Design of an aerospike nozzle for a hybrid rocket

Cedric O. Gould

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DESIGN OF AN AEROSPIKE NOZZLE FOR A HYBRID ROCKET

By

Cedric Oldfield Gould

A Thesis
Submitted to the Faculty of
Mississippi State University
In Partial Fulfillment of the Requirements
for the Degree of Master of Science
in Aerospace Engineering
in the Department of Aerospace Engineering

Mississippi State, Mississippi

August 2008
DESIGN OF AN AEROSPIKE NOZZLE FOR A HYBRID ROCKET

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This document describes the design of an axisymmetric aerospike nozzle to replace the conical converging-diverging nozzle of a commercially available hybrid rocket motor. The planar method of characteristics is used with isentropic flow assumptions to design the nozzle wall. Axisymmetric adjustments are made with quasi-one-dimensional flow approximations. Computational Fluid Dynamics (CFD) simulations verify these assumptions, and illustrate viscous effects within the flow. Nozzle truncations are also investigated. Development of a hybrid-rocket-specific data acquisition system is also detailed.
DEDICATION

This paper is dedicated to all of my friends and family who have supported me all along my journey pursuing my passion.
ACKNOWLEDGEMENTS

I would like to give thanks to my major professor, Dr. Keith Koenig, for all of his invaluable advice and support throughout my collegiate career. My unrelenting appreciation goes out to him.
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\( A_e \)  
cross-sectional area of the nozzle exit

\( A^* \)  
cross-sectional area of the nozzle throat

\( F \)  
thrust force

\( F_{avg} \)  
average thrust force

\( I \)  
impulse

\( I_{sp} \)  
specific impulse

\( M \)  
Mach number

\( M_e \)  
extit Mach number

\( m_{prop} \)  
propellant mass

\( \dot{m} \)  
mass flow rate

\( P_0 \)  
stagnation pressure

\( P_e \)  
pressure of the gases at the nozzle exit

\( P_\infty \)  
ambient atmospheric pressure

\( R_{gas} \)  
gas constant

\( r \)  
nozzle dimension from plane of symmetry or axis of revolution

\( r_e \)  
nozzle exit radius

\( T_0 \)  
stagnation temperature

\( T_e \)  
temperature of the gases at the nozzle exit

\( t_b \)  
burn time

\( V_e \)  
gas velocity at the nozzle exit

\( \gamma \)  
ratio of specific heats of exhaust gases

\( \nu \)  
Prandtl-Meyer function
CHAPTER I

INTRODUCTION

Since the 1950’s, rocket scientists have known that the “plug”, or more commonly referred “aerospike”, nozzle design has had the greatest potential for significantly improving the overall performance of chemical-propellant rocket motors throughout all phases of flight. Due to its unique ability to compensate for the varying atmospheric pressure throughout the launch and ascent phases of flight, the aerospike nozzle can recuperate approximately 50% of the remaining specific impulse gain possible between current conventional bell nozzles and the ideal, lossless nozzle (Whitmore, 2008). It is able to accomplish this by way of effectively increasing its expansion area ratio – a feat very difficult to accomplish with a bell nozzle – to match the engine’s operating pressure ratio. The magnitude of this performance gain could equate to several hundred, if not thousands of pounds of additional payload carrying capability, increased mass margin for other mission-critical hardware, or a reduction in required propellant mass. Other additional gains made by the aerospike design are that it is far more compact in size, weighs less, and would not require heavy gimbaling mechanisms required for thrust vector control as compared to a thrust-equivalent conventional rocket nozzle.

However, as with any new technological leap, the aerospike nozzle design has its own engineering challenges which must be overcome in order for a flight-worthy engine to be developed and utilized for a manned mission. Manufacturing complexities,
increased cooling requirements, and the lack of flight experience have all hindered the development of the aerospike nozzle concept; despite more than 50 years and $500 million in development expenditure costs (including small cold-flow models up to a 250,000 lbf thrust engine, 73 ground tests, and more than an hour in actual operation time) by Rocketdyne (now Boeing), NASA, and the United States Air Force. Other factors have impeded the off-and-on development of the aerospike concept, including the 2001 cancellation of Lockheed Martin’s X-33/VentureStar Single-Stage-To-Orbit (SSTO) project, which was slated to use a planar aerospike engine (O’Leary & Beck, 1992).

In contrast to the ever-present push for new technological development and advancement, the aerospike nozzle concept, in spite of its many potential benefits, had seemingly been removed from the forefront of rocket technology development and research. That is, until a group of students from California State University, Long Beach and the Garvey Spacecraft Corporation launched the world’s first known sounding rocket to use a liquid propellant aerospike nozzle in 2003 under the California Launch Vehicle Education Initiative (CALVEIN) program, putting the aerospike rocket nozzle concept in a positive light for the first time.

A good portion of recent rocket motor development work is focused on the use of hybrid propellants - ones that use either a gaseous or liquid oxidizer in combination with a solid fuel. Popular choices include hydroxyl-terminated polybutadiene (HTPB) and nitrous oxide (N₂O) as the fuel and oxidizer, respectively. Common discussion of this particular hybrid fuel combination typically leads to the “tire rubber and laughing gas”
joke; however, as testing and history has shown, these inexpensive chemicals hold themselves quite well against competing mixtures in many different performance-defining criteria. Note that the Ansari X-Prize was won in 2004 by the Scaled Composites entry SpaceShipOne, which was propelled by a hybrid motor using a very similar combination. This vehicle became the first privately funded vehicle to exceed the 100 km Kármán Line which is the internationally accepted boundary defining where outer space begins and the effective atmosphere ends.

Hybrid propellants have some very distinct advantages over their solid propellant cousins in that they are generally benign while in storage because they are physically separated from one another, and unlike liquid propellant engines, they do not require heavy pump mechanisms or heavily insulated and expensive storage tanks. The ignition process is also a deliberate one, requiring the two chemicals be in contact and the fuel-oxidizer mixture temperature be raised above 570°F – an environment somewhat difficult to create unintentionally.

The purpose of this project is to combine the advantageous performance features of the aerospike nozzle concept with safe and user-friendly hybrid propellants and determine whether the cost of nozzle development is worth the payoff of increased performance. For the sake of cost-effectiveness, the decision was made early on in the project to retrofit an existing commercially available hybrid rocket motor designed for use in the amateur sport of high-power rocketry. This would provide an inexpensive and proven system into which an aerospike nozzle could easily be added, and baseline performance data for comparison and evaluation be measured.
CHAPTER II
ENGINEERING METHODS

An array of engineering methods and tools were used to research the hybrid propellant-aerospike nozzle rocket motor combination including analytical, computational, and experimental. These tools are described here. First, the fundamentals of the method of characteristics (as it relates to the aerospike nozzle design program) are outlined. Then, a description of the selected commercially available hybrid rocket is provided. Next, computational fluid dynamics (CFD) simulations and results of the commercially available conical nozzle and the designed hybrid aerospike nozzle and variations of it are shown and compared. Finally, the design and development of the purpose-built data acquisition (DAQ) software package is detailed.

