that noise can have a predominant detrimental affect on closure measurements.

Research has also demonstrated that stress ratio and the magnitude of the maximum stress intensity factor in the load cycle $K_{max}$ affect fatigue crack closure (Shih, 1974). In addition, specimen thickness plays an important role in the measurement of crack closure (Bao, 1998). This is especially true when using remote measurement techniques, which fail to separate plane stress-plane strain variations through the
thickness. The majority of research has focused on through-cracks and plasticity-induced crack closure. However, there are instances when roughness or corrosion debris crack closure can dominate within a specimen (McEvily, 1997). Similarly, in the case of part through-cracks, the ability to measure the effect of crack closure becomes considerably more complex. Some researchers have tried to develop techniques using PMMA and optical interferometry (Troha, 1988) while others have used clever fractographic techniques to measure the effect of closure in semi-elliptical surface cracks (Putra, 1992).

2.2.4 Effect of Experimental Noise

Since crack closure effects are commonly measured in the laboratory the effect of experimental noise must be accounted for. In the general sense work has been done to differentiate the true data in any case from the noise being added (Trujillo, 1993). Prior to these analyzes the tools of cross-validation, dynamic programming, and other statistical methods were applied to purge noise from a data set (Trujillo, 1985). The combination of the aforementioned tools resulted in an algorithm that with the specification or calculation of a smoothing parameter would eliminate the effect of noise. The effect of noise has been shown to noticeably alter the crack opening load obtained from the ASTM compliance-offset technique (ASTM, 1999a) (Phillips, 1993). To that end the methods developed by Trujillo and Busby and been successfully applied to eliminate noise from load-displacement data prior to applying the compliance-offset technique (Daniewicz, 1999)
2.2.5 Analytical Crack Closure Modeling

Analytical modeling of fatigue crack closure has been valuable in the accurate prediction of fatigue crack growth rates. The simplest form of modeling stems from the wealth of experimental data that is available for curve-fitting and other techniques (Schijve, 1981). However, the most widely used analytical technique derives from the work of D. S. Dugdale (Dugdale, 1960). Dugdale measured the size of plastic zones in simple slotted plates. Years later it was found that the behavior observed in these plates was in effect cracks and their respective crack tip plastic zones. Several effective models, known commonly as strip-yield models, have developed from the work of Dugdale to predict the effects of plasticity induced crack closure.

The strip-yield model of J. C. Newman Jr. has been packaged in a publicly available code, FASTRAN, and used with considerable success in predicting crack closure effects (Newman, 1992). The model has developed the Dugdale theory to incorporate both 3-D constraint effects and crack closure behavior under spectrum loading (Newman, 1981). The strip-yield model not only allows for the prediction of closure effects but subsequent crack growth estimates in the presence of crack closure (Newman, 1999). The work of Gangloff et al. showed a large number of different alloys under various loading and environmental conditions comparing well with the FASTRAN model (Gangloff, 1994).

Several other researchers have produced similar models. The strip-yield model of Daniewicz et al. incorporates weight or influence functions to calculate the stress intensity factors in each geometry of interest (Daniewicz, 1994). The work has shown that along with the effects of crack closure, residual stresses may also be included into the
model. Furthermore, the strip-yield model has also been used to study the affect of geometry and stress ratio (Daniewicz, 1996). The model developed by Wang and Blom has furthered the strip-yield model to analyze plane stress-plane strain transitions (Wang, 1991).

Fortunately, strip-yield models are not the only analytical model available for obtaining crack opening loads. Recently both 2-D and 3-D finite element models have been successfully used to predict crack closure behavior (McClung, 1999). Comparisons between finite element analyses, FEA, and accepted strip-yield models have shown good correlation (McClung, 1994). In addition these models can be employed to analyze crack closure under either simple constant amplitude loading or more complex load shedding conditions (Skinner, 1999). Unfortunately the use of FEA involves computationally intensive calculations that either requires powerful computing resources or great patience.

2.3 Fatigue Crack Propagation

2.3.1 Paris Law

The development of fatigue crack propagation equations has greatly enhanced the ability to create damage tolerant designs. The most notable work has been performed by Paris and Erdogan (Paris, 1963). The propagation law developed is expressed in equation 1.1. This equation developed from the hypothesis that the driving force for a fatigue crack lies in the stress field near the crack tip (Paris, 1961). This near crack tip stress field has been characterized by Irwin using the stress intensity factor (Irwin, 1957). Thus the fatigue crack growth rate in the Paris/Erdogan equation is a direct function of the
stress intensity factor range. Today the fatigue crack growth rate equation is commonly called the Paris Law (Anderson, 1995).

2.3.2 Threshold Regime

Interest in fatigue crack growth rates below the Paris regime has forced researchers to develop different and more complex forms of testing. To obtain data in the threshold regime requires a load shedding technique. The two most widely accepted are fixed $R$ and constant $K_{\text{max}}$ testing. The numerous issues that must be addressed to perform a successful threshold test include $K$ gradient, initial $K_{\text{max}}$, prolonged test interruptions, precracking, residual stresses, and environmental conditions (Bush, 2000). The initial $K_{\text{max}}$ issue is of particular concern as researchers have gathered significantly different threshold growth rates with changes in $K_{\text{max}}$ (Newman, 2000). Newman et al. also demonstrated that an increase in crack tip voids results with increasing $K_{\text{max}}$ levels.

2.3.3 Variability of Test Results

Historically, variability or scatter in stress-life data has gained much notoriety. Unfortunately this is not the case with fatigue crack growth rate testing. It has been demonstrated that data scatter can affect the viability of using certain data sets in design scenarios (Schijve, 1994). Furthermore, factors in the lab inducing scatter may not be present in service life while factors influencing service life scatter may not be present in the lab. The benchmark of fatigue crack propagation scatter is the 68 replicate specimen database produced by Virkler et al. (Virkler, 1979).
The ASTM has taken a particular interest in fatigue crack growth rate variability and performed a round robin to examine causes and effects (Clark, 1975). The round robin test series was a comprehensive examination of various variables possibly contributing to growth rate scatter. A variability factor was defined as equation 2.1.

\[ VF = \frac{\max \left( \frac{da}{dN} \right)}{\min \left( \frac{da}{dN} \right)} \]  

(2.1)

Where the maximum and minimum growth rates are based on a 2 \( S_R \) criterion. Where \( S_R \) is the residual standard deviation. The values range from 1.31 to 2.93 for different specimen geometries and from 1.27 to 4.00 for different laboratories. Different data reduction techniques were also examined and variability factors associated with the techniques varied from 2.51 to 3.97. Clearly a large degree of scatter can occur in fatigue crack growth rates depending on different geometries, labs, and data reduction schemes.

Further research by other investigators has revealed that large scatter can be produced from the simple difference in natural and artificial cracks (Schijve, 1979). In addition, testing frequencies (Johnston, 1983) and environmental conditions (Shaw, 1981) have been shown to influence fatigue crack growth variability. The latter demonstrated a factor of seven spread in growth rates with drastic changes in relative humidity. Shaw et al. also calculated a small difference in statistical scatter when either normal, lognormal, or Weibull distributions are used. It should be noted that the majority of research has focused on multiple specimen scatter without an analysis of single specimen error distribution.
2.3.4 Variability Modeling

The modeling of variability is for the most part an advanced statistics problem and to that end has received considerable attention. Several models have used the work of Virkler and applied it to the coefficients in the Paris Law. Ostergaard and Hillberry analyzed various different regression techniques to find Paris coefficients from the Virkler data (Ostergaard, 1983). The technique most favored then accounted for scatter in subsequent use of the coefficients developed. Boganoff and Kozin applied a generalized B-model to the Virkler data (Boganoff, 1984). The B-model then predicted growth rates with a variability feature included. Ditlevsen and Olesen further analyzed the Virkler data by using maximum likelihood estimation to acquire the Paris coefficients (Ditlevsen, 1986). The Paris law was then blended with a stationary non-negative white noise process to allow the model to predict growth rates with variability added.

The addition of variability in the Paris coefficients seems to be the predominant approach to dealing with variability. Bastenaire et al. incorporated scatter into the Paris coefficients through the use of iterative convergence on specially integrated forms of the propagation law (Bastenaire, 1981). Engesvik and Moan used Monte Carlo simulations to estimate a set of Paris coefficients that are adjusted for variability (Engesvik, 1983). Of course the Paris law is not the only description of crack propagation. Ghonem and Dore constructed a model based on the analogy between a discontinuous Markovian stochastic process and the crack propagation process (Ghonem, 1987). However, the model was applied to the Forman equation (Forman, 1967) of fatigue crack growth. A more elaborate hyperbolic sine expression for fatigue crack growth has been developed and combined with a Gaussian random process to incorporate variability (Yang, 1983).
The models presented thus far have incorporated scatter into subsequent crack growth predictions; however, the ability to remove variability before the data is used for prediction has also been addressed. The use of least squares curve fitting to provide smooth $a$ vs. $N$ data has been demonstrated to improve future predicted results (McCartney, 1977). This assumes that the data reduction technique involved in acquiring $da/dN$ from the $a$ vs. $N$ data induces considerable variability. Other statistical work has also been applied to reduce the scatter in growth rates through the use of similar but different least squares techniques (Zheng, 1997).

2.4 Displacement and Crack Length Validation

Experimentally measuring crack opening loads and fatigue crack growth rates requires accurate and precise instrumentation. When using a remote mounted clip gage the displacements being recorded can be compared with analytically determined displacements (Roberts, 1969) or finite element solutions. The determined displacements are used directly in closure measurements and indirectly for fatigue crack growth rates. Expressions have been developed to relate remote displacements with specimen crack length through compliance methods (Saxena, 1978a). It is well known that these compliance methods tend to under-predict crack lengths. Modification factors to the modulus of elasticity are widely used to account for the effect (Hewitt, 1983) (Saxena, 1978a). These compliance methods have greatly enhanced the ability to perform tests in both the threshold and Paris regime with complete computer automation (Saxena, 1978b) (Donald, 1980). However, care must be taken to visually verify compliance crack lengths whenever possible.
CHAPTER III

EQUIPMENT, PROCEDURES, AND SUPPORT TESTING

3.1 Introduction

The current investigation requires the use of specialized equipment, careful procedures, and productive support testing. The equipment includes not only the testing apparatus but also data acquisition mechanisms and testing specimens. The procedures include the methods applied to the testing equipment and software inputs. Finally, the support testing incorporates all testing outside of the specimen data collected for the current work. Careful consideration and adhering to strict guidelines in these areas has led to a series of successful testing.

3.2 Equipment

3.2.1 Testing Apparatus

The most important piece of equipment used in this research is the SATEC TC-25 Uniframe test machine. The TC-25 is a servo-hydraulic test machine with a 25,000 lb. load cell. Recently the TC-25 has been upgraded with an Instron 8800 controller and Dell Dimension PC. A large compliment of testing software was included with the controller upgrade. Specifically written software for fatigue crack propagation and fracture toughness was included with more generic fatigue and static packages. To aid in
data acquisition, a SATEC DG-25 clip gage was supplied with the original load frame purchase. The gage allows for electronic crack length determination and more computer automated testing.

Several additional pieces of equipment were also acquired to aid in the testing. The first was a pair of pin and clevis fixtures machined per ASTM specification (ASTM, 1999a). The pin and clevis fixtures were designed to thread directly into the TC-25 frame; however, adaptors were created to allow use of the TC-25’s hydraulic grips. The addition of the adaptors allowed other researchers access to the machine, and less man-hours removing the hydraulic grips. Verification of electronically measured crack lengths was needed to ensure accurate testing, and was accomplished with a 10X Gaertner telemicroscope. To aid in the visual telemicroscope measurements a Dolan-Jenner MI-150 Fiber-Lite® is used to illuminate the polished specimen surface. A final piece of equipment was the steel knife edges mounted on the C(T) specimens to allow the DG-25 to be mounted.

