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Improving a second grade student's number sense: an instructional intervention

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IMPROVING A SECOND GRADE STUDENT'S NUMBER SENSE: DEVELOPING AN INSTRUCTION INTERVENTION

By

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The purpose of this qualitative case study was to help a second grade student, who struggled with mathematics but excelled in reading, to develop a conceptual understanding of number sense, using a teacher researcher-created intervention. The five-step, one-on-one intervention included the following: (1) use trade books to build mathematical knowledge and vocabulary (2) teacher modeling of concepts, (3) guided practice with manipulatives, (4) review using games and a "Fact Pack", and (5) journal writing to explain concepts. The Early Mathematics Assessment-3 (TEMA-3) was used as a pre- and post-test assessment the student's mathematical knowledge. Other data included transcriptions of audio taped intervention sessions, notes from video-taped intervention sessions, fieldnotes, and artifacts from the classroom and intervention sessions. Data sources were triangulated. On the TEMA-3 Pretest, the student scored at the first grade, fourth month (1.4) level. After 13 Lessons covered in 22 sessions (approximately 11 hours of one-on-one instruction), the student scored at the second

grade, second month (2.2) level. She also scored Proficient on the state curriculum test in mathematics. Four aspects of the intervention seem to help the student most in her development of number sense. They included the use of (1) a number line; (2) a number structure, which visually depicted the value of numbers; (3) trade books to provide an anchor for each skill and a memorable context; and (4) journaling. In addition, the data revealed that once the student understood the concept of ten’s and one’s, her ability to count and add extended to include numbers 1 to 100. Recommendations for students who excel in reading and writing, but struggle with mathematics, include the following: the use of trade books and writing may help them better understand mathematics concepts; review of mathematical concepts through enjoyable, meaningful games and the use of a Fact Pack are useful; the use a horizontal number line and number structure, which is consistent with left-to-right directionality of reading and writing, may help students better understand the concepts of more, less, before, and after; and consistent use of vocabulary during instruction may help students better understand number concepts.

DEDICATION

I would like to dedicate this research to my neighbor Francis McFarland, an outstanding writer, who helped me develop my writing skills when I was younger. Mrs. Mac spent many hours editing all my papers in high school as well as college and gave much encouragement.

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I would like to express my utmost appreciation for Dr. Dana Franz, my professor and committee member, for her constant encouragement, mathematical expertise, and her ideas developing my intervention framework. She dedicated many hours to helping me design my intervention and develop my mathematical ideas throughout my time at Mississippi State.

I would also like to thank Dr. Vivian Miller for serving on my committee and taking the time to share her knowledge in mathematics, benefiting my study tremendously.

Finally, I would like to thank my parents, for supporting me during these last four years I have been in school, financially and emotionally. I want to thank my giving mother for modeling great teaching, encouraging me to grow as an educator, and spending many hours reading my thesis. Thanks goes to my dad for his loving support,

for believing I could do this, insisting I continue my education and being the computer expert of the family.

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INTRODUCTION

Background of the Study

Students' mathematical knowledge begins at birth (Gersten, Russel, Chard, and David, 1999 & Griffin, 2004) and as they progress through pre-kindergarten to second grade. The National Council for Teaching Mathematics (NCTM) believes this is when students actually develop their foundation in mathematics (NCTM, 2000). Typically students progress through a natural stage of development in mathematics, first recognizing differences between two objects and three objects almost at birth, to understanding place value by age six (Griffin, 2004).

When students arrive at school, it is easy to recognize those who have number sense (Gersten, Russel, Chard, & David, 1999). Students that have good number sense usually recognize patterns, can invent different ways for finding an answer, recognize quantities in the real world, and recognize the magnitude of numbers (Gersten, Russel, Chard, & David, 1999). Students who do not have core concepts by age five or six typically struggle with mathematics.

One cause of lack of number sense is socio-economic status (Griffin, 2004). Students who come from low-income families are usually two years below middle in-come families (Griffin, 2004). Gersten, Jordan, and Flojo (2005) state that 96% of high-socioeconomic students could answer which number was bigger at the age of six, while only 18% of low-socioeconomic students could answer the question correctly. This proved number sense could be taught because the high-socio-economic students had exposure to these concepts (Gersten, Jordan, & Flojo, 2005; Griffin, 2004).

Gersten, Russel, Chard and David (1999) state that number sense activities in early instruction reduced failures in mathematics and if they were focused on in the beginning of math instruction, many students who struggle in mathematics would profit. Before instruction can take place, the depth of a child's number knowledge needs to be assessed. Various assessments of a child's number sense are available including *The Knowledge Number Test* (Gersten, Jordan, & Flojo, 2005) and the *The Early Mathematics Assessment-3* (*TEMA-3*) (Ginsbrg & Baroody, 2003).

Number sense is the key ingredient in the complex world of arithmetic giving students "fluidity and flexibility with numbers" (Gersten, Russell, Chard, & David, 1999, p.3) meaning instruction is needed for every student, including struggling students. Most students come to school with an informal knowledge of numbers from parents and siblings, from activities such as playing hopscotch or setting the table for dinner (Gersten, Russel, Chard, & David,1999). Other students who did not acquire this from home need formal instruction.

Students struggling with number sense need to understand the value of numbers (Gersten, Russel, Chard, & David, 1999) using various strategies to solve problems (Wright, Martland, Stafford, and Stanger, 2002) with fluency. In order to achieve automaticity in problem solving, researchers recommend teaching with manipulatives (NCTM, 2000), a number line, and games (Gersten, Jordan, & Flojo, 2005), as well as having students write (Burns, 1988; Yang, 2003) and draw (Yang, 2003) about what they are learning. To assess strengths and weaknesses of problem solving, researchers recommend using daily observations, classroom assessments, formal assessments, in-class work, and informal assessments in class (Sherman, Richardson, & Yard, 2005).

To help guide instruction for all students, especially those who struggle, the National Council for Teaching Mathematics (2000) set six principles for schools in America. The first is the equity principle in which excellence and high expectations are exhibited for students, no matter their background, personal characteristics, or physical challenges (NCTM, 2000).The second principle involves the curriculum, stating that it should be interconnected and cumulative, preparing students to solve problems in various situations such as school, home, or in the work place. Teachers are addressed in the third principle, where NCTM calls for a deep understanding of mathematics and constant professional development, including collaboration, analyzing, and discussing with other teachers as well as observing other teachers. The fourth principle, the learning principle, states “students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge” (p. 20). The fifth principal addresses assessments. According to NCTM, assessments should be on-going, giving information not only to teachers, but to the students as well, while also supporting their learning. The final principle is the technology principle, calling for the use of computers and calculators for learning mathematics (NCTM, 2000).

As I sought to teach, using the principles and standards set forth by NCTM, I realized I was unprepared to do so. When I completed my bachelor’s and master’s degrees, the majority of my classes were dedicated to reading instruction. To receive my bachelor's degree, I was required to take 12 hours in reading methods, plus an extra nine hours for a reading add-on certificate and only six hours in math methods and nine hours in math content. My master's degree in reading had 24 hours of reading instruction, with no instruction in mathematics. The Educational Specialist degree I was working on when I did this research consisted of 12 hours in reading instruction and three hours of instruction in mathematics. That totals 57 hours of instruction on teaching reading and 15 hours of instruction on teaching mathematics, which parallels the research on students who struggle in mathematics versus students who struggle in reading, which is six to one (Gersten, Russel, Chard & David, 1999, p. 25; Robinson, Menchetti, &Torgesen, 2002).

Due to a small number of courses in the field of mathematics, I lacked knowledge about teaching mathematics. The one thing I retained from courses was the need to use manipulatives (Burns, 1992; NCTM, 2000). However, when I initially tried this in my second grade classroom, I had little success. Several things probably contributed to my lack of success. First, I did not have enough manipulatives for all the students, so students were forced to share. When I did have enough manipulatives, my students threw them or played with them instead of using them to solve problems. The times they were not playing with them, the manipulatives seemed to confuse them more than help them. I soon put away the manipulatives and taught mathematics the way I was taught, with worksheet after worksheet, knowing researchers had found better ways.