This project is a continuation of work completed by the author as an undergraduate seminar research assignment. A planar cold-flow aerospike nozzle was designed, prototyped, and videographed using a common schlieren technique in an attempt to utilize and validate the use of the planar method of characteristics in the aforementioned application (Gould, 2006). The Mathcad® program written for this work was used here (and included in Appendix A) with inputs to account for the expected propellant combustion products, and is outlined below.
Method of Characteristics for Aerospike Nozzles

For a full derivation and discussion of the method of characteristics, the reader is referred to one of the many texts available, such as Hodge and Koenig (1995), Thompson (1972), or Liepmann and Roshko (1957). As the reader will see, the method of characteristics as it applies to the Prandtl-Meyer centered expansion for the aerospike design is simple to employ – much more so than for conventional bell nozzles where different flow regions and methods must be determined, accounted for and implemented, thus complicating the process. The following design procedure does not involve non-simple, simple-plus, free surface, or shock unit processes, therefore they are not included in this discussion.

In a bell nozzle, both the nozzle throat and exit planes are parallel. Therefore, if the flow is expanded in supersonic fashion and is uniform and parallel at the exit, the flow must first turn away from its initial flow angle, then turn back again to become parallel to the original angle. These two turns create simple-plus, simple-minus, and non-simple flow regions (Hamilton, 2003). However, with an aerospike nozzle, the throat and exit planes are not parallel, and the angle made between the two corresponds to the total flow turn angle required to accelerate the flow to its design exit Mach number. The single turn creates a simple-minus region whereby the flow is expanded and accelerated. This comparison can be seen in Figure 1. The reader will note that the dash-dotted lines in Figure 1 (a) and (b) represent each respective nozzle’s axis or plane of symmetry.

As with all fluid mechanics problems, the governing relations are those of conservation of mass, momentum, and energy. Though ultimately satisfied, the Prandtl-
Meyer function is derived merely with geometric foundation. In order for supersonic flow to be accelerated, the flow must be turn-expanded, and the Prandtl-Meyer function (Eq. 1) relates the amount of flow angular turn with the exit Mach number (Anderson, 2003). Therefore, given a desired flow exit Mach number, the corresponding total flow direction turn angle is known. Within the design program, the total turn angle is divided into a user-selected number of sections which are used in the point-slope-intersection marching method approach of the method of characteristics to solve for the nozzle wall defining coordinates.

\[
v(M) = \sqrt{\frac{y+1}{y-1}} \tan^{-1} \sqrt{\frac{y-1}{y+1} (M^2 - 1)} - \tan^{-1} \sqrt{M^2 - 1}
\]

(Eq. 1)

Figure 1 Supersonic Flow Expansion Comparison Between (a) Conventional Bell and (b) Aerospike Nozzles
Combining this information with gas properties, nozzle throat sizing, and an atmospheric design pressure altitude (Benson, 2006), the design program then calculates the required chamber pressure (Eq. 2), exhaust temperature (Eq. 3), speed of sound, and velocity, the required mass flow rate (Eq. 4), motor thrust (Eq. 5), and specific impulse (Eq. 6). The assumptions made within the design program are that the flow is accelerated isentropically, the fluid is inviscid, and that the fluid is calorically perfect.

\[ P_0 = P_\infty \left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\frac{\gamma}{\gamma-1}} \]  
\[ T_e = \frac{T_0}{1 + \frac{\gamma-1}{2} M_e^2} \]

\[ \dot{m} = \frac{p_0 A^* \sqrt{\frac{\gamma}{R_{gas}} \left(2 \frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}}}}{\sqrt{T_0}} \]  
\[ F = \dot{m} V_e + (P_e - P_\infty) A_e \]

\[ I_{sp} = \frac{F}{\dot{m} g} \]  

The reader will note that in the thrust equation (Eq. 5) the second term is zero for the atmospheric design condition where the pressure at the nozzle exit equals the atmospheric pressure at the design altitude. Interestingly, when the second term is non-zero (for example, a conventional nozzle operating off-design so that the exhaust pressure at the exit plane does not equal the ambient atmospheric pressure), less than optimum total thrust is generated due to incomplete expansion of the flow, and thus a disproportionately lower flow exit velocity is achieved. Recall that the primary purpose
and advantage of the aerospike design is to compensate for atmospheric pressure variations. It does so by effectively changing the nozzle exit area such that the second term goes to zero, or nearly zero, throughout the flight, thereby always optimizing the flow exit velocity. In an off-design condition where the nozzle operates at an atmospheric pressure different from the back pressure the nozzle was designed for, the jet plume exiting the nozzle will be smaller than design at altitudes below the design altitude, and larger if above. The exhaust plume flight progression can be seen in Figure 2 (O’Leary & Beck, 1992).

![Aerospike Exhaust Plume Variations](image)

**Figure 2** Aerospike Exhaust Plume Variations - (a) Sea Level, (b) Design Altitude, to (c) Above Design Altitude Atmospheric Pressures

The aforementioned gas property inputs are required to represent the combustion products expected from the hybrid rocket motor for use within the design program. These values are shown in Table 1 for the nitrous oxide/hydroxyl-terminated
polybutadiene (N₂O/HTPB) propellant combination consumed within the motor (Whitmore, 2008).

Table 1  Combustion Product Gas Properties of N₂O and HTPB

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<tr>
<td>γ</td>
<td>1.26</td>
</tr>
<tr>
<td>R_{\text{gas}}</td>
<td>1913 \left(\frac{ft}{s}\right)^2 \frac{\text{in}}{s}</td>
</tr>
<tr>
<td>T₀</td>
<td>5400 °R</td>
</tr>
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</table>

**Motor Hardware Description**

For the purposes of this work, the Contrail Rockets 54-mm diameter, 28-in long hybrid rocket motor hardware was selected for its low cost, availability, reusability, ignition system simplicity, and nozzle adaptability. Other brands of amateur high power hybrid rockets utilize one-time use nozzles and/or combustion chambers which would be much more difficult to couple with a new nozzle design and test repeatedly, while others use complicated ignition systems requiring both nitrous oxide and gaseous oxygen in order to start the motor. The chosen motor requires neither of these complications.

The Contrail Rockets motor uses a single, all-inclusive aluminum tube which acts as both the oxidizer tank and combustion chamber, separated by an internal floating-style bulkhead into which the oxidizer fill port/injector is installed. The oxidizer tank volume is contained by the upper bulkhead which also holds the oxidizer tank fill vent. Below the floating bulkhead is the fuel grain and nozzle. In order to contain the motor contents while under pressure and during operation, expansion snap rings are installed in grooves
on the inside surface of the aluminum tube on both ends - one above the upper bulkhead, and another below the nozzle (this design feature is important to note). These features also allow for ease of disassembly and cleaning after each motor firing. Figure 3 illustrates the Contrail Rockets motor assembly in a cut-away diagram.

![Contrail Rockets Hybrid Rocket Motor Assembly](image)

Figure 3 Contrail Rockets Hybrid Rocket Motor Assembly

This commercially available, off-the shelf (COTS) motor uses a conically shaped converging–diverging (30°/15°, half-angles) graphite nozzle. Easily manufactured, conical nozzles are known to be an inefficient means of expanding supersonic flow, as at the exit plane the flow is neither uniform nor parallel. This study utilizes the COTS nozzle as a baseline for performance comparison, and its geometry to establish estimates
of the remaining motor operating parameters such as the mass flow rate and combustion chamber stagnation pressure.