3.2.2 Test Specimens

The entire current research revolves around a 0.5 in. thick plate of AA 7075-T651. The composition of 7075-T651 is 5.6% Zn, 2.5% Mg, 1.6% Cu, and 0.23% Cr (Dowling, 1999). The T651 temper is a solution heat treatment with stress-relief by stretching and artificial aging. The specimens needed from the plate were coupons for tension testing and C(T) specimens for the fracture toughness and crack growth tests. The specimens were to comply as close as possible with current ASTM procedures (ASTM, 1999a) (ASTM, 1999b).
The tension specimens seen in Figure 3.1 were modified slightly from the ASTM specification to facilitate easier machining and easier mounting in the test machine. Since the plate was limited to 0.5 in. thickness, the C(T) specimen seen in Figure 3.2 was used for fracture toughness and crack propagation tests. Ideally a thicker specimen would aid in the fracture toughness testing but this was not available. All dimensions were verified after machining and the C(T)’s were polished with a 600/1500 grit sandpaper series and then polished with Never Dull.

Figure 3.1 Tension Test Specimens for AA 7075-T651 Plate

Figure 3.2 Specimen for Crack Propagation and Fracture Toughness
3.3 Testing Procedures

Proper test procedures are important in gaining accurate experimental data. The procedures used to develop the current data are from both the ASTM standards and a bit of trial and error. All of the tests aside from the tension test require a period of constant amplitude loading. Either the $da/dN$-32 or Single Axis Max [SAX] software, developed by Instron, was used to accomplish the loading. $da/dN$-32 allows a crack of fixed length to be grown in the specimen while SAX works with a cycle count criteria. Both require the maximum load $P_{max}$ and load ratio $R$ to be specified. In the case of $da/dN$-32, the compliance information must be supplied for automated crack length determination. A number of specimens were run under constant amplitude loads with $P_{max} = 2000$ lbs. and $R = 0.7$ with the $da/dN$-32 software. Cycle count, crack length, $da/dN$, and $\Delta K$ were stored in the output files to allow plotting and analyses.

The procedure to obtain crack opening loads consists of two stages. The first is constant amplitude loading at a $P_{max} = 1200$ lbs. and varying load ratios of 0.1, 0.2, and 0.3. The SAX software was used, since $da/dN$-32 had not been obtained, to cycle the specimen until the crack reached scribe marks on the specimen. The scribe marks denoted crack lengths of 1.0 and 1.5 in. Once at the crack lengths of interest the second stage of the procedure began. SAX was used to apply a slow ($\approx 0.1$Hz) low cycle ($\approx 60$) fatigue test that gathered large amounts ($\approx 1000$ points/cycle) of load versus displacement data. The cycle count was chosen to eliminate any transient behavior in the load-displacement data. The test was applied at both crack lengths and the subsequent data analyzed for the current investigation.
The threshold tests also consisted of a period of constant amplitude loading. The load ratio was 0.7 and $P_{max} = 2000$ lbs. Da/dN-32 was used to achieve the loading with an end crack length criterion of $a = 0.8$ in., measured from the load line. The da/dN-32 software was then changed to perform a constant $K_{max}$ test. This type of testing is governed by equation 3.1 and involves shedding load.

$$\Delta K = \Delta K_a \cdot \varepsilon^{[C(a-a_c)]}$$

(3.1)

Where $\Delta K$, $C$, and $a$ are the stress intensity factor range, $K$-gradient, and crack length, respectively. The constant $K_{max}$ test was allowed to run until load application stability deteriorated. The da/dN-32 software stored all the appropriate data throughout the test to be examined later.

The fracture toughness tests also consisted of constant amplitude loading period with $P_{max} = 1200$ lbs and $R = 0.1$. Using da/dN-32 allowed a crack length of approximately 1.5 in. to be placed in each specimen. Once at the crack length Instron software Klc was used to perform the test. A loading rate of 300 lbs/min and sampling interval of 10 lbs were specified in the software along with material properties and compliance information. The Klc software collected data until the specimen fractured and then per ASTM guidelines requested five fracture surface measurements and a reject/accept decision on the fracture surface condition (ASTM, 1999b). The data could then be reduced via ASTM recommendations.

The tension tests were performed in two groups. The first group consisted of four specimens loaded at 2500 lbs/min. A 2 in. extensometer was placed in the gage length to record the strain. Because the extensometer cannot withstand fracture it was removed at a strain of approximately 0.007 in/in. After the extensometer was removed the
specimens were loaded to fracture. The second test group of two specimens was loaded at 2500 lbs/min until the load reached a value about 2000 lbs below the load corresponding to a strain of 0.007 in/in. At this point the load rate was decreased to 500 lbs/min in hopes of gathering more data close to the yield stress.

3.4  Support Testing

3.4.1  Overview

Support testing is considered to be all testing outside of the main testing used for analyzes. These tests include the tension, fracture toughness, preliminary crack propagation, and clip gage calibration testing. Through these tests the above procedures were established. The tests allowed a close examination of software package capabilities as well as the machine’s limits.

3.4.2  Tension Tests

The tension tests performed on the AA 7075-T651 specimens were designed to reveal the materials ultimate strength, yield strength, and elastic modulus. The results of the first stage of tension tests are presented in Table 3.1.

Table 3.1 Results of AA 7075-T651 Tension Tests

<table>
<thead>
<tr>
<th>Spec. #</th>
<th>Load at 1° (lbs.)</th>
<th>Stress at 1° (psi)</th>
<th>Strain at 1° (µε)</th>
<th>Ultimate Load (lbs.)</th>
<th>Ultimate Stress (psi)</th>
<th>Modulus (Msi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17,270</td>
<td>69,625.1</td>
<td>6440</td>
<td>21,940</td>
<td>88,452.5</td>
<td>10.35</td>
</tr>
<tr>
<td>2</td>
<td>19,050</td>
<td>76,801.3</td>
<td>7720</td>
<td>21,910</td>
<td>88,331.6</td>
<td>10.53</td>
</tr>
<tr>
<td>3</td>
<td>18,460</td>
<td>74,422.7</td>
<td>7190</td>
<td>21,370</td>
<td>86,154.6</td>
<td>NA</td>
</tr>
<tr>
<td>4</td>
<td>17,540</td>
<td>70713.7</td>
<td>7100</td>
<td>21,290</td>
<td>85,832.1</td>
<td>NA</td>
</tr>
</tbody>
</table>

* Indicates point where extensometer was removed
It should be immediately clear that yield stress is not included in the above table. The reason is that the AA 7075-T651 has a yield stress around 77 ksi and the stresses at extensometer removal are below this expected yield stress. The stress vs. strain plot for specimen 2 is shown in Figure 3.3. The regression line is completely obscured reinforcing the theory that the extensometer was removed before the onset of yielding. The $R^2$ value for the regression line was 0.9992. As for the other values, an ultimate strength of 84 ksi and modulus of 10.5 Mpsi were expected.

![Figure 3.3 Stress vs. Strain for AA 7075-T651 [Specimen 2]](image)

The stage 2 specimens with a change in load rate and longer extensometer attachment were to hopefully reveal the yield strength. Unfortunately this was not possible for two reasons. First, when the loading rate was changed the software adjusted the sampling rate so that the same number of data points were outputted to the file.
Secondly, allowing the extensometer to stay on longer but without risk to the gage only increased the removal stress to 79 ksi, which is only marginally greater than the expected yield stress. It is the opinion of the author that to obtain an accurate yield stress either requires a fracture capable extensometer or the use of strain gages.

3.4.3 Fracture Toughness

Similar to the tension tests, the fracture toughness tests were designed to better understand the AA 7075-T651 plate being investigated. Fracture toughness $K_{lc}$ is an important material property in the field of linear elastic fracture mechanics LEFM. Four specimens were used in the testing series and the results of the tests are in Table 3.2. The validity criterion is based on the ASTM ratio $P_{\text{max}}/P_Q$ and plastic zone calculations.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_Q$</td>
<td>N/A</td>
<td>27.74</td>
<td>28.53</td>
<td>27.34</td>
</tr>
<tr>
<td>Validity</td>
<td>Failed- Mixed mode Fracture</td>
<td>Failed- Ratio</td>
<td>Failed- Ratio</td>
<td>Failed- Ratio</td>
</tr>
</tbody>
</table>

The results in the table are somewhat misleading in the sense that specimens 2, 3, and 4 all failed the ratio $P_{\text{max}}/P_Q$, which ASTM states as being grounds for not calculating $K_Q$ (ASTM, 1999b). In fact, the possibility of $K_Q$ having no relation to $K_{lc}$ may be possible when this ratio fails. In spite of this factor the author calculated a $K_Q$ value to at least determine if the value was in the region of other recorded values. It should be noted that fracture toughness is thickness dependant and asymptotically approaches a plane strain fracture toughness as much thicker specimens are used. The $K_{lc}$ value reported for AA 7075-T651 is 26.45 ksi\(\sqrt{\text{in}}\) for an 0.75 in thick specimen with an L-T orientation (USAF,
From this it is clear that the specimens tested in this series are close to the reported value but high as a result of insufficient thickness. The surfaces of the fracture toughness specimens were observed to have large shear lips at both edges and end. (Berry, 2001). Furthermore, the crack had visible waviness, which may be a product of material inhomogeneity. Overall the fracture surface reinforces the failed tests.

### 3.4.4 Preliminary Crack Propagation Tests

Prior to performing the main crack propagation tests and determining specimen geometry a number of tests with practice specimens were completed. The practice specimens had the holes, needed for pin and clevis loading, drilled in the wrong location. It was decided to create notches with a bandsaw and then load the specimens to observe their behavior. The first most notable observation was the crack initiation site at the end of the bandsaw cut. Because the cut ended in a predominantly square pattern the crack initiated from the upper corner of the square and headed upward in the specimen. After numerous cycles of loading the crack, which should have begun to straighten itself, continued on a curved path.

Two major changes were made to account for the above observations. First the C(T) specimens used in the main testing were supplied with sharp notches as per Figure 3.2. Secondly, the curved crack growth was clearly a product of misalignment in the load train. This required the design of a restraining device to hold the assembly together as it was installed into the test machine grips. Further preliminary tests were performed with improved bandsaw notches and better load train alignment. The results were quite good both in terms of crack initiation site and straightness of crack growth.
3.4.5 Clip Gage Calibration Tests

To perform accurate crack propagation tests and even fracture toughness tests requires a calibrated clip gage. The DG-25 gage had been supplied with the testing system before the Instron upgrade. It was initially assumed that the gage was calibrated when the upgrade was made but this was not so. The tests consisted of an uncracked test specimen loaded under constant amplitude loading till a substantial crack was present, and then load-displacement data was acquired from the gage and its accuracy determined. The first results revealed about a 43% error in crack length with the clip gage. These results were further verified by published work (Roberts, 1969) and linear elastic finite element results. Clearly this level of accuracy was totally unacceptable. Calibration was performed on the gage by the author and accuracy was improved to about 10% error. Considering this to be unacceptable still for future testing, an Instron technician next calibrated the gage. Currently electronic crack length measurements are within about a maximum of 1.5% error. Once successful calibration was achieved the main body of testing could be performed.
CHAPTER IV
CRACK OPENING LOAD MEASUREMENT

4.1 Introduction

Making a structure or component damage tolerant requires insight into the behavior of anticipated flaws. Damage tolerant design also requires knowledge of the effective portion of the loading cycle that contributes to crack propagation. The effective stress intensity factor range $\Delta K_{\text{eff}}$ is defined as:

$$\Delta K_{\text{eff}} \equiv K_{\text{max}} - K_{\text{o}}$$  \hspace{1cm} (4.1)

Where $K_{\text{o}}$ and $K_{\text{max}}$ are the stress-intensity factors associated with an opening load and maximum load, respectively.

The opening load quantifies the effect of plastically deformed material behind the crack tip, surface roughness along the crack surfaces, or corrosion debris. It is known that the opening load is a function of the load ratio $R$. As $R$ increases the opening load increases until the load ratio becomes high enough that the crack is fully open and $K_{\text{o}} = K_{\text{min}}$. As the load ratio increases the opening load should become more difficult to measure since the period of time the crack surfaces are in contact is shortened.