While pursuing my Educational Specialist degree and working outside the elementary school classroom, I observed two different first grade teachers who used manipulatives and taught, following the NCTM principles and standards for math instruction. The teachers modeled how to use manipulatives and gave time at the beginning of each year for the students to learn the correct way to use the manipulatives. The teachers modeled problem solving each day while the students sat around the edge of a rug and watched their teacher solve math problems with the manipulatives. The teachers communicated their thoughts, proved their answers, and asked the students to communicate with partners what they saw and were learning. Each child could see how the teacher was using the manipulatives and could participate through conversation, while the teacher modeled problem-solving with the manipulatives. The teacher also allowed the students to work with manipulatives in small groups while she walked around the room and guided the students' practice. Each child had the opportunity to problem solve using manipulatives with teacher supervision, recommended by leading math researcher Marilyn Burns (1988).

When I returned to the classroom in January of 2005, I mirrored my mathematics instruction after these two teachers and tried in incorporate the standards set by NCTM using a gradual release of responsibility model of teaching (Pearson & Gallagher, 2003). Though most of my students succeeded, one student continued to struggle with mathematics in my classroom.

Participant Observation and Selection

Amy (pseudonym) was a first grade student of mine who looped to second grade with me. She completed the assignments each week enough to pass the math test at the end of unit, but struggled throughout the week in her attitude and her class work. As I modeled and provided guided practice, she usually responded with the wrong answer or did not respond at all. Her behavior during reading lessons differed greatly; generally she eagerly raised her hand and participated in class discussions. During small group math work, however, she constantly required my help or that of a classmate. While she worked on her independent work, my assistant could hardly leave her side. Amy's mother worked with her each night and Amy returned her homework each morning. After much assistance, however she usually got a B or C on the Friday test and then could not complete a similar problem a few weeks later.

Purpose of the Study

The purpose of this study was to help Amy develop her conceptual understanding of number sense using a researcher-created intervention. The intervention was designed to meet NCTM standards as well as using the premise that building Amy's number sense would improve her overall math skills.

Research Question

The research question that guided my study was: How can I help Amy develop a conceptual understanding of number sense?

Instructional Framework for Intervention

I met with Amy two to three times a week for a series of 13 systematic lessons, over 22 sessions for about 13 hours involving manipulative work. Each lesson followed a systematic five-step process, based on standards of the NCTM (2000).

Step One involved Building Mathematical Knowledge and Vocabulary through Literature, where the student had little responsibility, except for listening or reading a book. The book modeled mathematical language and concepts in order to help Amy "recognize and apply mathematics in the contexts outside of mathematics" (NCTM, 2000, p. 64) and helped her to "translate mathematical representations to solve problems" (NCTM, 2000, p. 65). I began with a children's trade book to help build Amy's schema, help her relate mathematics to the world around her, see how mathematics is everywhere, not just digits on paper, and see how it is purposeful in life (Isenbarger & Baroody, 2001). The children's trade books many times contained the vocabulary articulated in mathematics such as more or less or were counting books that would help Amy to see numbers that were all around her, such as in nature or even the food that she ate. Hearing the vocabulary and seeing mathematics used in real-life situations such as grocery shopping, buying a present or counting groups of people would hopefully give mathematics a purpose and anchor mathematics vocabulary and skills to something concrete.

Step Two involved modeling realistic problems, usually taken from the text used in Step One, where the teacher had all of the responsibility and thought aloud how to solve the problem. NCTM (2000) urges teachers to use daily life situations to create mathematical problems for students to solve. Using the book as a setting for a problem allowed me an opportunity to model how I would solve the problem and helped me demonstrate the correct mathematical computations. NCTM (2000) encourages solving problems to solidify and extend students' knowledge in math. As students solve problems, NCTM wants them to reflect on their actions, organize them, and also reflect on their learning. Modeling allows the student to see problem solving and reflecting going on in hopes she will translate this into her own learning.

Step Three involved practicing with manipulatives to solidify her knowledge. The student began gaining responsibility while the teacher still possessed some responsibility as well. NCTM (as cited in Moyer, 2001) states that fluency comes from understanding and understanding is broadened when students can use concrete materials, such as manipulatives, to understand abstract mathematical concepts. Manipulatives were used in every problem and then gradually taken away as concepts became more automatic and the student was able to visualize a number line. Phillips and Crowell (1994) as well as Tarver and Jung (1995) found it critical that a mental number line was used for first grade students solving problems involving addition and subtracting, two areas Amy struggled with.

In Step Four, Amy reviewed concepts using board games, teacher-created games, and a “Fact Pact,” consisting of vocabulary words she was learning as well as math facts she was trying to become automatic with. During this time, the student worked independently, but she also had feedback on her answers. Pellegrino and Goldman (1997) found that extended practice with math facts led to automaticity allowing more attention for the student to complete other activities while trying to solve problems. If Amy was automatic with vocabulary and facts, she would be more likely to have the ability to apply her knowledge to different contexts and solve various problems.

Finally, Step Five, included journaling. Journaling allowed Amy to put mathematical concepts in her own words in order to solidify what she had learned using written language and to allow the researcher to assess Amy’s understanding of what had been taught. Amy applied her knowledge mostly in real life questions. NCTM has four goals for students as they master the process standard. These goals consist of having students organize their thoughts, communicate their thinking, analyze and apply strategies, as well as using mathematical language (NCTM, 2000). The journaling the researcher assess if Amy could do this.

Assumptions

Based on my interactions from January to December 2005, as Amy's first and second grade teacher, I believed that Amy did not have the foundation needed in number sense. She lacked knowledge of the value of numbers, prohibiting her from using problem solving skills or understanding mathematical phenomena. Amy could memorize digits but because she did not know the value of digits and could not grasp the concept of smaller and larger numbers, it was difficult for her to understand addition and subtraction. This also hindered her understanding of time, money, and the capacity of objects (such as a cup of water or two cups of water) because numbers were only digits, not values.

Amy had already received instruction on number values in kindergarten and first grade, but did not appear to understand. I was not sure why, but I believed it may have had to do with an emphasis on memorizing the digit and not having enough opportunities to work with manipulatives when learning the value of numbers. This is something most students grasp in kindergarten or early first grade and because she did not understand this, she was unable to apply strategies or comprehend addition and subtraction because she did not understand if the number was increasing or decreasing in value.

Methodology

In my research, I used a qualitative case study design, characterized as particularistic, descriptive, heuristic, and inductive. It focused on the particular situation in my classroom that involved the common scenario of a child who excels in reading, yet does not in math. I used in-depth descriptions of the intervention sessions and Amy's actions during the interventions to extend other's knowledge about teaching number sense. Finally, I used inductive reasoning to discover new relationships and discoveries involving number sense.

Summary

Chapter I provides the background information that led me to desire to study number sense with this particular student. I discuss my participant and how I selected her as well as the purpose of my study, my research question, and the instructional framework I used in my research. Finally, I discuss the methodology. The definitions of terms and the organization of the study follow.

Definition of Terms

During this study, the following definitions of the term apply.

> <u>Number Sense</u> – An innate knowledge of numbers that is drawn from all the different meanings of numbers (Liedtke & Werner, 1988). "A fixed body of knowledge involving numbers and their manipulation through rules and algorithms" (Griffin, 2004 p. 39). "A child's fluidity and flexibility with numbers, the sense of what numbers mean, and an ability to perform mental mathematics

and to look at the world and make comparisons" (Gersten & Chard, 1999, p. 19-20). "Number sense develops as students understand the size of numbers, develop multiple ways of thinking about and representing numbers, use numbers as references, and develop accurate perceptions about the effects of operations on numbers" (NCTM, 2000, p.80)

Organization of the Study

Chapter II presents my review of related literature and discusses how other educational researchers have come to the conclusions in the math field using research in the field of reading, how other math researchers define number sense and have worked with students that struggle with number sense, including research on the use of manipulatives and the teaching of strategies to gain automaticity as well as the need of verbalization in mathematics.

In Chapter III, I describe why I chose to use a qualitative research design. I describe the school and the participant in detail. I discuss the creation of the intervention as well as my use of fieldnotes and teacher reflections. I also describe how I collected the data using informal interviews, audiotaping, video taping, and how artifacts were collected throughout the course of the study. I also describe my coding techniques and my final categories and codes. Lastly, I address the ethical concerns of my study.

Chapter IV presents the findings of my research. I describe each intervention lesson and the results of the pre- and post-tests.