With motor hardware selected, the design program was used to propose an aerospike nozzle which would seamlessly and directly replace the COTS nozzle. Test data provided by the Tripoli Motor Testing (TMT) group (Holmes, 2007) (one of the national amateur high power rocketry governing bodies; included in Appendix B) presented insight to certain performance parameters not given by the manufacturer, and are summarized in Table 2.

Table 2 Contraill Rockets TMT Test Result Summary

<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>( m_{prop} )</td>
<td>0.0479 slug</td>
</tr>
<tr>
<td>( t_b )</td>
<td>2.62 s</td>
</tr>
<tr>
<td>( \dot{m} )</td>
<td>0.0183 ( \frac{slug}{s} )</td>
</tr>
<tr>
<td>( F_{avg} )</td>
<td>55.1 lbf</td>
</tr>
<tr>
<td>( l )</td>
<td>144.3 lbf ( \cdot ) s</td>
</tr>
<tr>
<td>( I_{sp} )</td>
<td>93.5 s</td>
</tr>
</tbody>
</table>

Prior to acquiring the TMT test data, however, an estimate of the combustion stagnation pressure was made at 575 psi, based upon the prescribed operating oxidizer tank pressure of 550–900 psi as recommended by the manufacturer with the understanding that in order for the self-pressurizing \( \text{N}_2\text{O} \) to flow into the combustion chamber, the oxidizer tank pressure must be greater than the combustion chamber pressure. Without an analysis of the injector itself, it was deemed a reasonable first-order
estimate. It is this stagnation pressure on which the aerospike nozzle designed herein is based. Conversely, rearranging the mass flow rate equation (Eq. 4) for stagnation pressure and inputting the TMT test data from Table 2 resulted in an average combustion stagnation pressure of 415 psi. Due to the significant difference between these two pressures and their resulting performance parameters, analytical comparisons of both the original estimated chamber pressure and test data average pressure cases are shown, but only CFD solutions of the 575 psi aerospike nozzle (and its truncation variants) and 415 psi stock nozzle have been completed. Table 3 summarizes the design input parameters used for the Mathcad nozzle design program to create the aerospike nozzle and shows the reverse-engineered performance parameters for the COTS nozzle. All calculations use ambient sea level pressure (14.7 psi) as the back pressure.

Table 3  Design Program Performance Parameter Comparison

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<th>Aerospike Design</th>
<th>COTS Nozzle</th>
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<tr>
<td>$P_0$ (psi)</td>
<td>575</td>
<td>415</td>
</tr>
<tr>
<td>$M_e$</td>
<td>2.953</td>
<td>2.762</td>
</tr>
<tr>
<td>$A_e / A^*$</td>
<td>5.35</td>
<td>4.26$^\dagger$</td>
</tr>
<tr>
<td>$F$ (lbf)</td>
<td>177.0</td>
<td>122.6</td>
</tr>
<tr>
<td>$\dot{m} \left( \frac{slug}{s} \right)$</td>
<td>0.0260</td>
<td>0.0183</td>
</tr>
<tr>
<td>$I_{sp}$ (s)</td>
<td>214.7</td>
<td>207.6</td>
</tr>
</tbody>
</table>

$^\dagger$ - actual area ratio = 7.46

The significant contrast between the TMT test results for the COTS motor in Table 2 and the design program prediction values in Table 3 leads one to question the
validity of either the testing procedure and results or the design program methodology. However, understanding that the motor design cannot supply a constant oxidizer tank pressure due to not having a regulated injector or a high pressure regulation system leads to a generally regressive and erratic motor burn. As the TMT thrust versus time data shows (see Appendix B), however, the very beginning of the burn (while the oxidizer tank pressure is still high) has instantaneous thrust values comparable to the design program performance predictions, as opposed to the burn duration-averaged data in Table 2. It is therefore assumed that the selected hardware can still be used to directly evaluate the success of the aerospike nozzle, though only for a short period of time after ignition.

**Computational Nozzle Simulations**

Prior to committing expensive manufacturing resources towards the production of the aerospike nozzle for testing, it was decided that CFD simulations should be run in an effort to better justify the design. This would provide for higher confidence in the success of the work being accomplished. A copy of ANSYS® Academic Teaching CFD was allotted by the Mechanical Engineering Department at Mississippi State University for this purpose.

**Planar Aerospike Analysis**

The first task undertaken was a check of the aerospike design program with the CFD program to recreate and compare results from the author’s prior work (Gould, 2006), which utilized a small, cold-flow planar aerospike nozzle. This step was taken because the nozzle was already designed and computer aided design (CAD) files of the nozzle wall shape were available. This also gave the author an opportunity to become
familiar with the use of the CFD software. Table 4 summarizes the design program inputs and results for the cold-flow aerospike solution and compares them to the CFD results. The reader will note that Table 4 shows proportionally smaller thrust and mass flow rates in the CFD solution than the design program prediction, yet the CFD-predicted nozzle specific impulse is almost exactly what was predicted by the design program. Figure 4, meanwhile, compares the nozzle exit plane velocity from the design program with the exit plane velocity distribution as computed by the CFD software. The similarity between the design program prediction and CFD solution is quite remarkable considering the simplicity of the design program utilized and the short amount of time the CFD simulation took to run on a laptop computer. It is this result that leads one to begin to believe in the applicability and usefulness in the method of characteristics for supersonic rocket nozzle design.

Table 4  Cold-Flow Aerospike Design Program and CFD Results Comparison

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Results</th>
<th>Aerospike Design</th>
<th>CFD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td></td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>$R_{gas}$</td>
<td>$F \ (lb)</td>
<td>33.9</td>
<td>27.9</td>
</tr>
<tr>
<td>$\dot{m} \ (\text{slug/s})$</td>
<td></td>
<td>0.0177</td>
<td>0.0146</td>
</tr>
<tr>
<td>$M_e$</td>
<td>$I_{sp} \ (s)$</td>
<td>59.4</td>
<td>59.4</td>
</tr>
<tr>
<td>$T_0$</td>
<td>$A_e/A'^*$</td>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>$P_0$</td>
<td></td>
<td>250 psi</td>
<td></td>
</tr>
</tbody>
</table>
One distinct advantage to a CFD simulation of a nozzle is the investigation of boundary layer effects and viscosity on nozzle performance – something not accomplished or taken into consideration through the method of characteristics. In this case, the effect of the boundary layer on the velocity component of total thrust is insignificant (< 0.05 lbf); however this is not to say that it is so in all circumstances, applications, configurations, or operating conditions. Over the entire normalized station range (with the exception of the boundary layer), the average exit velocity error from the design exit velocity (1912 ft/s) is 0.1%; taking the boundary layer into account, the average exit velocity error only increases to 1.2%. This concurs with the data in Table 4 in that the CFD-predicted thrust for the cold-flow aerospike is proportionately lower than
the design program prediction due to the reduced mass flow rate through the nozzle and not the exit plane velocity. Examination of the mass flow rate equation (Eq. 4) has led the author to conclude that the viscous effects on such a small nozzle (throat height of 0.1 in) have the negative consequence of reducing the effective nozzle throat area (as evidenced here) by nearly 18% - a very significant result when trying to design nozzles to meet certain specifications or mission needs. Such outcomes would not normally be anticipated by design engineers if this type of tool or data were not available. Figures 5-10 illustrate the CFD results of the cold-flow aerospike nozzle with contours of pressure (psi), temperature (°R), density (lbf·s²/in⁴), velocity (in/min), Mach number, and flow streamlines, respectively.