Furthermore, the development of standard test methods have relied heavily on the use of data with $R = 0.1$ (Phillips, 1993).
Experimentally measuring an opening load generally requires the use of a numerical method capable of reducing global load-displacement data to obtain a single opening load. Numerous data reduction techniques have been developed over the years to perform the opening load calculation (Donald, 1998) (Booth, 1988) (Allison, 1988) (Elber, 1975). These methods require some form of data reduction to find the point at which the load-displacement curve first becomes linear. The initial nonlinear region is associated with the crack opening process, and the crack opening load is defined to be the load at which the curve first becomes linear. If noise is present in the load-displacement data, as is always the case to a certain degree, then a data reduction technique, which is sensitive to noise, could make effective interpretation difficult.

Interpreting the opening load using most data reduction techniques can be highly subjective (Donald, 1988). To minimize the impact of noise, many experimenters have resorted to approximations such as offsets to help encourage agreement from test to test. An offset is simply a point shifted slightly from the point of interest to accommodate noise or other variations in the data. If effective comparisons between models and data are to be made, reliable experimental data with a minimal degree of noise must be obtained. Minimizing the impact of noise would, in turn, eliminate the use of offsets and other approximations.

One possible method to eliminate the problem of noise in load-displacement data is to smooth the data using a low-pass filter. Of course an electrical modification can be made to the test machine that enables the filtering of noise; however, this is not always feasible or desired. An alternate numerical approach using dynamic programming easily allows the data to be filtered (Daniewicz, 1999).
If only plasticity-induced closure is of interest, prediction of the crack opening load can be accomplished by the use of several methods including strip-yield models or finite element analyses (Daniewicz, 1994) (Skinner, 1999) (Newman, 1981) (McClung, 1999). The foremost advantage of using a model is that flaws in structures can be analyzed when experimentation is unreasonable due to size or expense. However, models for prediction of the opening load have not yet achieved full acceptance. In this paper, several models will be used to compare predictions with experimental data, allowing the models to be compared and judged.

4.2 Methodology

Experiments were run on AA 7075-T651 using standard C(T) specimens (ASTM, 1999a) and a crack mouth displacement clip gage. The yield strength, ultimate strength, and plane strain fracture toughness were approximately 77 ksi, 84 ksi, and 26 ksi respectively. The specimens had a width $W = 3.0$ inches and a thickness $B = 0.5$ inches. The crack length was 1.5 inches, measured from the load line. The maximum applied load was 1200 lbs and three load-ratios of 0.1, 0.2, and 0.3 were used. The loading was constant-amplitude and sinusoidal with a frequency of 12 Hz. Sufficient load versus displacement data was recorded at the crack length of interest. Using several data reduction techniques, the opening load was then determined for both raw and smoothed data. Multiple specimens were tested at each load ratio; however, since little differences were observed in the reduced data only results for a single specimen at each load ratio are presented.
When using two-dimensional strip-yield models, a constraint factor $\alpha$ must be specified. The constraint factor varies from 1 to 3, where $\bullet \Leftarrow 1$ represents a plane stress crack tip condition and $\alpha = 3$ represents plane strain. For $1 < \bullet < 3$, a finite thickness three-dimensional body may be approximated. A thickness criteria for plane strain may be written as:

$$t' \bullet 2.5 \left( \frac{K_{\text{max}}}{S_y} \right)^2$$

(4.2)

Where $K_{\text{max}}$ and $S_y$ are the stress intensity factor at maximum load and yield strength respectively. Using eqn. (4.2), a value of $t' = 0.075$ in was obtained. With $t' \ll B$, $\alpha = 3$ was chosen to model the plane strain conditions existing in the specimen. An $\bullet \Leftarrow 2$ was also utilized to give a pragmatic upper limit to the strip-yield model predictions as opposed to a value of $\bullet \Leftarrow 1$ which was judged excessively low given $t' \ll B$.

The strip-yield models used were FASTRAN (Newman, 1992) and a similar model FLAP (Daniewicz, 1994). FASTRAN computes opening loads for center-crack specimens and uses a $K$-analogy between the center-crack strip-yield model and the C(T) specimen to determine the C(T) opening loads. McClung has shown that this $K$-analogy approach correlated well with two-dimensional finite element models (McClung, 1994). FLAP considers the C(T) geometry directly through the use of a weight function based approach. The inputs to these models are the AA 7075-T651 material properties and the specimen geometry. The output varies from program to program, but each will output the crack opening load as a function of crack length.

As a more rigorous but computationally intensive alternative to a strip-yield model, plasticity-induced crack closure may be modeled using three-dimensional elastic-
plastic finite element analysis (McClung, 1999). This involves performing a series of load cycles, each of which is composed of two monotonic analyses and a crack advance. The model is incrementally loaded to the maximum applied load, at which time the crack tip nodes are released allowing the crack front to advance one elemental length, and the applied load is incrementally lowered until the minimum load is attained. Crack surface closure is modeled by changing the boundary conditions on the crack surface nodes. During unloading the crack surface nodal displacements are monitored, if a nodal displacement becomes negative the node is closed and a nodal fixity is applied to prevent crack surface penetration. Similarly, during loading the reaction forces on the closed nodes are monitored, and when a reaction force becomes positive the nodal fixity is removed and the node is open. The load cycles are repeated until the opening levels reach steady state. Once the opening levels have stabilized, a single opening level is calculated by averaging the opening levels along the crack front.

4.3 Compliance-Offset Technique

The first method investigated for determining an opening load from experimental data was the compliance-offset technique. This method, formalized by the American Society of Testing and Materials (ASTM, 1999a) uses multiple least squares fits for segments of the load-displacement loading curve to compute compliance \( C \). A fully open compliance \( C_o \) is also computed using the unloading portion of the load-displacement data. The compliance-offset is defined as:

\[
\text{compliance offset} \cdot \frac{C_o \cdot C}{C_o}
\]  

(4.3)
For small applied loads, the crack will be partially closed and exhibit a compliance $C < C_0$. Conversely, for large applied loads, crack tip plasticity will result in a compliance $C > C_0$. The load at which the compliance-offset reaches zero with $C = C_0$ is theoretically defined to be the opening load. To allow for the effect of noise, and to make the measurement more robust, an offset of 1% or 2% is often used instead of zero.

Looking at the results in Figure 4.1, a clear increase in opening load as $R$ increases is observed. Data were evaluated in both the raw and smoothed conditions, and smoothing was performed using a dynamic programming algorithm (Daniewicz, 1999) which requires specification of a smoothing parameter. The effect of smoothing was minimal. A wide range for the smoothing parameter [see (Daniewicz, 1999) for details] was investigated on all the load ratios until an optimum value was found. The optimum value was determined from graphical analyses and a single value used throughout. The smoothing parameter defines the cut-off frequency at which the data are filtered (Daniewicz, 1999). It should be noted that the use of least squares regression, which is an integral part of the compliance-offset technique, inherently smoothes the data when computing compliance for 10% increments along the loading curve. Consequently, it might be expected that additional data smoothing would have only marginal influence.
In terms of actual opening loads at a 1% or 2% offset, the smoothed and raw data give values that are essentially equal. The percent differences are on the order of 2-3%. It is the opinion of the authors that a 0% offset has more physical significance. Unfortunately, it is clear from Figure 4.1 that for load ratios above $R = 0.1$ it is not possible to obtain 0% offset values from the raw or smoothed data, because the data exhibit a distinct shift off of the 0% offset axis.

4.4 Reduced Crack Mouth Opening Displacement Technique

In an effort to investigate other possible means of obtaining an opening load from load-displacement data, the reduced crack mouth opening displacement [CMOD] technique, developed by Elber (Elber, 1975) was studied. The reduced CMOD technique
fits a single least squares line to the linear portion of the loading curve and then compares
displacements computed from this line with measured displacements to define a
displacement deviation. The applied load is then plotted as a function of the
displacement deviations. Researchers generally define the opening load as the point
where the aforementioned curve exhibits a vertical tangent [VTP] (Elber, 1975) (Ray,
1988) (Newman, 1999). However, a better estimation of the opening load might be when
the displacement deviation equals zero. For completeness both will be evaluated and
compared.

The data in Figure 4.2 demonstrates the effect of smoothing when using the
reduced CMOD technique. Without question if the reduced CMOD technique is used
then smoothing is very beneficial. However, this method appears to be largely unused in
present day research. The method’s apparent sensitivity to noise may have played a role
in this regard. With respect to the measured opening loads for the smoothed data, from
Figure 4.2 the zero deviation criterion gives a normalized load of approximately 0.280
whereas the VTP criterion gives a value of approximately 0.260. If smoothed
compliance-offset was the true value (= 0.225 at 1% offset) then the error for the zero
deviation value is 24.5% and the error for the VTP value is about 15%. Of course, the
selection of the VTP is highly subjective and could be as low as 0.230 and as high as
0.300. With this in mind it can be said that reduced CMOD tends to predict higher
opening loads than compliance-offset when using a nonzero offset. The $R = 0.2$ data
showed similar trends when compared with the $R=0.1$ data. However, the trends in the
data began to change for the $R = 0.3$ data. Figure 4.3 compares the smoothed and raw
data from the reduced CMOD method at $R = 0.3$. From an initial inspection the use of a
zero deviation appears to be erroneous. This is consistent with the effect load ratio had on compliance-offset. As the load ratio increases the data pulls away from the zero deviation line or 0% offset line such that only inaccurate results can be obtained from the specified criteria. It should be noted that the data plotted in Figures 4.2 and 4.3 were both smoothed using the same smoothing parameter value previously discussed. The data shown in Figure 4.3 would clearly benefit from additional smoothing, but a single smoothing parameter was utilized for all work reported herein. It should be noted that the VTP value obtained from the $R = 0.3$ data agreed well with compliance-offset at 2%.

![Figure 4.2 Effect of Smoothing Data When Using Reduced CMOD Method ($R=0.1$)](image-url)
Figure 4.3 Effect of Smoothing Data When Using the Reduced CMOD Method ($R=0.3$)

4.5 Modified Compliance-Offset Technique

Since the load ratio seems to have a significant affect on the ability of processed data to approach a zero offset or zero deviation, then some modification must be found to correct for the effect. Considering only the compliance-offset technique, two possible changes are of interest. The first change addresses the specified 25% segment of the unloading curve used to find the open-crack compliance $C_o$ (ASTM, 1999a). Since unloading is linear it seems reasonable that increasing this percentage would give a more accurate $C_o$, which in turn would give a more accurate compliance offset and opening load. The second possible change considers the 10% increments specified (ASTM, 1999a) on the loading curve to define the least squares compliance values $C$, which are then compared with $C_o$ to find the offset. Basically, the increment was investigated in
order to have a complete investigation of the various components of the compliance-offset technique that may contribute to the observed data shift.

To examine the effect of different segment lengths on the unloading curve, lengths of 50%, 75%, and 100% were investigated. The data in Figure 4.4 demonstrates the effect of different segment lengths when using smoothed data at $R = 0.3$. As the segment length increases the tendency of the curve to shift is decreased. This then enables a 0% offset normalized opening load of approximately 0.410 to be obtained when using a 100% segment length. The standard 25% segment (ASTM, 1999a), produces a data set shifted too far to allow for a 0% offset to be used. If having the ability to produce a 0% offset opening load is desirable, then it would appear that a segment length greater than 25% is needed, especially at load ratios above 0.1.

![Figure 4.4 Effect of Unloading Least Squares Segment Length When Using Compliance-Offset Method ($R=0.3$)](image)
Moving on to the $R = 0.1$ data shown in Figure 4.5, we see that the data does not exhibit any further shift as the segment length is increased. The fact that the data did not shift further is an important result as it shows that changing the segment length does not alter data that was already considered satisfactory. Furthermore, the opening load at a 0% offset did not change by any appreciable amount.