In Chapter V, I report my conclusions and recommendations to teachers who are working with children who struggle with understanding number sense. I also suggest possibilities for future research.

CHAPTER II

LITERATURE REVIEW

A major challenge for teachers is to provide effective instruction for students who struggle to learn to read or to understand mathematics concepts. Much research has been conducted to determine useful interventions to support readers who struggle. However, little research has been conducted to help students who struggle with mathematics. *The Journal of Special Education* reports, “the magnitude of research, curriculum development, and training in reading and language arts, as opposed to mathematics, would easily be of the order of 6 to 1” (Gersten, Russel, Chard & David, 1999, p. 25). Gersten, Russel, Chard and David (1999) suggest that research in mathematics, as it relates to number sense instruction, is just as important as research in reading, as it relates to phonics instruction. Furthermore, they posit that number sense lays the foundation for further learning in mathematics, just like the understanding of phonics lays a foundation for learning to read. They hypothesize that if the teaching of number sense was emphasized in the same way that phonics was in reading then mathematics instruction would be dramatically improved.

In this chapter, I explore the definitions of number sense and describe the theories of the development of number sense. I extend the analogous relationship between instruction in number sense and phonics. Furthermore, I review the literature that describes effective methods for teaching number sense and support the components of the instructional intervention I developed.

Number Sense

Griffin (2004, p. 39) defines number sense as “a fixed body of knowledge involving numbers and their manipulation through rules and algorithms.” Gersten, Russel, Chard and David’s (1999) definition adds specifics to the above definition. Number sense is “a child’s fluidity and flexibility with numbers, the sense of what numbers mean, and an ability to perform mental mathematics and to look at the world and make comparisons” (pp. 19-20).

Researchers posit that most students learn number sense early in life (Gersten, Russel, Chard, & David, 1999). Griffin (2004) reports that researchers in the fields of cognitive neuropsychology and infant cognition have found that, “human infants are born with brain structures that are specifically attuned to numerical quantities (p.40). Griffin (2004) describes the development of aspects of number sense in the first seven years of life. Infants can distinguish between two objects compared to several objects as early as the first few days of life. Around five months of age, children show surprise when two puppets behind a screen change to three, which demonstrates they recognize the change and difference between quantities. By six months of age, babies can distinguish between and match three sounds to three objects, making it obvious that the foundation for number sense is created in the first months of a child’s life. As infants become toddlers, they gain language skills and begin associating names with symbols, such as the name “five,” represented by the numeral 5.Toddlers also develop two schemas, one for quantity and one for counting. By age five, these two schemas begin to merge and five-year-olds can count without using manipulatives, and they begin to add. Around age seven, students' knowledge grows more complex, and they can distinguish between tens and ones, time on a clock, and different coin amounts (Griffin, 2004).

Just as children progress in mathematics through different stages, the development of phonological awareness and grapho-phonemic awareness progresses systematically. Gersten, Russel, Chard, and David (1999, p.1) state "the most notable advances in the learning disabilities field since the late 1970s have been in reading disabilities," which are the result of developing understandings about the importance of phonological awareness and how it contributes to building strong reading skills. Bear and Barone (1998) suggest that readers first hear sounds and then recognize that words are made up of individual sounds. Then they start to match these sounds with individual symbols (i.e., letters of the alphabet). When students are not competent with aspects of this progression, instructional interventions are often initiated. Gersten, Russel. Chard, and David (1999) believe that the development of number sense is as crucial for understanding mathematics as phonological awareness is for reading development.

The Development of Number Sense

The National Council for Teaching Mathematics (Yang, 2003) emphasizes the importance of teaching number sense in elementary schools because of its crucial role in laying a foundation for further development of mathematics. Yang further notes that "number sense is a way of thinking that should permeate all aspects of mathematics teaching and learning" (p. 12). Gersten, Jordan, and Flojo (2005) state that linkages with number sense "serve as essential tools for helping students to think about mathematical problems and to develop higher order insights when working on mathematical problems" (p. 297). They also recommend that it is important to teach number sense, especially the value of numbers and the use of a number line, for students who struggle with mathematics. NCTM suggests that number sense has five components: "developing number meaning; exploring number relationships with objects that can be manipulated; understanding the relative magnitudes of numbers; developing intuitions about the relative effect of performing operations on numbers; and developing referents for measures of common objects and situations in their environment" (Liedtke & Werner, 1998). Gersten, Jordan, and Flojo (2005) add to this with four characteristics of good number sense: estimating and determining value with fluency, the ability to distinguish unreasonable answers, the ability to solve problems mentally, and finally the ability to use strategies and know which is best to use when solving the problem.

Teaching Number (Wright, Martland, Stafford, & Stanger, 2002), a book about teaching number sense, defines six Stages of Early Arithmetical Learning (SEAL) and outlines a similar developmental framework as NCTM for describing early number sense. In Stage 0, referred to as Emergent Counting, students cannot count visible items. In Stage 1, Perceptual Counting, students can count visible items in collections and rows but not those that are concealed. In Stage 2, Figurative Counting, students count all objects in two groups, ignoring the fact that they already know how many is in one group, they count the group again.. In Stage 3, Initial Number Sequence, a child counts on from one number and only counts one group, when they already know how many are in the other group. In addition this is applied when students can count on as they are told the number of objects that are hidden from view In Stage 4, Intermediate Number Sequence, the child counts down to solve missing addends and chooses the most efficient way to solve the problem. Lastly, Stage 5, is Facile Number Sequence. In this stage, a child uses a variety of strategies, including skip counting. At this point, a child can add two digits and understands the inverse relationship of addition and subtraction.

The six learning stages have been correlated with five phases for teaching number sense. The first phase of instruction should be taught to Emergent learners. Instruction should include counting visible items in rows and groups. Perceptual learners move to Phase 2, where instruction would include counting with some items being covered. As learners become figurative learners, they move to Phase 3 of instruction where they learn to count on from a larger number and count backwards. In Phase 4, students learn to skip count and increase their knowledge of numbers to include all numbers to 1,000. In Phase 5, facile learners extend their knowledge to include two digit addition and subtraction and advance all the way to multiplication and division.

In Royer's (2003) book, *Mathematical Cognition*, he describes an additional concept related to number sense that must be taught. He suggests that when teaching about numbers, the goal is to help children understand that numbers represent values rather than a digit. Royer (2003) posits that an understanding of number sense is important because it permits a child to "interpret the world of quantity and number in increasingly sophisticated ways" (p. 11).

Students Who Struggle

Students who do not understand the core concepts described above by age five or six, typically struggle with mathematics. Research (Griffin, 2004) shows that students who come from low-income families are usually two years below middle in-come families in relation to their understandings of number sense. Gersten, Russel, Chard and David (1999) state that number sense activities in early instruction reduce failures in mathematics and if they were focused on in the beginning of math instruction, many students who struggle in mathematics would profit.

Gersten, Russel, Chard, and David (1999) and Kulak (1993) found a relationship between students with reading disabilities and students who struggle in math. They liken the learning of number sense to learning phonics. The International Reading Association states, "Teaching children to read involves more than helping them to recognize the combinations of sounds and letters that make up individual words. Understanding the meaning of text—words, numbers, and images, in print or in digital form—is a no less critical part of what it means to be literate in today's society" (IRA, 2005, p.1). Gersten, Russel, Chard, and David (1999) suggest that just as phonological awareness contributes to, but is not sufficient to guarantee fluent reading and comprehension, number sense is necessary, but not sufficient, for problem solving. In other words, students may identify the numerals 1, 2, and 3, but without an understanding of the value of the numbers, the numerals are merely squiggles on a page. Therefore, researchers, such as Gersten, Russel, Chard, and David (1999), conclude that if the teaching of number sense was emphasized in beginning math instruction like phonics is emphasized in early reading instruction, students with learning disabilities would benefit.

When conducting my own classroom research, typically the poor readers are weak in phonemic awareness. They also lack knowledge of grapho-phonic relationships (sound-letter relationships), which makes it difficult for the students to decode words and as they labor over the sounds of letters, they generally miss the entire purpose of the passage. Just as comprehension is one of the purposes in reading, math also requires comprehension. The understanding of numbers is not that two circles in the numeral are eight, like the letters c-a-t spells *cat*, but the *value* of the numeral is what matters (Royer, 2003). Gersten, Russell, Chard, and David (1999, p. 5) report the “focus on the devastating effects of weak automaticity on the ability to solve problems and understand mathematical concepts is a direct parallel to the reading research of the early 1980s, which demonstrated that students who are slow or plodding decoders tend to be poor comprehenders.”