Figure 5   Cold-Flow Planar Aerospike CFD Pressure Contours (psi)
Figure 6  Cold-Flow Planar Aerospike CFD Temperature Contours (°R)

Figure 7  Cold-Flow Planar Aerospike CFD Density Contours (lbf·s²/in⁴)
Figure 8  Cold-Flow Planar Aerospike CFD Velocity Contours (in/min)

Figure 9  Cold-Flow Planar Aerospike CFD Mach Contours
The reader will note the fascinating similarity in each of Figure 5-9’s physical property contour lines in the supersonic nozzle section to Figure 11, the original design program output which defined the nozzle wall coordinates. If overlaid with any of the aforementioned Figures, the flow characteristic lines in Figure 11 align nearly precisely, extending from the flow expansion corner to the end of the nozzle wall, thereby lending great confidence in the original theory behind the method of characteristics that along characteristic lines in a simple region, flow properties (p, ρ, T, etc.) are indeed continuous (Anderson, 2003).

With confidence and correlation established between the aerospike nozzle design program and the ANSYS CFD software, it was decided that a CFD solution of the Contrail Rockets COTS nozzle be generated next. This would provide insight to the
expected flow field and a baseline of performance parameters with which to compare to the aerospike nozzle’s simulation and performance data once completed.

Figure 11    Cold-Flow Planar Aerospike Nozzle Wall and Characteristics

**Axisymmetric Aerospike Analysis**

Table 5 summarizes the design program input parameters for an axisymmetric aerospike nozzle utilizing hybrid propellant product properties at the TMT test data time-averaged combustion chamber pressure of 415 psi, and compares the design program output to the TMT test data and CFD simulations of the COTS nozzle. As the table shows, the COTS nozzle has an area ratio of 7.46. Given these two important parameters, the COTS nozzle seems to be designed for an operating altitude of approximately 20,000 ft, and thus an equivalent back pressure of 6.6 psi. This information brings into question the design process and requirement which defined the COTS nozzle, as for this size motor used for the purpose intended (the sport of amateur high power rocketry), it is impossible for any rocket powered by this motor to reach the intended design operating condition of 20,000 ft. This result suggests that the
manufacturer of this motor may have had conflicting requirements for their rocket nozzle design. If the manufacturer were to design the best performing rocket motor for the customer, and thereby capture the market, the only way to do so would be to optimize the motor’s nozzle to perform best under the operating conditions expected during use.

Table 5  COTS Nozzle Design Program, Test Data, and CFD Results Comparison

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>1.26</td>
</tr>
<tr>
<td>$R_{\text{gas}}$</td>
<td>$\frac{(ft/s)^2}{g} \times \frac{F (\text{lb})}{\text{Design}}$</td>
</tr>
<tr>
<td>$\bar{R}_g$</td>
<td>$\frac{\text{slug}}{s}$ \times 0.01833</td>
</tr>
<tr>
<td>$M_e$</td>
<td>$l_{sp} (s)$</td>
</tr>
<tr>
<td>$T_0$</td>
<td>$5400 , ^\circ R$</td>
</tr>
<tr>
<td>$P_0$</td>
<td>$415 , \text{psi}$</td>
</tr>
</tbody>
</table>

It bears repeating that the actual motor test data from TMT have been averaged over the total burn time for the purpose of comparison. Unfortunately, due to the regressive burn experienced by the COTS motor because of the oxidizer pressure drop over time, the time-averaging of data paints a rather pessimistic picture of the motor’s performance. Closer examination of the TMT data shows an erratic but closer representation of the design program predicted performance values over approximately the first 0.10–0.15s of the total burn. It is believed that if the raw test data could be acquired and an analysis performed on this brief but representative portion of motor
operation where the combustion chamber pressure is much closer to the design program prediction occurs, a much better comparison between the two results could be made.

The reader will note that Figure 12 clearly shows an over-expanded COTS nozzle flow. This is evidenced by the fact that the exit plane velocity distribution is a significant percentage higher (~6.2%) over 96% of the nozzle exit face than the design condition prediction, and the pressure across the face is quite a bit lower than the ambient pressure (depicted by the negative normalized pressure difference). As discussed earlier, an examination of the rocket thrust equation (Eq. 5) with a non-zero second term leads to a significant divergence from the proposed design condition and maximum performance.

![Figure 12](image)

**Figure 12** COTS Nozzle CFD Exit Plane Velocity Comparison and Normalized Pressure Difference Distributions
Overexpanded nozzles have other drawbacks than reductions in efficiency or performance. For example, severely overexpanded flows are subject to the possible development of oblique or even normal shock waves within the confines of the nozzle itself, leading to significant structural or thermal/cooling issues. Shock waves of either type lead to discontinuous jumps in pressure and temperature, wreaking havoc on cooling systems, nozzle material ablation, and/or structural deformation or destruction of the nozzle itself. However, from the CFD flow results of the COTS nozzle contained within Figures 13-18, it is evident that the severity of overexpansion within it is insufficient to create such damaging effects. Note the consistent (but non-standard) units utilized in the cold-flow aerospike CFD results in Figures 5-9 and Figures 13-17 (pressure (psi), temperature (°R), density (lbf·s²/in⁴), velocity (in/min), and Mach number), respectively.

One important aspect to note about the CFD results of the COTS nozzle is the development of the downstream plume and resulting shock diamonds - typical of overexpanded supersonic nozzle flow. This phenomenon is common to rockets launching from a pad, i.e. the Space Shuttle main engines, or operating below design condition altitudes or pressure ratios. This particular flow pattern is most easily visualized in Figures 14, 16, and 17 - contour plots of temperature, velocity, and Mach number, respectively. These contour plots show the sudden and steep gradients induced by the oblique shock waves and the more gradual and continuous expansion fans in an interchanging pattern resembling a diamond pattern. Figure 18, on the other hand, shows with great detail, the interaction of the stagnant surrounding air with the primary nozzle flow and resulting shear layer. The air surrounding the plume is brought radially inward
Figure 13  COTS Nozzle CFD Pressure Contours (psi)

Figure 14  COTS Nozzle CFD Temperature Contours (°R)
Figure 15  COTS Nozzle CFD Density Contours (lbf·s²/in⁴)

Figure 16  COTS Nozzle CFD Velocity Contours (in/min)
Figure 17  COTS Nozzle CFD Mach Contours

Figure 18  COTS Nozzle CFD Streamlines
along the plume’s length then pulled along with the jet, entraining it. The reader will also note that in Figure 14, the ambient air appears to be $5400 \, ^\circ\text{R}$. This is due to the solution being an adiabatic one, neglecting any heat transfer effects on the resulting flow.

**CFD Model Design**

With a general performance picture of the COTS nozzle, the research now turns to the development of a model of an axisymmetric aerospike replacement nozzle to be used in a CFD study. This study will attempt to determine whether the performance increase is worth the effort to design and manufacture a flight nozzle. A description of the procedure to design the CFD model follows.