Figure 4.5 Effect of Unloading Least Squares Segment Length When Using Compliance Offset Method ($R = 0.1$)

Modifying the segment length, although it appears to offer a considerable improvement, has some physical limits that must be addressed. The ASTM recommended 25% segment was chosen to capture the open-crack compliance. If this value is increased to 100%, then no longer is an open-crack compliance being obtained but rather some average of open and closed compliance. Studies were conducted to determine at what percent of the total load range the unloading curve appears to become nonlinear. For $R = 0.1$ roughly 20% of the unloading curve is nonlinear. At $R = 0.2$, 7%
of the unloading curve is nonlinear, and finally at \( R = 0.3 \) only 3% of the unloading curve is nonlinear. Therefore, using a 100% segment is invalid for all three load ratios, but at \( R = 0.3 \) this is a very small error. It must be concluded that as load ratio increases, larger segment lengths can and should be used. The segment length size should not be greater than the linear portion of the unloading curve, but should be large enough to achieve a more physically significant 0% offset. A potential modification to the ASTM compliance-offset methodology would be to determine the extent of the linear portion of the unloading curve, and to use a segment length slightly smaller than this calculated amount.

The second compliance-offset change investigated was with regard to the 10% increments on the loading curve. It was of interest to observe what effect a smaller increment would have. Again, the goal in changing the increment was the elimination of the load ratio induced shift from a 0% offset. Several different increment values were examined and each demonstrated the same trend, consequently only the results for a 2% increment are shown in Figure 4.6.
From Figure 4.6, one obvious result is the need for smoothed data when lowering the increment size. However, when compared to Figure 4.1 (which used a 10% increment), the smaller 2% increment has not shifted the data toward a 0% offset but merely increased the points along the shifted curve. It should be noted that much of the raw data in Figure 4.6 was eliminated when the horizontal scale of Figure 4.1 was imposed on Figure 4.6. Of course the reduced increment may eliminate errors induced by interpolation when determining opening loads at precise offsets, but it does not aid in eliminating the data shift from a 0% offset line. To be complete it should be pointed out that changing the increment size when using the $R = 0.1$ and 0.2 data gave similar results.
4.6 Prediction of Opening Loads

The discussion thus far has been focused solely on experimental testing to determine a crack opening load. In this section comparisons to strip-yield model predictions and three-dimensional finite element model predictions will be made. Exact correlation with experimental results may not occur since corrosion debris and roughness induced closure may be present in the experimental data. These effects are not accounted for in the models presented.

Contained in Table 4.1 and Figure 4.7 is a summary of the results obtained from the various models and different experimental methods. Focusing only on the models, it becomes immediately clear that a significant deviation is present between the two different strip-yield model results. As a frame of reference to help compare the models a mean experimental line is given in Figure 4.7. The agreement between the mean experimental values and 3-D FEA is reasonable at $R = 0.2$ and $0.3$ but poor at $R = 0.1$. Through the range of load ratios, the strip-yield models follow the experimental values fairly well. The FASTRAN values are lower than the mean experimental at $R = 0.3$ but correlate well with results at the lower load ratios. FLAP results have a similar discrepancy that appears at $R = 0.1$, but agreement with mean experimental values resumes at $R = 0.2$ and $0.3$. Of course a mean experimental value is certainly not a feasible or recommended method of determining an opening load, but does allow for an effective evaluation of the models investigated.
Figure 4.7 Opening Load Results

<table>
<thead>
<tr>
<th>Table 4.1 Normalized Opening Load Results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>FLAP ($\alpha=3, \alpha=2$)</td>
</tr>
<tr>
<td>FASTRAN ($\alpha=3, \alpha=2$)</td>
</tr>
<tr>
<td>3-D FEA</td>
</tr>
<tr>
<td>Compliance-Offset</td>
</tr>
<tr>
<td>2% offset (Smooth)</td>
</tr>
<tr>
<td>Reduced CMOD</td>
</tr>
<tr>
<td>VTP (Smooth)</td>
</tr>
<tr>
<td>Modified Comp.-Off.</td>
</tr>
<tr>
<td>100% Segment</td>
</tr>
<tr>
<td>0% offset (Smooth)</td>
</tr>
<tr>
<td>Mean Opening Load</td>
</tr>
</tbody>
</table>
Studying the modeling results more closely requires a detailed consideration of
the inputs used by the models. As was stated previously, a primary input for the strip-
yield models was the constraint factor $\alpha$, and this was determined to be equal to 3 since
plane strain conditions dominated for the specimen loading. Clearly it can be seen that
using $\alpha = 3$ did not give good results for all the load ratios. In fact, the mean
experimental results exceeded our presumed FASTRAN upper bound of $\cdot = 2$ at $R = 0.3,$
while FLAP predicts too high an opening load at $R = 0.1$. Of course there may be other
factors at work influencing these results. In terms of speed and ease of use the strip-yield
models were very attractive.

The inputs required by the 3-D FEA were considerably more complex. The
commercial code used to perform the analyses was ANSYS 5.5.3. The material was
assumed to be an elastic-perfectly plastic aluminum alloy with modulus $E = 10.45$ Mpsi
and flow stress $\sigma_0 = 80.5$ ksi. The von-Mises yield criterion and the associated flow rule
were used. Furthermore, small deformation theory was employed. The crack front was
advanced one element size during each cycle with $da = 128.6$ $\mu$m. A loading equivalent
to that applied in the experimental work was incorporated into the models. The opening
load values obtained from the models were through thickness averages weighted by the
element lengths along the crack front. Figure 4.8 illustrates the variation in opening load
through the thickness. This variation suggests that the crack front is dominated by a
plane strain condition.
The number of elements present in the reversed plastic zone was one parameter that influenced the results. Because the reversed plastic zone size decreases with increasing $R$, a highly refined finite element mesh was needed to sufficiently discretize the reversed plastic zone size at the higher load ratios, resulting in long execution times. Figure 4.9 shows the final mesh used, which exhibits two planes of symmetry and contained 15,880 elements and 19,848 nodes. In the model interior the refined region provided approximately 11 elements in the reversed plastic zone at $R = 0.3$ and up to 15 elements at $R = 0.1$. The level of refinement at $R = 0.3$ was considered sufficient to capture the reversed plastic zone. It can then be assumed that the relatively low opening load prediction seen at $R = 0.1$ is not a result of inadequate mesh refinement.
In addition to the number of elements in the reversed plastic zone, other modeling issues were apparent. As can be seen in Figure 4.7 the 3-D FEA results demonstrated a linear trend, which was not present in the experimental or strip-yield results. The FEA modeling methodology was reviewed and studied closely, but no reason for this behavior was determined. Other researchers have modeled plasticity-induced closure using FEA (Chermahini, 1988) (Fleck, 1988), but the stress levels used in these investigations were significantly greater than those applied in this work to reduce mesh refinement requirements. Consequently, these other crack opening load results could not be compared with the results presented here.
CHAPTER V

CRACK OPENING LOAD UNCERTAINTY

5.1 Introduction

Experimental errors or uncertainty have long been studied. Statisticians have developed numerous methods to quantify how well a variable has been measured. Recently, engineering based methodologies have been developed and successfully applied in various experimental settings (Coleman, 1999). These methods combine sound statistical theory with practical engineering knowledge to quantify the uncertainty in a measured variable and calculated result.

Uncertainty analysis is applied not only to measured variables in an experiment but can be the groundwork for designing, constructing, debugging and executing a successful experimental program. Moreover, the methods with some variations have been included in ANSI/ASME and ISO publications (ANSI, 1998) (ISO, 1993). The advantages of uncertainty analysis include uncertainty percent contributions, efficient incorporation into data acquisition codes, and applicability to all areas of experimental engineering. All areas of experimentation, including highly complex data reduction equations, are well within the capabilities of uncertainty analysis.
5.2 Principles of Uncertainty Analysis

The following overview is taken largely from the work of Coleman and Steele (Coleman, 1999). The total uncertainty in a variable is a combination of systematic and random uncertainties. If a large number of measurements of variable $x_i$ are available, the random uncertainty can be estimated as

$$P_{x_i} \cdot 2S$$

(5.1)

where $P_{x_i}$ and $S$ are the random uncertainty and sample standard deviation. The exception to equation 1 is if variable $x_i$ is a mean value. In this situation the random uncertainty $P_{x_i}$ should be divided by $\sqrt{N}$ where $N$ is the number of values used in the calculation of the mean.

Typically, it is more common to be interested in determining the uncertainty in a calculated value $r$ composed of variables $x_1, x_2, \ldots, x_J$ which results in a random uncertainty

$$P_r \cdot \sqrt{\sum_{i=1}^{J} \theta_i^2 P_i^2}$$

(5.2)

where $\theta_i$ and $P_i$ are the sensitivity coefficient and random uncertainty for a variable $x_i$ with $\theta_i = \partial r / \partial x_i$.

Systematic uncertainties are those of a fixed-value nature and are sometimes referred to as bias. The calculation of systematic uncertainty is based on the assumption that the true magnitude of the fixed error lies within a confidence interval. The true error is unknown, but the limits of the confidence limit are taken as the estimate of the systematic uncertainty. The systematic uncertainties $(B_i)_k$ for the elemental fixed error
sources in each variable are estimated and are combined by root-sum-square to determine the systematic uncertainty for variable \( x \) as

\[
B_{i} = \sqrt[2]{\sum_{k=1}^{M} (B_{i})_{k}^{2}}
\]  

(5.3)

When systematic elemental errors for two different variables have the same source, the systematic uncertainties are correlated. Correlated systematic uncertainties are an important part of uncertainty analyzes. The calculation of a correlated systematic uncertainty revolves around determining the covariance estimator of variables \( x_{i} \) and \( x_{k} \). The covariance estimator \( B_{ik} \) can be approximated as

\[
B_{ik} = \sum_{\alpha=1}^{I} \left( B_{i} \right)_{\alpha} \left( B_{k} \right)_{\alpha}
\]  

(5.4)

where \( (B_{i})_{\alpha} \) and \( (B_{k})_{\alpha} \) are the elemental systematic uncertainties that are common for variables \( i \) and \( k \). Using the above approximation allows for the determination of a total systematic uncertainty of the result as

\[
B_{r} = \sqrt{\sum_{i=1}^{J} \theta_{i}^{2} B_{i}^{2} + 2 \sum_{i=1}^{J} \sum_{k=i+1}^{J} \theta_{i} \theta_{k} B_{ik}}
\]  

(5.5)

The proper development of random and systematic uncertainties is important. Using the best possible information and relying on previous experimental work to construct estimates is helpful. Once both random and systematic uncertainties are known, then a total uncertainty can be found as

\[
U_{r} = \sqrt{B_{r}^{2} + P_{r}^{2}}
\]  

(5.6)

A significant aspect of uncertainty analysis is determining which measured variables contribute to the total uncertainty and to what extent. This is important for potentially improving current research and developing more successful future testing.
The quantity used to assess the uncertainty for each variable is the uncertainty percent contribution defined as

\[ UPC_i \cdot \frac{(\bullet \bullet)^2 (B_i \text{ or } P_i)^2}{U_r^2} \]  

(5.7)

where the \( UPC_i \) is determined for all of the \( B_i \) and \( P_i \) components. Further details regarding uncertainty analysis may be found in (Coleman, 1999).

5.3 Uncertainty Analysis of Crack Opening Loads

5.3.1 Overview

The experimental determination of a crack opening load is a relatively simple process. Chapter IV discussed various data reduction techniques. To demonstrate the effectiveness of uncertainty methodologies the compliance-offset technique will be analyzed. For mathematical simplicity an ASTM round robin data (RRD) set was first used. This was done for two reasons. First, the RRD was composed of about half the data points as obtained in the current work allowing for much easier coding of the uncertainty calculations. Second, the RRD had a small maximum to minimum load range, which simplified both the coding as well as elemental uncertainty estimates. The major disadvantage to using RRD is that no information on the elemental uncertainties can be found since the data was collected at a different lab years ago. Since the purpose is to demonstrate the usefulness of uncertainty methodologies, elemental uncertainties from the authors test facilities were used on the RRD. Once the uncertainty analysis of the RRD was completed the coding was modified to reduce the data collected as part of this research.
5.3.2 Methodology

When employing the compliance-offset technique, a series of mean loads for a sequence of load ranges must be computed. The uncertainty analysis of a mean load appears trivial, but in reality is not. In the calculation of a mean load each data point is in fact a variable with its own uncertainty estimates. This requires finding both random and systematic uncertainty estimates for all the loads. In addition, since the same load cell measures each load the systematic uncertainties are all correlated. The use of powerful mathematical software enables these calculations to be made fairly fast once coded into the software.