Making the comparison of number sense and phonemic awareness, researchers see the need for number sense to be taught and the question arises, why do students struggle with this concept? In the book, *Teaching Children Who Struggle with Mathematics*, Sherman, Richardson, and Yard (2005) identify environmental factors and personal factors that may hinder the development of a child’s number sense. One environmental factor that may prohibit a child from developing number sense in early education is instruction that does not provide challenging questions, problem solving, reasoning, and connections to real-world situations (Sherman, Richardson, & Yard, 2005). Students who are simply taught to memorize, usually have difficulty recognizing and retaining math concepts. Another environmental factor is curricular materials. Often students who struggle are taught to memorize basic facts or procedures without understanding the concept, narrowing their view of mathematics and limiting their opportunities to reason and problem solve. The final environmental factor is the gap between the student and the material being taught. The mathematics content may be unrelated to the students for many reasons, such as a mismatch with a student's ability level, the student does not have the life experiences with a math concept, or the student may not attend school regularly (Sherman, Richardson, & Yard, 2005).

Sherman, Richardson, and Yard (2005), suggest that personal factors, which hinder a child's development of number sense include locus of control, a child's memory, attention span, or their understanding of the language used in mathematics. Gersten, Jordan, and Flojo (2005) add socio-economic status to the list of personal factors, stating that 96% of high-socioeconomic students could answer the question "which number was bigger?" at the age of six, while only 18% of low-socioeconomic students could answer the question correctly. This finding substantiated the assumption that number sense can be taught because the high-socio-economic students had exposure to these concepts (Gersten, Jordan, & Flojo, 2005).

Students with Math Disabilities

Although, students who struggle with number sense could have a disability, this is not always the case (Kulack, 1993; Mazzocoo & Myers, 2003). Some researchers believe students may simply be delayed in their development of mathematics concepts. Kulak (1993) defines a delay as when students process information in a way that is quantitatively different from other students, but they continue to go through the same stages as other students. Students with a disability demonstrate a qualitative difference in the way they learn math (Kulak, 1993). In Gersten, Russel, Chard and David's study (1999), they found that a delay may stem from a low socioeconomic status. It has been postulated that students who struggle may need more exposure to number sense because of the lack of exposure in their early years (Gersten, Russel, Chard & David, 1999; Griffin, 2004; Kulak, 1993).

Number Sense Instruction

Because number sense is so important, NCTM has called for instructional reforms (Isenbarger & Baroody, 2001) in a way educators teach number sense. Sherman, Richardson, and Yard (2005), suggest three of the six NCTM Principles to help increase number sense: the Equity Principle, the Teaching Principle, and the Learning Principle.

The Equity Principle includes the concept that all students can learn and need high expectations and strong support (NCTM, 2000; Sherman, Richardson, & Yard, 2005). Accommodations could include activities with manipulatives, games, or more time on specific tasks (NCTM, 2000; Sherman, Richardson, & Yard, 2005). Gersten, Jordan, and Flojo (2005) add to this list, the use of a number line, mathematical language, and direct instruction.

Phillips and Crowell (1994) as well as Tarver and Jung (1995) found that a mental number line was critical for solving problems involving addition and subtracting for students in first grade. Understanding that one number is closer to another on a number line gives students an ingredient needed to solve basic facts in mathematics (Gersten, Russell, Chard, & David, 1999)

Claudia Zaslavsky (2001) reports in her article, “Developing Number, What Can Other Cultures Tell Us?”, that children acquire number sense as they become familiar with numbers. As children work with the quantity of numbers using kinesthetic materials, the easier it is to recognize and understand the value of a number. Piaget (as cited in Moyer, 2001) also suggests children need concrete materials, such as manipulatives, to understand abstract mathematical concepts.

Cain-Caston’s (1996) research supports working with manipulatives during math instruction. Four third-grade classes were assessed using the California Achievement Test. Statistical tests were not used to determine a statistical difference, but the mean average of the two classes that used manipulatives scored two grade levels above their present grade, while the two classes that used only worksheets scored on grade level.

The Teaching Principle highlights the use of what students know to design curriculum that is authentic and challenging (NCTM, 2000; Sherman, Richardson, & Yard, 2005). Similarly, Yang (2003) found that relating instruction to daily life situations improved struggling students’ ability in mathematics. This relationship between their daily lives and mathematics gives numbers significance and helps students solidify the concept (Yang, 2003). Isenbarger and Baroody (2001) suggest that making mathematics purposeful, using worthwhile tasks makes it more interesting and creates a need to learn, in contrast to the purposelessness and redundancy of math worksheets. They also note that children are more likely to retain information when concepts are taught in a meaningful context rather than strictly through memorization (Isenbarger & Baroody, 2001).

Marilyn Burns (1988, 1995), a leading voice in the math field, recommends making students' learning realistic through the use of children's literature. Literature helps a student to be actively engaged and usually creates a realistic problem for the student to solve. She posits that literature is a bonus and also helps extend mathematics beyond arithmetic skills. These recommendations are consistent with research results related to schema theory in the 1980's. Students' background knowledge was found to be a major determiner of individuals' understandings (Harvey & Goudvis, 2000).

The Learning Principle calls for students to "learn mathematics with understanding, actively building new knowledge from experience and prior knowledge" (NCTM, 2000, p. 20). The goal is for students to make sense of mathematics themselves while answering questions and using strategies to solve problems (Sherman, Richardson, & Yard, 2005). In addition, it is important to teach three basic concepts: quantities, counting, and formal symbols while having the students solve problems (Griffin, 2004).

Marilyn Burns (1988) echoes the need for problem solving in her book, *A Collection of Math Lessons*, emphasizing that math instruction should help children learn to think and reason mathematically. Burns recommends that teachers approach mathematics with a view toward its usefulness. When students need to use concepts and skills they have learned to solve problems, this brings meaning to those concepts. Burns encourages teachers to build their lessons around problem solving and concepts students know and can extend. She recommends that students write or speak about their thinking during problem solving, to use manipulatives, and to work cooperatively. Futhermore, she suggests teachers create lessons that encourage automaticity through thoughtful engagements (e.g., problem solving) instead of worksheets.

Automaticity is helpful when solving problems, because it frees up brain power to solve difficult problems (Gersten, Russel, Chard & David, 1999; Gersten, Jordan, & Flojo, 2005). “The human mind has a limited capacity to process information, and if too much energy goes into figuring out what nine plus eight equals, little is left over to understand [other concepts]” (Gersten, Russel Chard, & David, 1999, p. 21).

The need for strategies in mathematics can be seen easier by looking at the developmental stages children go through when acquiring mathematics strategies. A strategy is defined as “procedures a child might use to solve various kinds of early number tasks” (Wright, Martland, Stafford, & Stanger, 2002, p.7). There are many different kinds of strategies and a particular child may use a multitude of them. According to Kulak (1993), children go through three stages as they master mathematic strategies. The first stage is “counting all” (Kulak, 1993, p. 669): a student counts each manipulative, finger, or picture that represents a number. In the second stage, “counting on,” (Kulak, 1993, p. 669) a student uses one addend (or number) and counts on to the next number (Kulak, 1993). As students advance into the third stage, they use the “min” strategy (Kulack, 1993, p. 669); students try to minimize their counting by counting on from the largest addend. As students progress, they begin to memorize facts using these strategies.

As mentioned earlier, memorizing facts contributes to automaticity. Pellegrino and Goldman (1987) found that a focus on the memorization of facts helped students with mathematical learning disabilities. Extended practice with math facts led to automaticity and allowed students to pay more attention to the actual problem rather than focusing on computations (Pellegrino & Goldman, 1997). However, the memorization of facts without understanding of the concept may also be a hindrance (Sherman, Richardson, & Yard, 2005).