The design procedure assumes that there is uniform flow at the throat and exit planes and that the flow throughout is isentropic. These assumptions allow the isentropic, quasi-one-dimensional $A/A^*$ relation to be used at the exit. Specification of the exit Mach number, here $2.95$, then gives $A_e/A^* = 5.35$ for $\gamma = 1.26$.

Next, the replacement aerospike nozzle must allow for the same mass flow rate and chamber conditions as the COTS nozzle. In order for this to occur the throat area for the aerospike must be the same as the COTS throat area. The COTS throat area along with the known $A_e/A^*$ ratio gives $A_e$. The COTS exit area is not used because it appears to be too large for the proper expansion at sea level air. For the full aerospike $A_e = \pi r_e^2$ and so the exit plane flow radius $r_e$, is now known. For the design case $r_2 = r_e$ and thus $r_2$ is known.
The throat for the aerospike forms a slice of a cone with the area found in Eq. 8.

\[ A^* = \frac{\pi(r_2^2 - r_1^2)}{\sin(\theta)} \]  

(8)

The angle \( \theta \) is found by realizing that the flow from throat to exit must turn an amount equal to the Prandtl-Meyer function (Eq. 1), corresponding to the exit Mach number, so that \( \theta = \pi/2 - \nu \). The \( A^* \) equation then gives \( r_1 \).

The locations of points 1 and 2 are now known. The location of point 3, given by length \( L \), and the shape of the wall from 1 to 3 are found using the method of characteristics. For the design here the planar method characteristics is used, although the axisymmetric method should actually be used since the nozzle is axisymmetric. The planar method is much simpler to apply and that is the reason for its selection. Because of the design method being used, flaws will most likely appear in the resulting flow field.
These flaws should be visible in the CFD solutions and qualitative assessment at least can be made of the effects they will have on nozzle performance.

Table 6 summarizes the design program input parameters and compares CFD results for the COTS nozzle and full aerospike at this chamber pressure. Interestingly, the COTS nozzle outperforms the designed aerospike nozzle in thrust, but at the expense of increased mass flow rate and thus, a significant specific impulse penalty. In rocket performance analyses, it is specific impulse which is most commonly used as the determining factor whether a particular nozzle or configuration is better or worse than another. On this basis, it is clear that the aerospike nozzle is better than the COTS conical nozzle design due to its thrust-specific fuel efficiency.

Table 6  
Aerospike Nozzle Design Program, COTS Nozzle CFD Results Comparison

<table>
<thead>
<tr>
<th>Inputs</th>
<th></th>
<th>Results</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>γ</td>
<td>1.26</td>
<td></td>
<td>Design</td>
<td></td>
</tr>
<tr>
<td>R_{gas}</td>
<td>1913 ((ft/s)^2)^{(s)}</td>
<td>F (lbf)</td>
<td>177.0</td>
<td>180.0</td>
</tr>
<tr>
<td>M_e</td>
<td>2.953</td>
<td>m_{stug} (slug/s)</td>
<td>0.0260</td>
<td>0.02855</td>
</tr>
<tr>
<td>T_0</td>
<td>5400 °R</td>
<td>A_e/A^*</td>
<td>5.35</td>
<td>7.46</td>
</tr>
<tr>
<td>P_0</td>
<td>575 psi</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Also from Table 6 it is clear that although the pressure ratio across the COTS nozzle has increased by a significant margin (28.2 vs. 39.1, or P_0 = 415 psi vs. 575 psi), the nozzle is still simulated at an ambient back pressure much higher than the area ratio.
would indicate. At a chamber pressure of 575 psi and an area ratio of 7.46, the nozzle would be optimized for an altitude of approximately 12,500 ft, or a back pressure of 9.2 psi. This situation is consistent with the results from Figure 12 for the COTS nozzle operating at 415 psi, though more closely approaching the design condition pressure. Simply put, the COTS nozzle operating at 575 psi chamber pressure and sea level back pressure exhibits proportionately less exit plane velocity overshoot from the design condition and less undershoot of the exit plane static pressure from ambient than the 415 psi chamber pressure case shown in Figure 12. These results remain consistent with an overexpanded nozzle flow.

Using the inputs listed in Table 6, the aerospike design program output the nozzle wall defining coordinates and expansion characteristics shown in Figure 20.

![Figure 20: COTS Replacement Aerospike Nozzle Wall and Characteristics](image)

Understanding that the design program was written to determine the appropriate planar aerospike wall for a given set of desired performance parameters, the program was used without compensation for axisymmetric expansion effects; except to say that the
area ratio for the planar solution was used to adjust for the desired axisymmetric exit Mach number. To investigate the quality of flow expansion, an exit plane velocity distribution was extracted from the CFD solution for the aerospike nozzle and compared to the design program predicted exit flow velocity, as viewed in Figure 21. Over the entire exit plane, the average exit velocity error from the predicted value is a mere 4%, which includes the significant turbulent boundary layer near the spike due to the nozzle wall not being designed for axisymmetric application. Over the upper 78% of the flow, the average error is only 0.7%. This information leads the author to believe that the use

![Aerospike Nozzle CFD Exit Plane Velocity Comparison](image)

Figure 21  Aerospike Nozzle CFD Exit Plane Velocity Comparison

of the planar aerospike nozzle wall in an axisymmetric application is a good first-order approximate design from which a designer could begin with. Another interesting flow feature of the aerospike nozzle captured in Figure 21 is the interaction of the primary jet
exhaust with the ambient surrounding air. It is this interaction which creates a shear flow layer surrounding the primary flow, entraining the ambient air. This interaction can be seen in Figure 21 near the upper end of the normalized radial station where the flow velocity slows from the design program predicted value. As Figures 22-27 show, the rudimentary adjustment of area ratio for desired exit Mach number did not substantially affect the flow expansion through the nozzle.

Now with a good indication of how the aerospike nozzle might perform under the design conditions, the manufacturability and thermal considerations of the nozzle had to be taken into account. Given the burn time expected for future tests (< 3 s), and the reusability of the COTS nozzle, specific thermal considerations for the nozzle are neglected. As one can imagine, a revolved nozzle curve such as the aerospike one above would become prohibitively thin and spindly, thus making the full nozzle quite difficult to make from a delicate material such as graphite. Also, in a full scale application, full length aerospike nozzles would become unreasonably heavy and large. One common solution to this dilemma is the use of truncated aerospike nozzles. Therefore, two different truncated aerospike nozzles are investigated – one of which is 40% of the full spike length, and the other, 20%.

As a result of truncating and maintaining the original nozzle throat spacing, an area known as the nozzle base is created. The base is, in the case of an axisymmetric aerospike nozzle, circular in cross section. If a planar aerospike were truncated, the base would be rectangular. Aerospike nozzle researchers typically fall along two general lines
Figure 22  Full Aerospike CFD Pressure Contours (psi)

Figure 23  Full Aerospike CFD Temperature Contours (°R)
Figure 24  Full Aerospike CFD Density Contours (lbf·s$^2$/in$^4$)

Figure 25  Full Aerospike CFD Velocity Contours (in/min)
Figure 26    Full Aerospike CFD Mach Contours

Figure 27    Full Aerospike CFD Streamlines
of thought as to how the base affects nozzle performance (O’Leary & Beck, 1992). Either the base is closed and a trapped, subsonic recirculation zone develops, or, as in a liquid propellant motor, the turbopump exhaust is vented through the base area, creating a certain amount of thrust. In both instances, the idea is to recuperate as much of the thrust lost due to the truncation of the nozzle, and thus, the incomplete expansion of the flow. The research herein assumes a closed base. The CFD contour results for the 40% truncated aerospike nozzle are contained in Figures 32-37, while the 20% nozzle results in Figures 38-43. Each set shows that an entrapped, subsonic recirculation zone is, indeed, formed.