![Figure 5.1 Compliance-Offset Reduced Data](image)

Figure 5.1 shows a standard plot of compliance-offset reduced data. Each point is composed of an offset and mean load. The offsets are calculated using
\[
\text{Offset} \cdot \frac{C_O \cdot C_n}{C_O} \cdot 100\%
\]

(5.8)

Where the \(C_n\)'s and \(C_O\) are simple regression slopes. ASTM dictates the data sets comprising the regression slopes for \(C_n\) values and the mean loads. These data sets are 10% of the loading data with a 5% overlap repeated for the entire load range. The data sets begin at 90-95% of the maximum load. It should be noted that the load-displacement data is composed of both loading and unloading data. The slope \(C_O\) or open crack compliance is calculated from >25% of the unloading data. This is not as ASTM recommends but rather from the results of the research reported in Chapter IV.

Each time load-displacement data is reduced by the compliance-offset technique, 19 mean load offset pairs are generated. This results in performing a complete uncertainty analysis 19 times. For a given load range there may be anywhere from 400 to 1000 loads in the data. Estimating the random uncertainty in each load would require a separate constant load test with a subsequent standard deviation calculation. To eliminate this overwhelming task it was assumed that the random error would not change significantly in a \(\pm\) 100 lb. range, therefore, several loads within the load range and their standard deviations were used to represent the random uncertainty for all loads.

The elemental systematic uncertainty estimates were obtained from SATEC/Instron publications specific to the TC-25 test machine. All of these estimates were on a percent of reading basis, which allowed for easy use. The elemental systematic uncertainties are hysteresis, linearity, and threshold with values of 2.5%, 1.5%, and 2.0% of the load reading, respectively. It was decided to use the mean loads as the load reading in these elemental estimates. Preliminary calculations demonstrated only a small difference in total systematic uncertainty by making this simplification.
5.3.3 Application to Round Robin and Current Data

The results of the uncertainty analysis performed on the RRD are presented in Figure 5.2. The uncertainties for each mean load in Figure 5.2 range from 5.5 to 6.0%. This implies that given the elemental uncertainty estimates used in the analysis, a high crack opening load would have essentially the same error regardless of offset. Of course, as was pointed out earlier, each load used in the calculation of a mean is in fact a variable with its own uncertainty. Determining how much each uncertainty component contributes to the total uncertainty is important in identifying the most significant sources of error. For example take the point with a 1.86% offset; this mean load has 11 individual loads used in its calculation. The percent uncertainty contributions from the random, systematic, and correlated systematic uncertainties are 2.5, 4.5, and 93.0% respectively. Clearly the correlated systematic uncertainty is dominating the total uncertainty. This was expected, since the uncertainty analysis was performed on a mean, where all the sensitivity coefficients are positive typically gives similar results. Positive sensitivity coefficients cause the correlated systematic uncertainty to inflate. An example of the opposite trend is when dealing with a difference, where the sensitivity coefficients alternate signs, and reduce the correlated systematic uncertainty. Similar uncertainty results for the other points were obtained.
Once the uncertainty analysis was completed on the RRD, the code was modified to deal with the larger current data sets. Figure 5.3 illustrates the uncertainty results obtained from the current compliance-offset tests. The most immediate difference between the current and round robin data is the load range. Because of this the mean loads calculated in the current work are composed of 40 values. The effect of this increase can be seen in the uncertainty percent contributions which are now random =0.4%, systematic = 1.3%, and correlated systematic uncertainty = 98.3%. These values are obtained from the point with offset = 0.034 and mean load = 339.2 lb. Clearly, with the increase in data points, the correlated systematic uncertainty dominates to a greater degree. Unlike the RRD, which showed a slight variation in total uncertainty, the current work is almost perfectly constant at 5.0%. In fact, the standard deviation of total uncertainty for all 19 mean loads is 0.006%. Table 5.1 contains all the results.
Figure 5.3 Uncertainty in Crack Opening Load via Compliance-Offset Technique on Current Data

Table 5.1 Uncertainty Analysis Results for Current Work

<table>
<thead>
<tr>
<th>Mean Load – lbs.</th>
<th>Offset - %</th>
<th>Total Uncertainty – lbs.</th>
<th>Total Uncertainty - %</th>
<th>Random Uncertainty(^\circ) – lbs.</th>
<th>Systematic Uncertainty(^\circ) – lbs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1100.4774</td>
<td>-1.2419</td>
<td>55.3847</td>
<td>5.0329</td>
<td>6.3950</td>
<td>39.9080</td>
</tr>
<tr>
<td>1059.5545</td>
<td>-0.9317</td>
<td>53.3259</td>
<td>5.0324</td>
<td>6.3950</td>
<td>37.4610</td>
</tr>
<tr>
<td>1009.4024</td>
<td>-0.6614</td>
<td>50.7976</td>
<td>5.0325</td>
<td>4.5240</td>
<td>35.6880</td>
</tr>
<tr>
<td>952.0279</td>
<td>-0.7016</td>
<td>47.9108</td>
<td>5.0323</td>
<td>4.5240</td>
<td>33.6590</td>
</tr>
<tr>
<td>888.4346</td>
<td>-1.0626</td>
<td>44.7084</td>
<td>5.0322</td>
<td>3.2490</td>
<td>31.4110</td>
</tr>
<tr>
<td>820.0835</td>
<td>-0.3840</td>
<td>41.2679</td>
<td>5.0322</td>
<td>2.4900</td>
<td>28.9940</td>
</tr>
<tr>
<td>748.2653</td>
<td>-0.0980</td>
<td>37.6543</td>
<td>5.0322</td>
<td>2.4900</td>
<td>26.4550</td>
</tr>
<tr>
<td>672.2381</td>
<td>-0.2711</td>
<td>33.8281</td>
<td>5.0327</td>
<td>2.0140</td>
<td>23.7670</td>
</tr>
<tr>
<td>593.5028</td>
<td>-0.8553</td>
<td>29.8690</td>
<td>5.0329</td>
<td>3.1770</td>
<td>20.9830</td>
</tr>
<tr>
<td>510.9880</td>
<td>-0.1475</td>
<td>25.7176</td>
<td>5.0366</td>
<td>3.1770</td>
<td>18.0660</td>
</tr>
<tr>
<td>424.4610</td>
<td>-0.3716</td>
<td>21.3784</td>
<td>5.0422</td>
<td>5.7540</td>
<td>15.0070</td>
</tr>
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<td>339.1999</td>
<td>0.0335</td>
<td>17.1031</td>
<td>5.0355</td>
<td>6.8170</td>
<td>11.9930</td>
</tr>
<tr>
<td>257.2112</td>
<td>1.5213</td>
<td>12.9518</td>
<td>5.0397</td>
<td>3.0340</td>
<td>9.0940</td>
</tr>
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<td>187.8040</td>
<td>5.3702</td>
<td>9.4648</td>
<td>5.0441</td>
<td>3.2830</td>
<td>6.6400</td>
</tr>
<tr>
<td>150.3547</td>
<td>12.1846</td>
<td>7.5840</td>
<td>5.0461</td>
<td>3.2830</td>
<td>5.3160</td>
</tr>
<tr>
<td>139.0948</td>
<td>16.3895</td>
<td>7.0189</td>
<td>5.0472</td>
<td>3.2830</td>
<td>4.9180</td>
</tr>
<tr>
<td>133.7981</td>
<td>14.2330</td>
<td>6.7531</td>
<td>5.0481</td>
<td>3.2830</td>
<td>4.7300</td>
</tr>
<tr>
<td>130.1161</td>
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<td>6.5684</td>
<td>5.0488</td>
<td>3.2830</td>
<td>4.6000</td>
</tr>
<tr>
<td>127.5638</td>
<td>29.8949</td>
<td>6.4404</td>
<td>5.0329</td>
<td>3.2830</td>
<td>4.5100</td>
</tr>
</tbody>
</table>

\(^\circ\) These values are for individual loads being inputted to mean calculations
From the results presented in Table 5.1, there are some observations to be made. The first and most important observation pertains to the variability in random uncertainty. Clearly some values are higher than others, which only became apparent in the final analysis stages. After investigating the data sets it became apparent that this variability is a direct result of test frequency. The tests with high random uncertainties had high frequencies. In hindsight the uncertainty reported could be improved by using low frequency tests only. Of course, looking at the total uncertainty in percent, this difference is negligible when compared with the correlated systematic uncertainty. The second observation also pertains to the random uncertainty values. It should be obvious that the values do not change below a mean load of 187 lbs., this is due to low load limitations induced from specimen configuration and load cell difficulties. However the point is reinforced that this does not appear to have an effect in light of the correlated systematic uncertainty. Even with the many approximations made in these analyses the methodologies show that they are useful, informative, and a necessity in modern testing.
CHAPTER VI

FATIGUE CRACK PROPAGATION

6.1 Introduction

A major thrust of the research was to obtain and analyze fatigue crack growth rate data for AA 7075-T651. The majority of the data was to be acquired without the affects of fatigue crack closure present. This required running the tests at high load ratios. To gather as much data as possible, two types of tests, one at constant load and the other at constant $K_{max}$ were performed. The constant load tests utilized a fixed maximum and minimum load and generated increasing $da/dN$ data from a point in the Paris regime all the way to fracture. The constant $K_{max}$ tests involved a load shedding and produced decreasing $da/dN$ data from within the Paris regime down to the mid threshold regime.

Numerous post-processing objectives were designed for this closure-free data. The first was to perform a series of regression routines to find the best linear fit for the data. One method of determining the best fit was using an analytical strip-yield model that requires the regression coefficients as model inputs and computes the crack growth (Daniewicz, 1994). The experimental and predicted crack growth data are then compared and a quality assessment made. The second objective for the closure-free data is to experimentally obtain low $R$ data and, using the crack opening load from these tests, collapse the low $R$ data onto the closure-free data. The collapse of this data should result
in a single da/dN vs ∆Keff curve. The final post-processing objective for all the data was to compare the data with Forman and Walker eqs. using accepted coefficients for AA 7075-T6 and previous experimental data.

6.2 Closure-Free Crack Growth Data

6.2.1 Raw Data Acquisition

The closure-free data consisted of 5 constant load tests and 3 constant K_{max} tests. The constant load tests were at a P_{max} = 2000 lbs. and R = 0.7 while the constant K_{max} tests were at a K_{max} = 12 ksi√in and an initial R = 0.7. Figure 6.1 displays the results. The transition from the Paris regime to Region III growth can be seen in the figure. Unfortunately, the transition into the threshold region is not apparent. Due to load instability ending the threshold tests, this data does not extend into the threshold region.

Figure 6.1 da/dN vs. ∆K for AA 7075-T651
The point has been made that this data is not affected by fatigue crack closure because the minimum load is greater than the crack opening load. This is known because it was proven experimentally using the compliance-offset technique, the results are Figure 6.2. The $R = 0.7$ data does not appear to show any change in compliance over the narrow load range. It is concluded that $R = 0.7$ must be closure-free as stated.

![Figure 6.2 Compliance-Offset Results](image)

Casting the closure-free data in $da/dN$ vs. $\Delta K$ fashion and then plotting on log-log coordinates may mask some of the important components present in fatigue crack growth. Since variability is one important component, it is convenient to plot $a$ vs. $N$, which is done in Figure 6.3, and observe the amount of variability present in the data. The major disadvantage to plotting $a$ vs. $N$ is the necessary elimination of threshold testing data from the plots. These tests accumulate large cycle counts and very small changes in crack length, therefore distorting the entire plot. Threshold data is rarely, if
ever plotted in an $a$ vs. $N$ fashion and is not done so here. From Figure 6.3 a clear perspective for specimen variability is presented. A detailed look at single and multiple specimen error will be presented in a subsequent chapter.