Verbalizing orally or in writing can also improve problem solving skills. Marilyn Burns (1992) found that writing strengthens students' learning by allowing the students to sort out their thoughts, clarify, and define what it is they are thinking. Yang (2005) found diary writing in mathematics developed number sense. Along with the writing, children included their drawings and written communication of how the students solved problems. After the diary writing project, 90% of the students wrote that number sense was important, writing helped them understand there was more than one way to solve a problem, and writing helped them think and reason. Yang (2005) and Burns (1995) note that diary writing help teachers assess students' understanding of concepts.

Assessments

Sherman (2005) recommends assessing students' strengths and weaknesses through daily observations, classroom assessments, formal assessments, in-class work, and informal assessments in class. One formal assessment is the *Number Knowledge Test* (Gersten, Jordan, & Flojo, 2005). *The Number Knowledge Test*, which assesses children's knowledge of basic arithmetic concepts and operations and their depth of understanding. Other tests are available such as the *TEMA-3*, a more extensive test of number knowledge (Ginsburg & Baroody, 2003)

Summary

In summary, through the research of phonemic awareness, we learn that number sense is teachable and students develop it in stages (Gersten, Jordan, & Flojo, 2005; Griffin, 2004). Students struggling in number sense need to understand the value of numbers (Gersten & Chard, 1999), using various strategies to solve problems (Wright, Martland, Stafford, and Stanger, 2002) with fluency. In order to achieve automaticity in problem solving, researchers recommend teaching with manipulatives (NCTM, 2000), a number line, and games (Gersten, Jordan, and Flojo, 2005), as well as having students write (Burns, 1988; Yang, 2005) and draw (Yang, 2005) about what they are learning to solidify problem solving. To assess strengths and weaknesses of problem solving, researchers recommend using daily observations, classroom assessments, in-class work, and informal assessments in class (Sherman, 2005). Various formal assessments of a child's number sense are available including *The Knowledge Number Test* (Gersten, Jordan, & Flojo, 2005) and the *TEMA-3* (Ginsbrg & Baroody, 2003).

CHAPTER THREE

METHODOLOGY

Introduction

In Chapter III, I describe the qualitative case study design used for this study. I explain the rationale for this study and describe the participant. I also describe the intervention used in the study and the data collection methods. Finally, I describe how the data were analyzed, including my coding procedures and the codes and categories I used. Finally, I discuss ethical issues.

Research Design

I used a qualitative case study design to investigate how I could improve a second grader's understanding of number sense, using a researcher-created intervention. According to Merriam (1988), a qualitative case study is appropriate to use when a specific phenomenon, such as a person, is under investigation. The qualitative case study design is usually selected because a situation the researcher has encountered has raised a concern (Merriam, 1998). She also states that the problem may be selected because it is interesting and one may want to gain a greater understanding of the situation (Merriam, 1988). Using the case study method, the researcher gathers data in various ways, including testing, interviewing, and observation (Merriam, 1998). “The case study seeks holistic description and explanation” (Merriam, 1998, p. 10).

Case studies can be defined by four characteristics: particularistic, descriptive, heuristic, and inductive (Merriam, 1998). Particularistic means that the case focuses on a particular situation and it is important because it may reveal something about a situation (Merriam, 1998). In this case study, I focused on trying to understand Amy’s knowledge of number sense and design instruction to improve her understanding of number sense as well as enhance her overall mathematical ability.

Case studies are descriptive because the final product goes in-depth about the situation (Merriam, 1998). For 14 weeks (minus four weeks for spring break, Easter break, state testing preparation, and state testing), I studied Amy’s developing knowledge of number sense while using an intervention. I used many sources of information to gain insight into Amy’s abilities and used thick description to describe the intervention and her developing mathematical understanding.

Case studies are heuristic because they describe the researcher’s understanding of a situation and broaden the reader’s understanding of the participant. In this study, I describe Amy’s developing number sense and provide readers with a new understanding about how to teach number sense, especially to students who struggle.

Finally, case studies are inductive because the researcher relies on inductive reasoning while examining the data. As I analyzed the data, I drew tentative inferences and assigned preliminary codes to the data. My goal was to discover new relationships and understandings about how to effectively teach number sense to a second grade child who struggled when learning mathematics rather than confirm or deny a predetermined hypothesis.

Context of Study

This is a qualitative case study of a second grade student with mathematical knowledge on a beginning first grade level. As Amy's first grade teacher from January 2005 – May 2005, I observed Amy during classroom instruction. During this time, I saw Amy's frustration with mathematics each day and her attempts to mask her lack of skills with stomach aches and headaches. As the months passed, I developed a relationship with Amy and began planning my research. From August 2005 – January 2005, I developed my intervention plan and finalized the design of my case study. I collected data for the study while working with Amy one-on-one using the systematic intervention plan from mid-February 2006 to mid-May 2006, meeting with Amy two to three times a week for about 30 – 45 minutes each session.

In the following section, I describe the context of the study, including the school, mathematics instruction in my classroom and the setting where the intervention took place. I also describe Amy and her knowledge of mathematics at the beginning of the study and the intervention I developed to work with Amy one-on-one.

The School

Fairfield Elementary School (pseudonym) is nestled in northeast Mississippi in a rural community. According to the Mississippi Southern Association of the Colleges and Schools Council and Accreditation and School Improvement 2004-2005 report, the school has approximately 500 students: 256 first graders and 237 second graders. At the time of the study, it had 15 first grade classrooms and 13 second grade classrooms, with the number of students in each classroom ranging from 16 to 24. Because the school has at least 40% of students on free and reduced lunch, it is considered a Title I school, which entitles high poverty schools to extra resources from the federal government to help improve instruction and to insure that lower income and minority students have the same opportunities as other children (MDE, 2003). Approximately 80% of the students in Fairfield Elementary are African-American, 19% are White, and less than 1% are Hispanic (MDE, 2003).

Classroom Instruction

I returned to Fairfield Elementary to teach first grade after being away from the classroom for a year and a half, in January of 2005, when the previous teacher moved out of town. My first grade classroom had one teaching assistant and 22 students: 18 African-Americans and 4 Whites. It had 17 girls and 8 boys. Two-thirds (14 students) had an "A" average in math, five students had a "B" average, and three students had a "C" average at the end of the year. After completing the academic year with these students, my teaching assistant and I looped to second grade with most of the same students. My second grade class consisted of 24 students: 15 of my first grade students and 9 students, who had other first grade teachers. All students were African American and there were 19 girls and 5 boys.

My classroom instruction followed the state-mandated, allotted times for each subject. We spent about 90 minutes on math instruction. The instructional time was divided into three stages: modeling, hands-on small group instruction, and independent practice. I began every math lesson on the rug where I modeled a skill as the students sat around the edge of the rectangular rug and I showed students how to solve problems with manipulatives. During this initial, whole class instruction, I also included time to review basic skills, often using the calendar. Examples of activities include discussions, which stemmed from the day's date and number of days we had attended school. We also reviewed the months, counted how many days we had been in school, found patterns, used coins to show the number of days in a particular month, and other activities. After whole class instruction, the students moved to their desks and began practicing skills independently, using the math workbook provided by the school district. As students worked independently, I called two or three students to various small tables, where I went over instructions for hands-on activities that utilized manipulatives found in plastic containers. Half of the students worked with manipulatives in small groups, playing games and solving problems located in the plastic containers. The teaching assistant monitored the students who were working at their desks, assisting them one-on-one as needed. During this time, I monitored the work of students who were solving problems using manipulatives and assisted them as needed. As one group finished their tasks with manipulatives, I called another group to complete this activity. This process continued until all groups had participated in the hands-on activity involving manipulatives. The small group format worked well because it allowed time for me to meet with a few students at a time. It also gave students hands-on practice with manipulatives each day, allowing me the opportunity to observe and assist students who needed extra help.

Setting of the Intervention

Each intervention took place in the creative writing lab of my school. The room was primarily used for small groups of students to come and publish books using software on the seven computers along the wall. A large table ran across the back wall that was used for putting books together. Another wall housed bookshelves full of books and other supplies used by the creative writing teacher to assemble the books. In the far left corner was a blue rug and a rocking chair where students would gather to read. Amy and I met here for each session. I set up the camera in the far left corner of the rug and it stayed there for the entire time we met with each other. Our supplies stayed on one of the bookshelves.