Table 7 compares the CFD results from the full, 40%, and 20% truncated nozzles to the design program predictions. These results show the anticipated decline in specific impulse with nozzle truncation due to the increasing expansion inefficiency. As anticipated, the full spike nozzle outperforms either of the truncation variations based solely on thrust performance. However, if the aerospike designed were for a full scale rocket, an installed engine weight component and additional considerations would need to be evaluated against the decrease in performance in order to make an intelligent,

<table>
<thead>
<tr>
<th>Table 7</th>
<th>CFD Performance Comparisons of Aerospike Variations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Design</td>
</tr>
<tr>
<td>$F$ ($\text{lbf}$)</td>
<td>177.0</td>
</tr>
<tr>
<td>$\dot{m} \left( \frac{\text{slug}}{s} \right)$</td>
<td>0.0260</td>
</tr>
<tr>
<td>$I_{sp}$ ($\text{s}$)</td>
<td>214.7</td>
</tr>
</tbody>
</table>
engineering decision as to the nozzle shape with which to proceed. Fortunately, the selection of this aerospike nozzle variation comes down to an issue of manufacturability. Considering the size of the nozzle required to fit within the COTS nozzle space, the exit diameter of the aerospike nozzle is a mere 30 mm. As a tradeoff between the thin, spindly, and difficult to manufacture but best performing full aerospike and the most easily manufactured, but worst performing 20% truncated spike, the 40% aerospike was selected as the nozzle to pursue to replace the COTS nozzle.

As before, exit velocity and normalized pressure difference plots are created for the 40% and 20% truncated nozzles. These plots are generated for each truncation at their respective actual nozzle exits (Figures 28 and 29, respectively), and at the full spike theoretical exits (Figures 30 and 31, respectively). This is done to understand the effect of the base flow on the primary exhaust flow. These figures show a consistent trend regarding the incomplete expansion of the nozzle flow. Lower than design exiting velocity and a non-planar remaining pressure distribution very clearly show that at the corner of the base, the flow will again turn-expand toward the nozzle axis. This secondary turn-expansion is followed by an oblique shock; required to turn the flow back towards the axial direction. The reader will note the size and angular differences of the secondary turn-expansion, base flows, and oblique shockwaves, respectively, between the 40% and 20% cases in Figures 32-43. Proportionate and increasing in angle, these results are related to the amount of nozzle truncation.
Figure 28   40% Truncated Aerospike Nozzle CFD Actual Exit Plane Velocity Comparison and Normalized Pressure Difference Distributions

Figure 29   20% Truncated Aerospike Nozzle CFD Actual Exit Plane Velocity Comparison and Normalized Pressure Difference Distributions
Figure 30  40% Truncated Aerospike Nozzle CFD Theoretical Exit Plane Velocity Comparison

Figure 31  20% Truncated Aerospike Nozzle CFD Theoretical Exit Plane Velocity Comparison
Figure 32  40% Truncated Aerospike CFD Pressure Contours (psi)

Figure 33  40% Truncated Aerospike CFD Temperature Contours (°R)
Figure 34  40% Truncated Aerospike CFD Density Contours (lbf·s²/in⁴)

Figure 35  40% Truncated Aerospike CFD Velocity Contours (in/min)
Figure 36 40% Truncated Aerospike CFD Mach Contours

Figure 37 40% Truncated Aerospike CFD Streamlines
Figure 38  20% Truncated Aerospike CFD Pressure Contours (psi)

Figure 39  20% Truncated Aerospike CFD Temperature Contours (°R)
Figure 40  
20% Truncated Aerospike CFD Density Contours (lbf·s²/in⁴)

Figure 41  
20% Truncated Aerospike CFD Velocity Contours (in/min)
Figure 42  
20% Truncated Aerospike CFD Mach Contours

Figure 43  
20% Truncated Aerospike CFD Streamlines
Data Acquisition System Description

Having selected an aerospike nozzle design to compare against the baseline COTS nozzle, a method and system was developed in order to gather thrust, oxidizer tank pressure, and combustion chamber pressure over the length of the motor burn. System expansion and an accommodation for different thrust-class motors were also considered. National Instruments (NI) LabVIEW® was selected as the data acquisition (DAQ) software used in conjunction with an NI USB 6251/SCB-68 DAQ hardware combination. LabVIEW and the NI DAQ hardware were supplied by the Aerospace Engineering Department at Mississippi State University. Keeping in mind the sport-intended application of the COTS motor, ground support equipment is required to successfully fill and fire these motors. To fill this role, the Pratt Hobbies Universal Launch System (ULS) was selected because of motor manufacturer recommendation and low cost. In order to automate the motor start and DAQ processes - oxidizer fill-up, countdown, ignition, and data sampling - integration of the NI DAQ and ULS hardware was also accomplished. An electrical system schematic of the DAQ hardware integration can be found in Appendix C.

LabVIEW is a graphical user interface (GUI) programming language intended for engineers and scientists, with programs stored in virtual instruments (VIs). These VIs are broken into two parts – the front page, which contains the user indicators, controls, and displays – and the block diagram, where all of the programming logic is set. Figure 44 shows the front page to the Rocket Motor Test Stand DAQ VI, while Figure 45 is the block diagram. When the VI is executed, the program first waits on a series of user inputs relating to whether the user is ready to begin the sequence, and if the checklist of
pre-test preparations is complete. This prevents the program from beginning to fill the oxidizer tank or attempt to ignite the motor prematurely. Once the user has acknowledged that they are ready to begin the test, the VI outputs a command signal from the DAQ hardware to a transistor. This, in turn, completes the circuit containing the oxidizer tank fill solenoid and filling commences. Meanwhile, the program monitors the rising oxidizer tank pressure in order to determine whether the tank has reached its maximum limit. The user executing the test has the option of overriding the filling process or aborting the test altogether at any time.

![Image](image_url)

Figure 44   NI LabVIEW Rocket Motor Test Stand DAQ VI Front Page
Once either the user or program has determined that the oxidizer tank has been filled, the fill solenoid circuit is opened, and a ten second countdown commences on the VI front page. When time equals zero, the igniter solenoid circuit is then activated for three seconds to ensure ignition and time continues to count up, all while sampling and recording data. If the user decides to abort the test, both the igniter and fill circuits are opened and the oxidizer dump solenoid is closed, purging the oxidizer from the motor. If the test is successful, the user then clicks the “Test Complete” icon, and selects where the output data is to be saved.

Figure 45 shows the graphical programming structure used to create the DAQ VI. Along the top are structured frames defining the main program sequence. The programming loop in the bottom center defines the real-time data acquisition, display, and abort checking algorithms.