![Graph showing crack growth as $a$ vs. $N$ in AA 7075-T651](image)

Figure 6.3 Crack Growth as $a$ vs. $N$ in AA 7075-T651

6.2.2 Data Regression

The most significant use for the closure free data is to perform a regression routine that will best fit the data, such that this data may then be used in a predictive model. However, only models designed to predict crack growth utilizing $\Delta K_{\text{eff}}$ may be considered. The use of the effective loading eliminates most stress ratio effects and should precipitate better agreement with real world crack growth. Furthermore, employing $\Delta K_{\text{eff}}$ forces experimental acquisition of closure-free data to be the only material characterization required.
The methods used to find an appropriate regression equation vary widely. The two methods chosen were a linear and a bilinear fit (in log-log coordinates) of all Region II data. Region II data from all eight tests were regressed to find the single linear fit. The bilinear fit was composed of the same data but split between the two test types forcing an upper and lower linear fit. Newman et al. have shown that the use of multiple linear fits can more accurately represent fatigue crack growth data (Newman, 1994).

Figure 6.4 Regression Results

The results of the two regression routines are shown in Figure 6.4. The three regression lines are significantly different but yet capture the appropriate data. It should be noted that even though all three lines extend through the data range they are not all valid in this range. The upper and lower bilinear fits are valid only for the smaller range that they were fit to. The calculated regression coefficients for each line are given in Table 6.1.
Table 6.1 Regression Coefficients for Closure-Free Crack Growth

<table>
<thead>
<tr>
<th></th>
<th>Single Linear Fit</th>
<th>Upper Bilinear</th>
<th>Lower Bilinear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope – $m$</td>
<td>3.819</td>
<td>4.304</td>
<td>3.364</td>
</tr>
<tr>
<td>Intercept – $C$ – in/cycle / ksi*n</td>
<td>1.306E-8</td>
<td>5.526E-9</td>
<td>2.429E-8</td>
</tr>
</tbody>
</table>

Measuring the quality of these regression fits is important before using the coefficients for design purposes. A reliable but subjective way of determining the quality is to use the coefficients in a predictive model. The model can generate $a$ vs. $N$ data which should compare well with the experimental data in Figure 6.3 if the coefficients are worthwhile. These comparisons were made and the results are presented in a later section. However, a form of quality analysis that belongs within this section is the examination of residuals. A residual is nothing more than the difference between the predicted and actual $y$ values and is plotted against the corresponding $x$ values. Considering the regression performed here the $y$ values are $\log(da/dN)$ and the $x$ values $\log(\Delta K)$. The residuals are plotted in Figure 6.5.
The residuals plot reveals several interesting and important characteristics. First it appears that the linear approximation is good in the constant $K_{\text{max}}$ regime and fitted well by the lower bilinear fit. The behavior in the constant load regime is much worse. The linear approximation does not appear accurate for either fit. It also appears that possibly all of the Region III data was not completely eliminated. Determining this transition is difficult and subjective. Possibly another linear fit belongs within this portion of the data to make a trilinear fit. Regardless of these concerns, the bilinear fit is judged superior by the residual analysis and is an effective way of dealing with fatigue crack growth rate data.
6.2.3 $\Delta K_{eff}$ Collapse for $R = 0.1$ and 0.3 Data

A prevailing concept within the fatigue community is that when fatigue crack growth rate data, regardless of load ratio, is plotted as $da/dN$ vs. $\Delta K_{eff}$ a single curve emerges for a given material and environment. This has been shown to be especially accurate in the Paris regime but perhaps untrue as threshold is approached (Bray, 1999). The procedure for converting an entire set of $da/dN$ vs. $\Delta K$ data into $da/dN$ vs. $\Delta K_{eff}$ data involves

\[ \Delta K_{eff} \cdot U \Delta K = \frac{1 \cdot \frac{P_o}{P_{max}}}{1 \cdot \frac{R}{R}} \Delta K \]  

(6.1)

Where $P_o$ is the crack opening load. Equation 6.1 employs the assumption that the normalized crack opening load is constant for all values of crack length. In some materials/geometries the approximation would be completely invalid. However, from previous experimental results the assumption is valid within the current field of tests. Of course equation 6.1 can be used on a point-to-point basis if the crack opening load was known for each value of $\Delta K$.

To examine some related issues specific to collapsing data onto a single effective curve, several tests were performed. An $R = 0.1$ test was run with an initial $K_{max}$ that was 60% of the $R = 0.7$ $K_{max}$. In addition, an $R = 0.3$ test was performed with an identical $K_{max}$ as was used in the $R = 0.7$ testing. The premise is to observe if $K_{max}$ affects the ability to collapse the data. The crack opening loads used to collapse the data were experimentally measured using the modified compliance-offset technique discussed in Chapter IV. The results of the tests and the collapse of the data are Figure 6.6.
Figure 6.6 Conversion of Low $R$ Data into $\Delta K_{eff}$ Data

The first observation relevant to Figure 6.6 is that much of the Region III data has been removed. As for the collapse of the data, it appears that the $\Delta K_{eff}$ concept is sound and that stress ratio effects can be accounted for by the use of $\Delta K_{eff}$. However, this statement is made with some reservations. Only after numerous tweaks to the load-displacement data and compliance-offset technique was a suitable crack opening load obtained. Having observed successful application of the $\Delta K_{eff}$ theory in other research, intense scrutiny of the measured crack opening loads was initiated and with additional effort the appropriate values were obtained. The possibility that the crack opening loads used were manipulated to a point of questionable accuracy is noted but given the unreliable nature of remote crack closure techniques the author can only assume the values are correct. One point of interest to note is that there appears to be no $K_{max}$ effect on the results.
6.3 Fatigue Crack Growth Predictions

6.3.1 Analytical Strip Yield Model

The ideas and concepts developed throughout the preceding chapters and sections are only important if they can be successfully implemented in the solution of a real engineering problem. This is best made possible by developing an analytical model that can use the regression coefficients and predict crack growth under any loading. For the strip-yield model used here the prevailing idea used is that $\Delta K_{\text{eff}}$ is the fatigue crack driving force (Daniewicz, 1994). The strip-yield model requires $P_{\text{max}}$, $R$, the flow stress $\sigma_o$, a constraint factor $\alpha$, specimen geometry, appropriate weight function, and closure-free regression coefficients. The model computes a $da$ value then determines a $K_{\text{max}}$ and $K_o$. Using these stress intensity factors a $\Delta K_{\text{eff}}$ can be found and with regression coefficients a resulting $dN$. The process is repeated until the final crack length of interest is reached. For the compact tension specimen, the crack length is limited to $a/w \leq 0.5$ due to weight function limitations. The only subjective input is the constraint factor $\alpha$, which approximates 3-D effects. The values range from a plane stress $\alpha = 1$ to a plane strain $\alpha = 3$. It should be obvious that experimental tests and service components are somewhere in between these values but where in between is unknown. For this reason, without better information, it is convenient to place upper and lower constraint bounds on predictions.

The strip-yield model was used to predict crack growth at $R = 0.1$, 0.3, and 0.7. The analyzes were performed with plane stress and plane strain bounds. The best set of regression coefficients were determined both from the residual analysis results previously
discussed and from the $R = 0.7$ prediction results. These $R = 0.7$ prediction results are presented in Figure 6.7. The plot shows the clear superiority in using a bilinear fit versus a single linear fit. When using the bilinear fit a good agreement with experimental results is achieved. It should be noted that there is no significant difference between the two bounding results because there is no crack closure present in the predictions.

![Figure 6.7 Predicted and Experimental Crack Growth at $R = 0.7$](image)

Predictions with crack closure that is most likely present in service loading are important. Since the $R = 0.1$ and 0.3 tests used in the preceding section contain crack closure it is relevant to compare analytical predictions with these results and this is done in Figure 6.8. These are somewhat mixed results. The $R = 0.1$ data falls within the constraint bounds but the $R = 0.3$ data does not. However, the plane stress $R = 0.3$ prediction does follow the data until about a 1.0 inch crack length. Due to a large initial $\Delta K$ the $R = 0.3$ test is considered to be an extreme loading test. From crack initiation to
failure took only 6 hours while an $R = 0.1$ test required approximately 34 hours. This does not justify the poor prediction but may be a contributing factor.

![Graph showing predicted and experimental crack growth at $R = 0.1$ and $0.3$.](image)

Figure 6.8 Predicted and Experimental Crack Growth at $R = 0.1$ and $0.3$

### 6.3.2 Alternate Crack Growth Equations and Experimental Data

As a quality measure and verification process it was desired to compare the current results with alternate crack growth expressions that use 7075-T6 data and experimental data generated by Hudson (Hudson, 1969). Exact correlation with the alternate crack growth equations is not expected but some measure of quality and similar trends are expected. The two expressions chosen are the Walker equation and Forman equation. These are explained and expressed in section 1.1. The coefficients used in these equations were taken from (Dowling, 1999). The results of the Walker equation
are presented in Figure 6.9 and the Forman equation in Figure 6.10 with the experimental data of Hudson in both figures.

Figure 6.9 Walker Equation and Experimental Crack Growth Data
Figure 6.10 Forman Equation and Experimental Crack Growth Data

From the two figures it appears that the Walker equation correlates better than the Forman. This is true especially for the $R = 0.1$ and $0.3$ curves but both appear to similarly agree at $R = 0.7$. Regardless, the Forman equation still agrees better than was expected. Interestingly the Forman equation appears to contain the nonlinear failure behavior at $R = 0.7$. Had the $R = 0.1$ and $0.3$ been plotted to an extended $\Delta K$ and higher $\frac{da}{dN}$ the same behavior would occur. It can be assumed that in terms of the Walker equation the T651 treatment does not significantly affect crack growth when compared with the T6 treatment. This is a somewhat more difficult assumption to make based on the Forman equation results.

The stunning result lies in the excellent agreement with the $R = 0.7$ experimental data of Hudson. This data was generated using 7075-T6 but clearly compares well with the T651. Comparing the two experimental data sets makes the assumption concerning
the differing heat treatments seem an obvious conclusion. Overall the agreement is above expectations and adds credibility to the current experimental crack growth data.
CHAPTER VII

FATIGUE CRACK PROPAGATION UNCERTAINTY

7.1 Introduction

Since fatigue crack growth rate data is the foundation for successful service life prediction, a quantification of errors and variability is needed. With this in mind uncertainty methodologies are applied to Chapter 6 data to evaluate single specimen error while alternate statistical methods are applied to the test groups to evaluate multiple specimen error. The premise behind single specimen error is that the measuring techniques are the only contributing source of uncertainty. In contrast, multiple specimen error can be affected by geometry, microstructure, environment, and frequency changes in combination with the single specimen uncertainties already present. The majority of work in this area has focused on multiple specimen error. The uncertainties in crack length $a$, growth rate $da/dN$, and stress intensity factor range $\Delta K$ have been computed and compared with multiple specimen error measures to evaluate the current data and enlighten the research community to the benefits of uncertainty analyzes.
7.2 Methodologies

7.2.1 Single Specimen Uncertainty

The uncertainty analysis for a single fatigue crack growth specimen is a careful application of uncertainty principles using several data reduction equations. Since \( \frac{da}{dN} \) and \( \Delta K \) use load, displacement, and geometry measurements indirectly, the uncertainty analysis is more difficult. Both \( \frac{da}{dN} \) and \( \Delta K \) are functions of the crack length \( a \), which is a function of the compliance, which finally is a function of the load, specimen thickness, and crack mouth opening displacement CMOD. Extensive use of the chain rule was required to determine the appropriate sensitivity coefficients. From ASTM Test Method for Measurement of Fatigue Crack Growth Rates E647 (ASTM, 1999a) the compliance factor \( u \) is defined as

\[
    u \cdot \left[ \left( \frac{E v B}{P} \right)^{\frac{1}{2}} + 1 \right]^{-1}
\]  

where \( E \), \( v \), \( B \), and \( P \) are the elastic modulus, CMOD, specimen thickness, and applied load respectively. Having the compliance factor allows the crack length to be calculated as

\[
    a \cdot w \left( C_o \cdot C_1 u \cdot C_2 u^2 \cdot C_3 u^3 \cdot C_4 u^4 \cdot C_5 u^5 \right)
\]  

where \( w \) and \( C_i \) are the specimen width and compliance coefficients, respectively. The compliance coefficients were taken from (Saxena, 1978a). At this point the calculations for \( \Delta K \) and \( \frac{da}{dN} \) deviate and must be addressed separately.