Selection

Even though several students in my class struggled with mathematics, I could easily determine why they were not excelling, except for Amy. One student who experienced difficulty was pulled out of class for two hours during mathematics instruction to attend a behavior modification program. Consequently, she had to make up the work with me or my assistant during free time. However, there was not enough free time to provide her with the amount of mathematics instruction she really needed. She also struggled in reading, for many of the same reasons. In addition, she did not have anyone to help her at home. The second student struggled in both reading and math and did not pass first grade, which meant she did not loop to second grade with the other students. In addition, this student did not seem motivated to learn nor did she receive help from home. Her parents did not come to school, even to pick up her report cards.

Amy was different from the two students mentioned above. Her mother was very involved in her education. Amy did her homework and performed well in school in reading and writing, but not as well in mathematics. Amy did not demonstrate an understanding of mathematics concepts typically mastered by six-year-olds (See Royer, 2003). At age seven, Amy had not started to integrate her quantity schema with her counting schema and the number line. She had not started to understand that when numbers increased on a number line, the quantity is larger and that as numbers decrease, they are lower in quantity. She had not begun to solve addition or subtraction problems without manipulatives, and she could not solve problems by counting up and down the number sequence. Futhermore, Amy had not begun to solve problems presented in hypothetical situations. Amy was not able to use her counting skills in a wide range of different contexts, such as determining time, the value of money, or the weight of an object.

I approached the special education teacher at our school to find out how I might help Amy. She told me that it was typical for students with a mathematics disability to do well in reading and poor in mathematics. I inquired about special education testing to see if Amy had a mathematics learning disability. The teacher told me that Amy would have to fail mathematics before she could be considered for testing. Because she was not failing, I looked elsewhere for explanations for Amy's difficulties and ideas for how I could help her.

Amy typically made a B or C on her weekly math papers and received extra help almost daily from my assistant. A sample of her weekly grades from the third-nine weeks of first grade are as follows: 75, 85, 90, 78, 88, and 85. Across the four nine weeks of first grade, her math grades were similar--mostly averaging about an 85, which was barely a B on the grading scale which was 100 – 93% A; 92 - 85% B; 84-75% C; 74 – 70 D; 69% or below resulted in failing. Amy seemed to retain information long enough for the test on Friday, but when she needed to recall information from another lesson or for cumulative review tests, she did not do as well. Her three lowest grades above (75, 78, 88) were from math tests that included a cumulative review of skills previously taught.

As I worked with Amy, I realized that she was could work the problems immediately after we studied the skill, but she did not understand the concepts well enough to apply them to other mathematical situations, leading me to believe she had only memorized the procedures for solving the problem, but did not understand the actual concept. Because she did not understand the concept, she would forget the steps to solve the problem and could not use the concept to help her solve other mathematics problems. Amy's difficulties in math were puzzling to me because Amy did not experience difficulties in reading. She had help at home, and I thought she was receiving adequate instruction at school. I thought that one-on-one instruction, which was designed to especially to meet Amy's needs might benefit her.

Description of the Participant

Amy (pseudonym) was seven years, eight months old at the beginning of the study. I had known her one year and one month. I first met Amy when I became her teacher in January of 2005. At the time she was six years old and would turn seven in May of 2005. She was a younger first grade student compared to others in my class, who turned seven in August or September, almost eight months before Amy. The next year, Amy continued to be my student, as she and other students looped to second grade with me as their teacher. Amy lived with her mother and brother, five years her senior. She spent every afternoon after school with her grandmother.

Amy was a petite African-American girl. She was about average height compared to other girls in the class. Her hair was usually arranged in multiple braided ponytails, typically accented with colorful bows at the top of her ponytails to match her outfit. She wore starched school uniforms of khaki pants and a white or navy blue shirt, typically from name-brand fashion stores. Amy wore light pink glasses for far-sightedness, but that was her only existing health condition. She had the typical smile of first and second graders, which constantly changed due to missing front teeth.

Amy had been enrolled in school for five years prior to second grade. She went to two years of preschool, one year of Headstart, one year in public kindergarten, and one year in first grade, where she earned mostly A's and B's and worked very hard.

Amy's mother, who had a high school education, valued education and wanted Amy to do well in school. Her mother regularly visited the classroom and showed a great deal of support for her. Amy's mother was aware of her difficulties in math and approached me weekly about how she was doing. Her mother also expressed her concern to me about how she could help Amy more with her math homework each night.

Amy's mother brought her to school each day and walked her to the room, which was unusual because most students rode a bus or their parents dropped them off at the front door of the school. Every day Amy's mother came to the room, she spoke to both my assistant and me, sometimes staying up to 30 minutes and other times staying long enough to just say a quick hello. Every Friday I sent spelling words home for students to study and each Monday morning Amy brought back her homework, which was correct and completed in her neatest handwriting. Each night she read a book with her mother and completed a math homework worksheet.

Amy came to school each morning with a smile on her face. She also smiled often during the day and usually asked to sit by me at lunch. She never entered or left the room without giving me a hug. She followed the rules of the classroom, rarely breaking them, unless she was talking during inappropriate times. Amy was well-liked among her classmates, with no particular group of friends. She seemed to be friends with everyone, choosing to play with different groups when they played outside. She had one best friend who she liked to sit with at lunch and play with at recess, but the two of them played with many others on the swings, when writing on the sidewalk with chalk, when cheering in the open field or just playing on the jungle gym. When allowed to sit by Amy in class, her friend would often help her with her independent seat work. As our seating arrangements changed, other students also helped Amy with her math assignments.

In first grade, Amy was in the most advanced reading group in my class. She did not struggle with any reading skill and was reading above first grade level according to Accelerated Reading STAR Test. She scored an “A” on her reading test each week and seemed to enjoy reading, often choosing to read on her own when she finished her work early. During reading she raised her hand to come to the board, answer questions or to read aloud for the class. In first grade, she knew all of the required sight words and showed signs of possessing a well-developed reading and writing vocabulary. At the end of first grade she read about one grade level above first grade level, according to the Analytical Reading Inventory and the Accelerated Reading STAR Test.

Amy met her Accelerated Reading goal each nine weeks of first and second grade and met the school goal for points each nine weeks of both grades. At the end of second grade, she read over a year above second grade level according to the STAR Test.

While working on math-related tasks in the classroom, Amy depended heavily on her classmates. I sat with her most of the time when her small group was playing games using manipulatives and re-taught skills she just did not seem to grasp. When she was not working with me, she spent all of her independent time with my assistant. There were three students who typically needed extra help, and my assistant worked with them as soon as whole class instruction was over. If Amy was left to work alone, she usually put her head down, cried, or asked to go to the restroom and stayed gone for long periods of time. If she did attempt the work, she usually answered most of the problems incorrectly. Oftentimes her answers were not logical. For example, when adding two numbers, the sum would be smaller in value than one of the digits she was adding.

During calendar math in first grade, I noticed a change in Amy's normally happy and eager demeanor. More days than not, she sat on the rug and stared out the window. This demeanor continued into second grade and was a problem most days. During this time, she rarely raised her hand to answer mathematical questions, and when her name was randomly chosen, she just sat there and did not answer the question. Similarly, if I called on her to see if she was listening, she would sit and look at me, or if she did attempt to answer, the answer was usually wrong.

In the classroom, I spent a week teaching a new skill or concept. The next two weeks, we reviewed the concepts, and students practiced the skill independently. After working three weeks on a particular mathematical concept, Amy usually made an average grade on her test--a C and sometimes a B. However, Amy did not demonstrate understanding of these concepts after a test. I noticed that if there was a question that reviewed a concept on a daily practice worksheet or if I asked Amy a question that required her to use a concept she had learned previously, most times she could not complete the problem, even though she may have answered similar problems correctly on the weekly test. She did not seem to remember a particular math concept after she was tested on it.

The Creation of the Intervention

In the spring of 2005, I was enrolled in a graduate course that focused on math instruction for elementary school-age children. The course was grounded in the NCTM standards, with many applications and lessons by Marilyn Burns. I discussed my concerns and questions about Amy and her struggles with my professor. I asked her if there were any intervention programs designed for students who struggle with mathematics, yet excel in reading. She was unaware of any, so we looked at several methods textbooks. We were unable to find any useful information. Knowing my Master's degree was in reading education, she asked what I would do with a student who struggled in reading. I explained that I would begin with a read aloud, after building background knowledge and introducing vocabulary related to the book. A phonemic awareness or phonics lesson, based on the student's needs, would follow and then the student would read a text on his or her reading level. From that text, I would choose new sight words to teach the student in isolation, due to the inability to decode the words. Then, we would review sounds and words the student had learned previously, or choose a familiar book to reread for fluency work. To conclude the lesson, the student would write in a journal about a self-selected topic. I would use the journal writing to assess the student's application of word attack skills he or she had learned previously and to determine what skills the student still needed to learn.