The system developed offers an up-to-date, real-time, simple to use, and expandable hybrid rocket motor data acquisition package. It also provides the Aerospace Engineering Department at Mississippi State University an additional laboratory facility to be used for undergraduate seminar projects and graduate propulsion system-related research.
Figure 45  NI LabVIEW Rocket Motor Test Stand DAQ VI Block Diagram
CHAPTER III

TEST RESULTS

Having prepared the data acquisition system and performance estimates, an experimental performance baseline of the COTS motor and nozzle was attempted. However, due to a manufacturing defect in the aluminum motor casing snap ring grooves, sudden catastrophic failure of the motor ensued during the oxidizer filling phase. Post-failure analysis shows that the defect in the motor case manufacture involved having the internal snap ring groove placed axially too close to the exterior thrust washer snap ring groove. This left too little material to resist the internal pressure of the oxidizer tank. Unfortunately, the case failure damaged the motor beyond repair and without sufficient remaining time in the study to procure a replacement. (Contrail Rockets has accepted responsibility for this defect and its consequences. They have replaced all of the motor components.) Therefore, experimental data of the designed aerospike were not obtained.

To better understand the failure a non-linear finite element analysis (FEA) of the motor has been done. Figure 46 shows the Unigraphics® NX Nastran® FEA mesh grid and von Mises equivalent stress distribution for the failed motor casing in (a) and (b), respectively. The model of the motor case utilized measurements from the casing in order to get an accurate and scale representation of what caused the failure. The wall is 2.35 mm thick 6061-T6 aluminum and the snap ring grooves measure 1.6 mm wide by 0.95 mm deep. A fixed constraint was applied at the upper casing boundary while the
applied load (2400 lbf) was placed upon the lower inside snap ring groove, representing the maximum possible nitrous oxide pressure available (800 psi) at the time of the failure. The fixed constraint was placed sufficiently far away such that any interactions were deemed negligible to the critical solution zone between the snap ring grooves, but not so far away as to create an unnecessarily large mesh and solution. The shallow stress gradient spanning across the motor tube wall thickness indicates that the peak stress in the upper right corner of the inside (lower left) snap ring groove was exceeded; thereby creating a stress riser which in turn zippered across the remaining wall material, severing the pressure vessel containment system.

Figure 46  Failed Motor Casing FEA (a) Grid Mesh, Constraints, and Loading, and (b) von Mises Equivalent Stress Distribution (psi)
Similarly, an FEA of the corrected Contrail Rockets motor casing was completed. Figure 47 shows the mesh grid and von Mises equivalent stress distribution in (a) and (b), respectively. The corrected motor casing has a 5 mm spacing between the interior and exterior snap ring grooves, significantly reducing the induced stress through the wall. Note that although high local stresses are present, a rapidly decreasing stress gradient surrounds the inner snap ring groove. This arrests failure from occurring in the motor casing. This analysis confirms original post-failure suspicions of the original casing being improperly manufactured.

Figure 47 Corrected Motor Casing FEA (a) Grid Mesh, Constraints, and Loading, and (b) von Mises Equivalent Stress Distribution (psi)
CHAPTER IV
CONCLUDING REMARKS

Through the course of the research presented herein, results from analytical engineering concepts and methods coincided with higher-fidelity numerical tools, establishing a useful engineering design or analysis path for aerospike rocket nozzles. As history shows, the ability to accurately estimate or predict performance earlier in the design phase of any project results in lower development costs, thereby greatly increasing the probability for success. Following these lines, the progression from the method of characteristics program to the CFD results yielded significant performance prediction agreement. With experimental data, validation of these methods will be complete.

However unfortunate, the motor case failure presented reminds us of a lesson of great importance – that of safety. Although hybrid rocket motors are considered by many to be the safest of propellant options, the failure that occurred during testing reinforces the respect and vigilance that one must have while working with such dangerous systems.

Future work on this project should include experimental testing for both the COTS and aerospike nozzles. The experimental results and results given herein should then be compared and analyzed. Any discrepancies should be investigated further, and if possible, a set of nozzle simulations containing larger, refined grid meshes should be generated. Flight tests for each nozzle design should also be attempted.
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Mathcad, Software Package, Ver. 13.0, Mathsoft Engineering & Education, Inc.


Unigraphics NX 5, Software Package, Ver. 5.0.0.25, Siemens Product Lifecycle Management Software, Plano, TX, 2007.


APPENDIX A

MATHCAD PLANAR AEROSPIKE NOZZLE DESIGN PROGRAM
Planar Aerospike Nozzle Design Program

\[ A_h^{\text{design}} = 18 \]

Design Altitude

\[ W_{\text{engine}} = 1ui \]

"Width" or Length of Engine

\[ L_{\text{throat}} = 1.4\text{mm} \]

Throat height (opening measurement)

\[ \gamma = 1.26 \]

Ratio of Specific Heats

\[ R_{\text{gas}} = 320 \frac{\text{J}}{\text{kgK}} \]

Gas Constant

\[ M_a = 2.933 \]

Design Exit Mach Number

\[ n_c = 30 \]

Number of Coordinates to generate

---

**Earth Atmospheric Model**

NASA Glenn Research Center

Earth Atmospheric Model

http://www.grc.nasa.gov/WWW/K-12/airplane/atmos.html

\[ T_E(h) = \begin{cases} 
205.05 + 0.00164 h + 459.7 & \text{if } h > 82345 \\
-70 + 459.7 & \text{if } 36152 \leq h \leq 82345 \\
59 - 0.00356 h + 459.7 & \text{if } h < 36152 
\end{cases} \frac{\text{R}}{\text{lb} \cdot \text{ft}^2} \]

\[ p_E(h) = \begin{cases} 
51.57 \left( \frac{T_E(h)}{389.98 \text{R}} \right)^{-11.332} & \text{if } h > 82345 \\
473.1 \cdot \exp(1.73 - 0.000046 h) & \text{if } 36152 \leq h \leq 82345 \\
2116 \left( \frac{T_E(h)}{518.5 \text{R}} \right)^{5.256} & \text{if } h < 36152 
\end{cases} \frac{\text{lbf}}{\text{slug}} \]

\[ p_E(h) = \frac{p_E(h)}{1718 \cdot T_E(h)} \frac{\text{slug}}{\text{ft}^3} \]

\[ P_{\text{design}} = P_E(A_h^{\text{design}}) \]

\[ T_0 = T_E(A_h^{\text{design}}) \]

**Earth Atmospheric Model**

\[ P_{\text{design}} = 14.7 \text{ psi} \]