The calculation of \( \Delta K \) begins with the determination of the \( C(T) \) geometry factor

\( F \) from E647 expressed as
\[
F \cdot \frac{2 \cdot \frac{a}{w}}{\left(1 \cdot \frac{a}{w}\right)^{\frac{1}{2}}} \left[ 0.886 \cdot \frac{4.64 a}{w} \cdot 3.32 \left(\frac{a}{w}\right)^2 + 14.72 \left(\frac{a}{w}\right)^3 - 5.6 \left(\frac{a}{w}\right)^4 \right] \tag{7.3}
\]

The stress intensity factor range \(\Delta K\) is then calculated as

\[
\Delta K = \frac{P_{\text{max}} \cdot (1 \cdot R)}{B \sqrt{w}} F \tag{7.4}
\]

The load ratio \(R\) is assumed to be without uncertainty, and for purposes of these analyses, \(P = P_{\text{max}}\). The calculation of \(da/dN\) is less complicated since the calculation involves only the crack length \(a\) and cycle count \(N\). However, there are numerous methods used to reduce the \(a\) vs. \(N\) data, some numerically complex and others elementary in nature. The software used to gather the experimental data employed a 7-point incremental polynomial method. However, for mathematical simplicity, a modified secant was used to find the uncertainty in \(da/dN\) with

\[
\frac{da}{dN_i} = \frac{a_{i-1} - a_{i-7}}{N_i - N_{i-7}} \tag{7.5}
\]

where \(i\) must be greater than 7. With a suitable method of determining \(da/dN\) having been established, a conventional uncertainty analysis was next performed using the appropriate sensitivity coefficients and component uncertainties. The cycle count \(N\) was assumed to be without uncertainty.

### 7.2.2 Multiple Specimen Uncertainty

The analysis of multiple specimens is much less complex than the preceding concepts and follows very conventional ideas. An ASTM round robin on fatigue crack
propagation organized by Clark and Hudak first developed concepts similar to those
employed here (Clark, 1975). A detailed discussion was presented in section 2.3.3. A
significant modification to the Clark and Hudak variability factor must be made in light
of the data generated in this research. First, instead of using a residual standard deviation
$S_R$, a sample standard deviation was employed. A residual standard deviation will in
theory under-estimate the random part of the data when not using the residual degrees of
freedom, which may significantly differ from the degrees of freedom used to calculate a
sample standard deviation. It is believed that Clark and Hudak used the residual degrees
of freedom and therefore the methods employed herein should be a justifiable
comparison. Secondly, Clark and Hudak developed their variability factor with large
quantities of data and therefore are justified in using a $2S_R$ criterion. In the current
research a maximum of only five tests was performed and therefore a $2S$ criterion to
estimate a 95% confidence interval is invalid. For the current work an appropriate
student $t$ value was used depending on the degrees of freedom present in each sample
standard deviation $S$. It has been proven that the use of 2 is valid for greater than 9
degrees of freedom, for values less than 9 an appropriate student $t$ must be used
(Coleman, 1999).

Once the appropriate student $t$ is used, variability factors can be calculated for
three cases. The first is for $da/dN$ values at a fixed value of $\Delta K$. Of course every test will
not contain the exact value of $\Delta K$ but using the closest possible $\Delta K$ is presumed
sufficient. The second variability factor is for values of $\Delta K$ at a fixed value of $da/dN$.
The same problem with exact values exists and is addressed in a similar manner. The
final variability factor is for the crack lengths at fixed cycle counts. Using these three
variability factors should appropriately measure the multiple specimen error. Once these variability factors are known they can be compared with equivalent uncertainty values and the results from Clark and Hudak.

7.3 Results

7.3.1 Single Specimen Uncertainty

To correctly perform an uncertainty analysis requires accurate estimates of elemental uncertainties. The load and CMOD random uncertainty estimates were made from previous test data generated using constant load conditions and equation 1. The tests were only run for a short period so no crack extension was achieved. The load systematic uncertainty estimates were a combination of reported values for the test machine (linearity = 1.5% of reading, hysteresis = 2.0% of reading, and threshold = 2.5% of reading) and a previous data estimate for calibration at ± 89 N (20 lb). The CMOD systematic uncertainty estimates included a reported value of 0.25% of reading for linearity and a calibration estimate from previous data at 0.00254 mm (0.0001 in). The specimen thickness was determined using a micrometer. The random uncertainty was calculated using equation 1, and the systematic uncertainty taken from the manufacturers information at 0.0076 mm (0.0003 in) for calibration and 0.018 mm (0.0007 in) for linearity. The width measurements were made using a laboratory ruler. The random uncertainty was found via equation 1 and the systematic uncertainty estimated at 0.254 mm (0.01 in).

Application of uncertainty principles to the fatigue crack propagation data revealed several interesting and relevant conclusions. The uncertainty results for $da/dN$
and $\Delta K$ are illustrated in Figure 7.1. The first point to be made concerning Figure 7.1 is that the log-log scale partially distorts the uncertainty bands, which are symmetrical. The uncertainty values $U_{x}/x$ for $da/dN$ range from 3.5% to 16%, and crack growth rate was determined using a modified secant. The systematic uncertainties for the loads and CMODs are correlated and this influences the nature of the $da/dN$ uncertainty. Also, the specimen thickness and width were the same for all crack length measurements. Because of the correlated systematic uncertainties and the fact that the modified secant calculation involves a difference, the total systematic uncertainty in $da/dN$ ranges from 1% to 20% of the total uncertainty. This forces the total uncertainty in $da/dN$ to be predominately random. The $\Delta K$ uncertainties range from 2.5% to 4.0%, with a mean of 3.1%. The composition of the $\Delta K$ uncertainty at $a/w$ of 0.6 was roughly 90% systematic and 10% random with no correlated systematic uncertainties present. The uncertainty percent contribution results are listed in Table 7.1.
Table 7.1 ΔK Uncertainty Contributions

<table>
<thead>
<tr>
<th>a/w</th>
<th>B %</th>
<th>P %</th>
<th>B %</th>
<th>P %</th>
<th>B %</th>
<th>P %</th>
<th>B %</th>
<th>P %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>0.15</td>
<td>1.76</td>
<td>46.9</td>
<td>0.12</td>
<td>0.10</td>
<td>0.45</td>
<td>29.3</td>
<td>21.2</td>
</tr>
<tr>
<td>0.35</td>
<td>0.30</td>
<td>3.50</td>
<td>93.4</td>
<td>0.22</td>
<td>0.31</td>
<td>1.37</td>
<td>0.92</td>
<td>0.22</td>
</tr>
</tbody>
</table>

With regard to the notation used in Table 7.1, B is the systematic uncertainty for variable x and P is the random uncertainty for variable x. The distribution of uncertainties changed significantly with crack length, with the total uncertainty changing by 70%. The systematic uncertainty in load is a large value. As previously discussed, both the test machine documentation and previous experimental work led to the estimates of these uncertainties, which may be too high in light of its dominating influence. However, the reason for the change in BP is not due to load but rather CMOD. Since this contribution affects all three crack growth variables it will be investigated in a later section.

The results in Figure 7.1 clearly show that in our laboratory fatigue crack growth rates can be measured to within a 3.5% uncertainty but that the uncertainty may become as high as 16%, while ΔK can be measured consistently to within 3%. Although it may exist, after a thorough literature review reported data of this kind was not found. Such information should prove useful when seeking to improve conventional testing methods.

Fatigue crack growth can also be expressed as a vs. N and this is important since a better characterization of variability is gained. Figure 7.2 illustrates the a vs. N data for the five constant load tests along with the uncertainties in crack length.
The uncertainty for the crack length varied from 1.2 to 6.5%. The way in which uncertainty appears to be a function of crack length again revolves around CMOD. A powerful capability of uncertainty methodologies is the analysis of error contributions and acquisition of sensitivity coefficients. The combination of these two tools allows the total uncertainty to be partitioned and the dominating factors revealed. As was already stated, CMOD was the dominating factor controlling all three uncertainty calculations as shown in Figure 7.3.
A CMOD increase causes the total uncertainty in $a$ and $da/dN$ to decrease while causing the total uncertainty in $\Delta K$ to increase. The data are clustered at the lower displacements because constant $K_{max}$ testing essentially keeps CMOD low and constant. To further investigate the CMOD effect, changes in uncertainty components and sensitivity coefficients were monitored as CMOD increased. In the case of crack length, the systematic uncertainty increased only marginally, while the sensitivity coefficient $\partial u/\partial v$ increased by 500%. This sensitivity coefficient is, because of extensive chain rule use, present in all three uncertainty calculations. Since $\partial u/\partial v$ is involved with functions containing other sensitivity coefficients, the influence of $\partial u/\partial v$ changes. Only through the use of uncertainty analyses can these types of observations be made, and the quantification of crack propagation uncertainty accomplished.
7.3.2 Multiple Specimen Uncertainty

The determination of variability factors for \( a, \Delta K \), and \( da/dN \) will provide insight into the behavior of these variables. Table 7.2 lists the \( da/dN \) variability factors for fixed values of \( \Delta K \), where the first four entries are for the constant load tests and the remaining three are for the constant \( K_{max} \) tests.

<table>
<thead>
<tr>
<th>( \Delta K ) MPa√m</th>
<th>( \Delta K ) Range MPa√m</th>
<th>Mean ( da/dN ) ( 10^{-6} ) mm/cycle</th>
<th>( S_{da/dN} ) ( 10^{-6} ) mm/cycle</th>
<th>( VF )</th>
<th>( V ) %</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.40</td>
<td>4.40 – 4.41</td>
<td>60.15</td>
<td>15.42</td>
<td>5.926</td>
<td>71.12</td>
</tr>
<tr>
<td>6.59</td>
<td>6.60 – 6.62</td>
<td>257.05</td>
<td>45.85</td>
<td>2.961</td>
<td>49.51</td>
</tr>
<tr>
<td>8.79</td>
<td>8.77 – 8.81</td>
<td>1387.86</td>
<td>313.94</td>
<td>4.373</td>
<td>62.78</td>
</tr>
<tr>
<td>10.99</td>
<td>11.1 – 11.21</td>
<td>5064.76</td>
<td>1291.59</td>
<td>5.848</td>
<td>70.79</td>
</tr>
<tr>
<td>3.30</td>
<td>3.30 – 3.31</td>
<td>26.01</td>
<td>3.69</td>
<td>4.132</td>
<td>61.03</td>
</tr>
<tr>
<td>2.47</td>
<td>2.44 – 2.47</td>
<td>8.56</td>
<td>1.01</td>
<td>3.076</td>
<td>50.93</td>
</tr>
<tr>
<td>1.92</td>
<td>1.89 – 1.93</td>
<td>3.82</td>
<td>0.47</td>
<td>3.252</td>
<td>52.96</td>
</tr>
</tbody>
</table>

The \( V \) variable in Table 7.2 is determined much like a percent random uncertainty and is calculated as

\[
V = \frac{tS_{da/dN}}{da/dN} \tag{7.6}
\]

These \( V \) values are generally much higher than the reported percent uncertainties calculated for a single specimen. The variability factors don’t compare as well with the work of Clark and Hudak as expected. This is primarily due to the student \( t \) value, which is 2.776 for constant load tests and 4.303 for the constant \( K_{max} \) tests. Clark and Hudak reported a variability factor for incremental polynomial reduced data of 2.51 and 2.93 for all 0.25-inch WOL specimens. Interestingly, if the reported standard deviations remained constant for a total of 10 tests for each loading condition, allowing a \( t = 2 \) to be used, the
variability factors would match almost perfectly. However, it is difficult to determine how representative this sample is with respect to a larger sample relative to the population. It is the opinion of the authors that increasing the number of tests would only marginally effect the scatter, therefore making the variability in current fatigue crack growth rates similar to those reported in 1975 by Clark and Hudak.