My professor took notes as I described the reading intervention and then suggested I consider developing a math intervention using a similar format and applying what NCTM and mathematics professionals recommend for effective mathematics instruction. Because Amy excelled in reading, I thought that integrating books into a math intervention would build on Amy's strengths.

Instructional Intervention

The instructional intervention I developed followed a systematic, five-step process, based on standards of the National Council of Teachers of Mathematics and explained in *Number and Operations* section of *Principles and Standards for School Mathematics* (NCTM, 2000).

Step 1: Building Mathematical Knowledge and Vocabulary Through Literature. NCTM (2000) states, "Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge" (p.75). To help Amy make connections with her life (Burns, 1998) and to understand how mathematics is relevant to her life, I selected children's trade books that used mathematics vocabulary and content that would help Amy develop number sense. NCTM (2000) states that making mathematics real for children helps them to see a need for math and understand it better. In addition, literature provides a context in which characters and readers use math concepts in situations outside a school context. I also chose to use trade books to link math to reading, which was a pleasurable activity to Amy and one in which she excelled.

During Step 1, I read a trade book to Amy, helping her “recognize and apply mathematics in the contexts outside of mathematics” (NCTM, 2000, p. 64). The purpose was to expose her to mathematics vocabulary (thus building background knowledge) and show her how mathematics could be used in a realistic context, so she could understand the usefulness of mathematics. It also gave a context for a mathematical problem used in Step 2. During Step 1, Amy did not engage in mathematics-related activities. Her responsibility was to listen to (and enjoy) the read aloud.

Step 2: Modeling. According to NCTM (2000), “the teacher’s role in choosing worthwhile problems and mathematical tasks is crucial” (p. 53). I used trade books to create meaningful problems. I also used manipulatives related to each story to model how to solve problems similar to those presented in a book. Cass, Cates, Smith, and Jackson (2003) report numerous studies providing evidence that manipulatives help students who struggle learning mathematics skills. One study reported that 100% of three learning disabled students identified place value with 80% accuracy after learning with manipulatives (Cass, Cates, Smith, & Jackson, 2003). Another report showed 90% accuracy of all the students learned to recognize the value, compare the numbers, and add them using manipulatives (Cass, Cates, Smith, & Jackson, 2003)

An example of how I used trade books and manipulatives follows. *The Oreo® Counting Book* (Lucas & Raymond, 1999) is about a little girl who has ten Oreos®. Each time a page is turned, she eats one Oreo®. I used Oreo® cookies to model for Amy how I solved problems such as, “Amy had six Oreos®. Her brother ate one. How many are left?” and to demonstrate correct mathematical computations. Later in the lesson, I modeled how to use Unifix® cubes to solve similar problems to solidify and extend students’ knowledge (NCTM, 2000). Then I progressed to the use of the number line.

Step 3: Guided Practice. NCTM (2000) states, “meaningful practice is necessary to develop fluency” (p. 87). Gerstan, Jordan, and Flojo (2005) recommend that one goal of early interventions is to build fluency and accuracy with basic combinations of numbers. In order to build fluency after I modeled problem solving using manipulatives, Amy had the opportunity to use manipulatives to practice solving problems with my guidance. For example, in the lesson with the *Oreo® Counting Book*, Amy solved problems with Oreo® cookies to determine how many Oreos® were left after taking one away from various amounts. With my guidance, she continued to practice counting different amounts of Oreos® and taking one away. We moved from using Oreos® to using black Unifix® cubes, making the skill more abstract. Finally, Amy used the number line, with my guidance, to practice solving similar problems.

I also incorporated games as a context for Amy to practice her developing skills. In a research article titled, “Playing Games and Learning Mathematics,” Peters (1998) found that the games were popular, motivated the lower achievers, and kept students involved. To motivate Amy, we played games to practice using skills.

Step 4: Review. To allow active learning that builds understanding leading to the automaticity of numbers, I played many games with Amy. Because Amy demonstrated in class that she did not seem to remember skills she learned from one week to the next, I incorporated much review of concepts we learned in each lesson. For example, after Amy had learned to add and subtract, we played *Hi, Ho Cherrio*™ to review the concepts. In the game, she added or took away cherries from a tree as she solved problems. We also compared who had more cherries in a bucket and who had more cherries on their tree.

In addition to playing games to review concepts, I also developed what I called a "fact pack." The fact pack consisted of index cards with vocabulary words, examples of problems, and diagrams we had used in previous lessons. I added cards to the fact pack as lessons progressed. For example, after using Oreos® to practice the skill of taking one away, I added the following card to the fact pack: "less – less means fewer, we take away cubes, the number gets smaller". During each lesson, we reviewed this card as well as cards with other math vocabulary, math facts, or words problems taught previously. I would show her a vocabulary word and she would explain to me what it meant.

Step 5: Assessment. NCTM (2000) says, "assessments should support the learning of important mathematics and furnish useful information to both teachers and students" (p. 75). The final step was assessing Amy's understanding of the concepts she learned during the intervention. Although assessment is the last step in the intervention, I used on-going assessment throughout each step. I observed Amy closely as we interacted during each intervention session to determine what kinds of support to provide her. For example, if I noticed that Amy was having difficulty in the Guided Practice step, I would go back to Step 2 and model again. On the other hand, if I noticed she needed less support during the guided practice step, I would give Amy opportunities to solve problems on her own. Thus, on-going assessment guided my decisions during each lesson.

At the end of each lesson (which extended over two to three days), I used a written assessment to assess Amy's understanding of a concept. I chose to use a math journal format for the assessment step. My assumption was that if Amy could verbalize what a concept meant or how to solve the problem, then this would be an indication that she understood the concept we had been studying. For example, after reading *The Oreo®* *Counting Book*, I asked Amy to write how she would solve "one less than eight." This gave me an opportunity to see if she knew how to subtract one and to give Amy an opportunity to express in her own words how she would solve the problem.(See Table 3.1 for a summary of the intervention.)

Table 3.1

Instructional Intervention

Steps	Instruction
1. Building Mathematical Knowledge and Vocabulary Through Literature	Read a trade book aloud to Amy to provide a realistic context for applying mathematics concepts
2. Modeling	Use manipulatives to demonstrate the concept introduced in the book read in Step 1
3. Guided Practice	Child practices solving problems with variety of manipulatives. Use mathematical terminology.
4. Review	Play games and review concepts in fact pack
5. Assessment	Write in math journal an explanation of a concept.

The instructional framework described above was used systematically. However, one session did not consist of each step. Typically during one session, I started with reading a trade book and modeled how to solve a problem related to the book. In the next session, Amy practiced problem solving using manipulatives, reviewed using games and the fact pack, and then wrote in her journal. However, if I found that Amy needed more modeling after we have begun practicing, for example, I revisited a previous step. We progressed through the steps as time permitted and according to Amy’s understanding in each session.

In the following sections, I explain how I gained access to the research site. Next, I describe my roles as teacher and researcher. Then, I describe the data collection methods in detail and the analysis procedures.

Gaining Entry to Research Site

Bogdan and Biklen (1998) suggest that establishing good relationships with the gate keepers of the research site and potential participants helps the researcher gain access and consent to conduct a study. They recommend the following five guiding questions that the researcher should answer when approaching principals and other individuals who are gate keepers for a research site: What is the researcher actually going to do? Will the researcher be disruptive to normal classroom routines? What will the researcher do with the findings? Why is the researcher interested in researching in a particular context? What will the participants get out of the study?

The first gate keeper in my study was the superintendent of the school district. I had known the superintendent for many years because he had been the principal of my high school when I graduated. He later became the district superintendent. I used the five questions to guide my explanation of my intervention plan to the superintendent in person, and he approved my study.