Ambient Design Condition Pressure based on Design Altitude

---

If desiring to design an engine for outer space application, enable these evaluations and input desired values.
\[ T_0 = 5400 \, \text{R} \quad \text{Ambient Design Condition Temperature based on Design Altitude} \]

\[ P_0 = P_{\text{design}} \left( 1 + \frac{\gamma - 1}{2} M_e^2 \right)^{\frac{\gamma}{\gamma - 1}} \quad \text{Stagnation Pressure of Combustion Chamber} \]

\[ T_{\text{exit}} = \frac{T_0}{1 + \frac{\gamma - 1}{2} M_e^2} \quad \text{Stagnation Temperature of Combustion Chamber} \]

\[ A_{\text{star}} = L_{\text{throat}} \cdot W_{\text{engine}} \quad \text{Nozzle Throat Area} \]

\[ m_{\text{dot}} = \frac{P_0 A_{\text{star}}}{\sqrt{T_0}} \left( \frac{\gamma}{\rho_{\text{gas}}} \left( \frac{2}{\gamma + 1} \right) \right)^{\frac{\gamma + 1}{\gamma - 1}} \quad \text{Mass Flow Rate of Engine} \]

\[ a_e = \sqrt{\gamma R_{\text{gas}} T_{\text{exit}}} \quad \text{Speed of Sound in Exhaust} \]

\[ V_e = M_e a_e \quad \text{Exit Velocity} \]

\[ \text{Design Thrust} = m_{\text{dot}} V_e \quad \text{Designed Thrust Output at Design Condition} \]

\[ T_{\text{SFC}} = \frac{m_{\text{dot}} \delta}{\text{Design Thrust}} \quad \text{Thrust-Specific Fuel Consumption} \]

\[ I_{\text{sp}} = \frac{1}{T_{\text{SFC}}} \quad \text{Specific Impulse} \]

\[ P_0 = 578.5 \, \text{psia} \quad \text{Required Combustion Chamber Pressure} \]

\[ \frac{P_0}{P_g(\Omega)} = 39.33 \quad \text{atmospheres} \]

\[ T_0 = 5400 \, \text{R} \quad \text{Stagnation Temperature of Combustion Chamber} \]

\[ T_{\text{exit}} = 2531 \, \text{R} \quad \text{Exhaust Gas Temperature} \]

\[ A_{\text{star}} = 0.0551 \, \text{in}^2 \quad \text{Nozzle Throat Area} \]

58
\( \dot{m} = 0.00655 \ \text{slug/s} \) Mass Flow Rate of Engine

\( \text{Design Thrust} = 47.7 \text{lbf} \) Designed Thrust Output at Design Condition

\( \text{TSFC} = 0.00441 \ \text{lb/lb} \) Thrust Specific Fuel Consumption

\( I_{sp} = 227 \text{ s} \) Specific Impulse

**Nozzle Design**

\[
v(M) = \sqrt{\frac{\gamma + 1}{\gamma - 1}} \cdot \text{atan}\left(\frac{\gamma - 1}{\gamma + 1} \cdot \sqrt{M^2 - 1}\right) - \text{atan}(\sqrt{M^2 - 1}) - \text{atan}\left(\frac{L_{\text{thrust}}}{\text{STI} \cdot \text{of} \cdot L_{\text{thrust}}/\text{in}}\right)
\]

\[
v(M_0) = 57.461 \text{ deg} \quad v_{\text{final}} = v(M_0) \quad \text{Total Turn Angle}
\]

\[
F_{v}(\dot{\phi}) = \sqrt{\frac{\gamma + 1}{\gamma - 1}} \cdot \text{atan}\left(\frac{\gamma - 1}{\gamma + 1} \cdot \sqrt{2x^2 - 1}\right) \quad \text{atan}(\sqrt{2x^2 - 1})
\]

*Given:* \( F_{v}(\dot{\phi}) - v = 0 \quad M_v(x, v) \Rightarrow F_{v}(\dot{\phi})

\[
x_{y_{0,0}} = 0 \quad x_{y_{0,1}} = 0
\]

\[
x_{y_{1,0}} = -\cos(v(M_0) - 90 \text{ deg}) \cdot L_{\text{thrust}} \quad x_{y_{1,1}} = \sin(v(M_0) - 90 \text{ deg}) \cdot L_{\text{thrust}}
\]

\[
\theta_{\text{init}} = -v_{\text{final}} \quad \theta_{\text{init}} = -57.461 \text{ deg} \quad \nu_{\text{init}} = 0 \text{ deg} \quad \Delta \theta = \frac{v_{\text{final}}}{nc - 1} \quad \Delta \theta = 1.98 \text{ deg}
\]

\[
i = 1, nc
\]

\[
\theta_i = \theta_{\text{init}} + \Delta \theta \cdot (i - 1) \quad v_i = v_{\text{init}} + \Delta \theta \cdot (i - 1) \quad M_i = M_{v1, v1} \quad \mu_i = \sin\left(\frac{1}{M_i}\right)
\]

\[
C_i = \theta_i - \mu_i
\]

\[
\delta = \begin{cases} 
\theta_i & \text{if } i = 1 \\
\theta_i - \frac{1}{2} \cdot (\theta_{i-1} + \theta_i) & \text{if } i > 1 
\end{cases}
\]

\[
\text{slope}_{\theta_i} = \tan(C_i) \quad \text{slope}_{\delta_i} = \tan(\delta_i)
\]
\[
xy := \begin{cases}
\text{for } i \in 2..nc \\
xy_{0,0} = x_{y-1,1} + \text{slope}_{i,1} x_{y-1,0} - \text{slope}_{i,0} x_{y,0} \\
x_{y,1} \leftarrow xy_{0,1} + \text{slope}_{i,0} (xy_{i,0} - xy_{0,0}) \\
x_i \leftarrow xy_{i,0} \\
y_i \leftarrow xy_{i,1} \\
ex_{i+1} = 0.5 \max(x) \\
exy_{i+1} = 0
\end{cases}
\]

\[
\text{for } i \in 2..nc \\
x_{2,i} \leftarrow xy_{0,0} \\
x_{2,i+1} \leftarrow xy_{i,0} \\
y_{2,i} \leftarrow xy_{0,1} \\
y_{2,i+1} \leftarrow xy_{i,1}
\]
APPENDIX B

TRIPOLI MOTOR TESTING RESULTS FOR COTS NOZZLE
January 22, 2007

Mr. Thomas Sanders  
49 North Blvd., Suite #2  
Lake Havasu City, AZ 86403

Dear Mr. Sanders,

The Contrail Rockets J245BG rocket motor was tested on 28-29 January 2006 and is in compliance with the NFPA 1125. The motor is certified indefinitely (Review due, Jun 2011) for hobby rocketry use by the members of the Tripoli Rocketry Association and any associations holding a reciprocal certification agreement.

<table>
<thead>
<tr>
<th>Motor Manufacturer</th>
<th>Contrail Rockets</th>
<th>Test Date</th>
<th>28-29 Jan 06</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor Designation</td>
<td>J245DG</td>
<td>Certified Until</td>
<td>Indefinitely</td>
</tr>
<tr>
<td>TMT Metric Design</td>
<td>J245 (0%)</td>
<td>Samples per Second</td>
<td>480</td>
</tr>
<tr>
<td>Metric Dimensions</td>
<td>54 X 719 MM</td>
<td>Burn Time</td>
<td>2.62 seconds</td>
</tr>
<tr>
<td>Total Weight</td>
<td>1550 g</td>
<td>Total Impulse</td>
<td>943.5 N</td>
</tr>
<tr>
<td>Recovery Weight</td>
<td>1451.5 g</td>
<td>Maximum Thrust</td>
<td>581.13 N</td>
</tr>
<tr>
<td>Fuel Grain Weight</td>
<td>450 G</td>
<td>Average Thrust</td>
<td>245.3 N</td>
</tr>
<tr>
<td>Nitrous Oxide Volume</td>
<td>530 cc</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Sincerely,

H. Paul Holmes  
Tripoli Motor Testing Chair
APPENDIX C

DATA ACQUISITION SYSTEM, GROUND SUPPORT HARDWARE
INTEGRATION ELECTRICAL SCHEMATIC