Continuing an investigation into multiple specimen uncertainty necessitates an analysis of $\Delta K$ at fixed $da/dN$ values. Table 7.3 lists the variability information computed.

<table>
<thead>
<tr>
<th>$da/dN$ $10^6$ mm/cycle</th>
<th>$da/dN$ Range $10^6$ mm/cycle</th>
<th>Mean $\Delta K$ MPa$\sqrt{m}$</th>
<th>$S_{\Delta K}$ MPa$\sqrt{m}$</th>
<th>$VF$</th>
<th>$V$ %</th>
</tr>
</thead>
<tbody>
<tr>
<td>7620.00</td>
<td>5740.40 – 8661.4</td>
<td>11.01</td>
<td>0.688</td>
<td>1.420</td>
<td>17.37</td>
</tr>
<tr>
<td>1270.00</td>
<td>1206.50 – 1325.88</td>
<td>8.84</td>
<td>0.234</td>
<td>1.158</td>
<td>7.34</td>
</tr>
<tr>
<td>177.80</td>
<td>176.53 – 180.59</td>
<td>5.85</td>
<td>0.147</td>
<td>1.150</td>
<td>6.96</td>
</tr>
<tr>
<td>50.80</td>
<td>50.29 – 51.82</td>
<td>4.20</td>
<td>0.120</td>
<td>1.171</td>
<td>7.88</td>
</tr>
<tr>
<td>20.32</td>
<td>20.07 – 20.83</td>
<td>2.95</td>
<td>0.140</td>
<td>1.514</td>
<td>20.43</td>
</tr>
<tr>
<td>10.16</td>
<td>9.91 – 10.41</td>
<td>2.58</td>
<td>0.068</td>
<td>1.258</td>
<td>11.44</td>
</tr>
<tr>
<td>5.08</td>
<td>4.83 – 5.33</td>
<td>2.25</td>
<td>0.095</td>
<td>1.444</td>
<td>18.16</td>
</tr>
</tbody>
</table>

Clearly the level of variability in $\Delta K$ is much less than that found in $da/dN$. As with $da/dN$, the $V$ values calculated for $\Delta K$ are higher than the single specimen uncertainties. This may indicate the influence of specimen geometry, microstructure, or environment. Regardless of the source of the multiple specimen uncertainty, it was expected that multiple specimen uncertainties would be larger than single specimen uncertainties, because the multiple specimen uncertainties are composed of single specimen uncertainties and other influential factors.
The variability trends in $\Delta K$ are further reinforced by the multiple specimen uncertainty seen in crack length $a$. Table 7.4 lists the crack length variability results.

Table 7.4 Variability in $a$ at Fixed $N$ Values

<table>
<thead>
<tr>
<th>$N \times 10^3$</th>
<th>$N$ Range</th>
<th>Mean $a$ in.</th>
<th>$S_a$ in.</th>
<th>$VF$</th>
<th>$V$ %</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>74.1 - 75.5</td>
<td>23.60</td>
<td>0.33</td>
<td>1.094</td>
<td>4.48</td>
</tr>
<tr>
<td>100</td>
<td>99.7 - 101</td>
<td>25.30</td>
<td>0.46</td>
<td>1.120</td>
<td>5.66</td>
</tr>
<tr>
<td>125</td>
<td>125 - 126</td>
<td>27.31</td>
<td>0.61</td>
<td>1.152</td>
<td>7.05</td>
</tr>
<tr>
<td>150</td>
<td>150 - 151</td>
<td>29.97</td>
<td>1.07</td>
<td>1.258</td>
<td>11.44</td>
</tr>
<tr>
<td>175</td>
<td>174 - 176</td>
<td>33.96</td>
<td>2.01</td>
<td>1.462</td>
<td>18.77</td>
</tr>
<tr>
<td>190</td>
<td>189 - 190</td>
<td>39.62</td>
<td>6.96</td>
<td>3.531</td>
<td>55.86</td>
</tr>
</tbody>
</table>

Only constant load data was used in Table 7.4 since the constant $K_{max}$ data often exhibits no appreciable change in $a$ for large $N$ values. Concerning the results, the variability in crack length obviously increases with cycle count, which may also be seen in Figure 2. The lower cycle count $V$ values are close to but still higher than calculated single specimen uncertainties. At approximately 150,000 cycles and above multiple specimen uncertainty becomes dominant.
CHAPTER VIII
SUMMARY AND CONCLUSIONS

8.1 Thesis Summary

This work has addressed numerous facets of fatigue crack propagation in AA 7075-T651. A complete development of fatigue crack propagation principles and a thorough review of relevant research was presented. In addition, the details of various support tests, which greatly enhanced the ability to gather successful data, were covered. Several data reduction techniques and analytical models were applied to crack opening load measurements and predictions to explore parameters of interest affecting crack opening loads. The principles of uncertainty analysis were applied to the compliance-offset method and uncertainty bands calculated. A respectable amount of fatigue crack propagation data was acquired at various load ratios, the majority of which was generated without the affect of crack closure. The application of practical regression techniques and the use of $\Delta K_{\text{eff}}$ were used to characterize the data for predictions and to develop a single intrinsic fatigue crack propagation curve. With these regression coefficients, successful crack propagation predictions were made using a strip-yield model at several load ratios. Finally, a detailed single and multiple specimen error analysis was performed on the closure-free fatigue crack propagation data to develop uncertainty bands and quantify scatter.
8.2 Conclusions

8.2.1 Determination and Prediction of Crack Opening Loads

Using various experimental data reduction techniques and several analytical models, crack opening loads for AA 7075-T651 C(T) specimens were determined in Chapter IV. A modified compliance-offset technique is recommended to eliminate high load ratio shift effects and allow the use of a physically realistic 0% offset as opposed to the 1 or 2% offsets typically used. The modified compliance-offset technique uses an increased unloading segment instead of the currently accepted (ASTM, 1999a) 25% segment length to measure the open crack compliance. Furthermore, the use of smoothed data is recommended to aid in the quality of measurements using a 0% offset since noise is always present to some degree. The use of these enhancements and alternatives may help when comparing model results and future experimental work.

Results from the analytical models evaluated suggest further research is needed in this area. The use of higher load ratios made the 3-D FEA modeling process difficult. The effect of the number of elements in the reversed plastic zone should be further investigated in an attempt to find criteria governing the amount of refinement necessary to accurately capture crack opening behavior. With regard to the influence of load ratio on opening load, a discrepancy when using the strip-yield models was observed. The predicted values exhibited a different functional relationship with load ratio when compared to that seen from both the finite element analyses and the experiments. Nonetheless, the relative ease of use and satisfactory accuracy of the strip-yield models
should eventually increase their acceptance within the fatigue and fracture mechanics community.

8.2.2 Uncertainty in Crack Opening Loads

The uncertainty in crack opening loads was determined through the use of practical uncertainty expressions. An uncertainty band was found for all 19 mean loads that are calculated in the compliance-offset technique. The uncertainty analyses were performed on both the author’s data as well as round robin data generated from another laboratory. The percent uncertainties ranged from 5.5 to 6.0% for the round robin data and remained constant at 5% for the current data. The most relevant conclusion from the uncertainty analyses was in the breakdown of total uncertainty. For the round robin data systematic uncertainty accounted for 93% of the total and 98% in the current work. This was due largely in part to correlated systematic uncertainty terms. From this, any minor discrepancies with random uncertainty are completely unnoticed. More importantly is the successful application of uncertainty methodologies to a standard experimental data reduction technique to produce a measure of error in the calculated results.

8.2.3 Fatigue Crack Propagation Data

Several analyses were performed on the closure-free $R = 0.7$ data and the other low $R$ crack propagation data. First, two regression routines were performed on the $R = 0.7$ data to determine the best-fit line. From these analyses it was determined that a bilinear fit is superior to a single linear fit. While a bilinear fit was chosen, the data could have been easily broken down further to three or four straight-line segments that likely
would have been an improvement over the bilinear fit. Second, the $R = 0.1$ and 0.3 data were reduced using $\Delta K_{eff}$ in the hopes of obtaining a single curve for the $R = 0.1, 0.3$ and 0.7 data. The single curve was achieved but only by obtaining questionable if not invalid crack opening loads. It seems prudent, in light of successful $\Delta K_{eff}$ application by other researchers, to assume that an error exists in either the crack growth data or opening loads measured. There is a possibility that an error does not exist but it seems unlikely. The limited tests were also designed to investigate the affect of $K_{max}$. After the analyses no apparent affect was observed.

The third analysis performed with the closure-free crack growth data was the use of regression coefficients in a strip-yield model capable of predicting crack growth with $\Delta K_{eff}$ as the primary driving force. The model gave $a$ vs. $N$ that compared well with experimental data generated using $R = 0.7$ and 0.1; however, comparisons with the $R = 0.3$ data was poor. The failure to predict $R = 0.3$ growth is attributed to a high initial $\Delta K$ chosen for the test.

The final results developed from the fatigue crack propagation data were a comparison to crack propagation equations and alternate experimental data. The crack propagation equations of Walker and Forman using AA 7075-T6 coefficients compared quite well with the current data. Moreover, the experimental data of Hudson also compared well. These results suggest that no difference exists between the T651 and T6 tempers when considering fatigue crack propagation.
8.2.4 Uncertainty in Fatigue Crack Propagation Data

Uncertainty methodologies were applied to fatigue crack growth rates, stress intensity factor ranges, and crack lengths in an effort to understand single specimen error. In addition, a statistical analysis of these same quantities over multiple specimens was also performed to quantify multiple specimen scatter. The single specimen uncertainty analysis revealed uncertainties in $da/dN$ ranging from 3.5 to 16%. With regard to the uncertainty in $\Delta K$, the values were lower at 2 to 4% over the range of crack growth studied. The uncertainties in crack length were also relatively small and ranged from 1 to 6.5%. A clear dependence on crack mouth opening displacement was found in all three uncertainty calculations. As the CMOD increased the total uncertainty in crack length and crack growth rate decreased while the uncertainty in stress intensity factor range increased. The reason for this was found in the sensitivity of these variables to changes in CMOD. A result such as this demonstrates the benefits of uncertainty analysis and can be used to improve future testing recommendations.

The second stage of the analysis, which focused on multiple specimen error, reinforced the single specimen uncertainties. The analysis revolved around the computation of variability factors and percent variability values. Variability factors for $da/dN$ ranged from 2.96 to 5.93 while the percent variability values were consistently higher than the single specimen uncertainties. This is important since multiple specimen uncertainty should contain as a subset the single specimen uncertainty. Comparing the variability factors to previous data generated by Clark and Hudak initially was poor. However, when assuming that the variability calculated in this small sample would not increase as specimens tested increased, the variability factors agree very well with the previous work. The variability factors and percent variability values for $\Delta K$ and $a$ essentially yielded similar results. A clear increase in crack length variability with cycle count was determined as expected from data plots.
8.3 Recommendations

The first and foremost recommendation for future work would be the testing of a larger specimen group. This would greatly improve the regression and multiple specimen error analyses. Another important area of improvement is the acquisition of suitable crack opening loads for $\Delta K_{eff}$ plotting. Assuming the data herein is without error some better method to find crack opening loads must be found. However, a larger set of tests may reveal an unknown error in the current work. To improve single and multiple specimen error calculations an environment control is recommended to eliminate temperature and humidity effects. An important issue for furthering the single specimen uncertainty analyses is the propagation of uncertainties through a 7-point incremental polynomial method to see if a decrease in uncertainty occurs. Moreover, the set of entire calculations should be programmed in such a manner that uncertainties can be calculated for all values of $\Delta K$. Overall the current work has addressed many issues but could be expanded to better characterize crack propagation in AA 7075-T651.
REFERENCES


Berry, 2001  Private communication with J. T. Berry, Professor, Mechanical Engineering Department, Mississippi State University.

Booth, 1988

Bray, 1999

Bush, 2000

Chen, 1991

Chermahini, 1988

Clark, 1975

Coleman, 1999

Daniewicz, 1999

Daniewicz, 1996

Daniewicz, 1994
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Wang, 1991  

Yang, 1983  

Zheng, 1997  