To gain permission to conduct the study in my school and classroom, I approached my principal and answered the same five questions. I explained that I would be working one-on-one with one of my students who was struggling in math and hoped to develop a math intervention that would benefit struggling students. I explained that my findings would be published in my thesis and that they would be submitted to an educational journal. I assured her that I would use pseudonyms for the student and school, thus protecting the privacy of all involved. I also explained how I chose to select a student from my classroom because she was having difficulties and needed intervention work in math, just like other students received in reading. Finally, I explained how the school would benefit. One-on-one instruction would improve Amy's abilities in math and hopefully affect state test scores, which is how the school gained accreditation and received some financial assistance.

My principal was not concerned that I would be leaving my classroom to help Amy, because I had an assistant who stayed in the classroom and guided the students' learning while I worked with Amy. The principal asked if Amy would miss any special instruction (i.e., music, physical education, library time). I guaranteed her that Amy would not miss music or physical education, however, at times she might miss library time. Because Amy would have opportunities to check out books from the library at other times during the day, my principal was satisfied with the arrangement.

For the intervention, I wanted to use a room in the school where I could set up the video camera and leave it. I also needed a place where Amy and I would not be interrupted during the intervention sessions. I asked my principal for permission to use the Creative Writing Lab each morning. However, the room I requested was used on occasion by a teacher who worked there with English Language Learners, I turned in my schedule to the office to not disrupt the tutoring of the English Language Learners or anyone else's schedule while working with Amy. The principal approved my study.

Next, I gained permission to work with Amy from her mother and from Amy herself. Amy was a student of mine and I had already established rapport with her mother through daily interactions as she dropped off Amy at school. She was very involved in the classroom and helped Amy at home with her homework. She wanted to help Amy in any way that she could. I had previously mentioned my concern about Amy's difficulties in math. Her mother was willing for me to help Amy as long as it did not prevent her from going to physical education, a class Amy loved to attend.

Amy also readily consented to work with me one-on-one when I assured her she would not miss physical education. She was excited about working with me by herself, because I had not worked with Amy individually. The only students I worked with individually were those who struggled with reading and Amy excelled in reading.

After I received consent from all the parties, I received approval from the university's Institutional Review Board to conduct my study.

Role of the Teacher-Researcher

Bogdan and Biklen (1998) describe two levels of participation by a researcher. A researcher can be a complete observer--one who does not participate in the activities but "looks at the scene, literally or figuratively, through a one way mirror" (Bogdan & Biklen, 1998, p. 81). The opposite of a complete observer is a researcher who is completely involved at the research site and acts as a participant. I was Amy's intervention teacher, therefore, I was a full participant in the research. In this case, I was a teacher researcher. In each lesson, I read aloud a math-related trade book, modeled problem solving, guided Amy's problem solving, observed Amy as she practiced solving problems independently, gave her feedback, engaged in on-going assessment to analyze

Amy's understandings, and planned subsequent lessons. At the same time, I collected data for later analysis.

Informal Interviews

I collected data using the guidelines of Bogdan and Biklen (1998) in the book, *Qualitative Research in Education*. First I conducted informal interviews with Amy, Amy's mother, and kindergarten teacher to find out about Amy's previous mathematics instruction and possible difficulties she may have experienced previously. I asked Amy questions about her interests to help guide my selection of books and manipulatives. I thought it was important to choose contexts for teaching math concepts that interested Amy to increase my chances of connecting with her and making mathematics learning more meaningful to her.

I had several purposes for interviewing Amy's mother. First, I wanted to find out about if anyone else in their family had struggled with mathematics. I also wanted to know what kind of instruction Amy was receiving at home with her homework and to formally document her mother's concerns and thoughts about Amy's difficulties with mathematics. Amy's kindergarten teacher was interviewed to give me a picture of the type of instruction Amy received with numbers as she established her number sense.

TEMA-3

"The *TEMA-3* is a norm-referenced, reliable, and valid test of early mathematical ability that is appropriate for children of ages 3 years 0 months through 8 years 11 months" (Ginsburg & Baroody, 2003, p. 5). The test comes in two forms, consisting of 72 items each, each parallel to the other. From the *TEMA-3* a researcher can determine “a raw score, age equivalent, grade equivalent, percentile rank, and Math Ability Score (standard score)” (Ginsburg & Baroody, 2003, p.5)

Ginsburg and Baroody list five purposes for the *TEMA-3*, however, I used it for only two purposes. First, I used Form A of the *TEMA-3* to “identify specific strengths and weaknesses in [Amy’s] mathematical thinking” (p.6). Secondly, Form B was used as documentation of Amy’s progress after the intervention had taken place, showing areas of growth for her.

Video Taped Data

Bogdan and Biklen (1998) state, “photography has been closely aligned with qualitative research and can be used in many different ways” (p. 141). I used video taping to record in detail what both Amy and I did during the intervention sessions. I also used the video tapes to clarify any questions I had about the transcribed lessons and to add descriptions about what Amy and I did during each session

Audiotaped Data

In addition to video taping each intervention session, each lesson was audiotaped and transcribed. The sessions were audiotaped to provide me with an easier way to transcribe the sessions. Amy wore a small microphone connected to a tape recorder during the intervention sessions. Amy thought the microphone was fun to wear, often reminding me she needed to put it on, as well as helping me turn the tape recorder on and off at the beginning and end of the sessions.

Fieldnotes

Bogdan and Biklen (1998) describe two kinds of fieldnotes: descriptive and reflective. Descriptive fieldnotes provide pictures through words that describe the people, setting, actions, and conversations that are observed by the researcher. Reflective fieldnotes are records of the frame of mind of the researcher, who records ideas and concerns that come up during the study and during data analysis.

I kept descriptive and reflective fieldnotes to document the intervention sessions with Amy. Bogdan and Biklin recommend that fieldnotes should include a portrait of the participant, reconstruction of dialogue, description of the physical setting, depiction of activities, and the participant/observer's behaviors. To help keep objective and detailed records (Bogdan & Biklin, 1998), I audiotaped and video recorded each session. I hired a secretary to transcribe each audiotape. After I received each transcription, I watched the video tapes and made comments about Amy's actions and my actions on those transcriptions. I wrote a detailed description of the setting in these transcriptions and documented any changes in the setting each time we met. I wrote a description of each session's activities in my lesson plans before meeting with her and then added notes after the lesson to show any additions or deletions from the plans due to comprehension of the material being taught.

The format of the transcripts I used was modeled after suggestions by Bogdan and Biklin. A capital letter "T" and a colon were placed before the teacher's words, and the uppercase letter "S" followed by a colon were placed before Amy's words. To help keep transcriptions organized, I used a header that documented the session number, date, and page number. The lines of the transcriptions were double-spaced, leaving a larger margin on the right-hand side of the page for writing notes during coding. I also numbered the lines of the transcriptions so I could easily identify data that related to codes.

I used reflective notes, designated with "TC" for teacher's comment, to document my thoughts after the sessions and while watching the videotapes. As I wrote down the actual events, I speculated about Amy's attitude toward math, clarified the transcriptions, noted what I would change about the next session, and my speculations about why Amy was understanding (and not understanding) the skills and concepts I was teaching. I added reflective notes to the transcriptions of each lesson. I also made reflective notes on the lesson plan form after each session to express my thoughts about how the session went and my feelings about how the intervention went, what I would change about my teaching during the intervention, and the direction I wanted to go in the future.
A sample of teacher comments follows.

> TC: Amy was excited about the game, *Hi Ho Cherrio*™. She grasped the word *more* as we compared the two buckets of cherries. However, she really did not answer questions about which bucket had *less* correctly. Next time I may use the number line to compare the numbers and help her see which bucket has less.

Artifacts

After assessing Amy's number sense using the *TEMA-3* and her interests, I wrote a lesson plan, using this information, for our first session. After each session, I designed a plan for the next instructional time, based on what I learned about Amy's developing skills. Other artifacts I used to assess Amy's math understandings included entries from

Amy's math journal that she wrote during Step 5 (see Appendix A), workbook pages completed for practice in the classroom, and test papers from her class. I also used her test papers to determine if she was using the math skills she was learning during the intervention sessions.

Data Analysis

On-going data analysis took place during and after data collection (Merriam, 1998). When the data collection was complete, I triangulated data sources: the transcriptions, the fieldnotes from the video tapes, lesson plans, and student artifacts. From the data, I looked for patterns in the data that would help me understand what instructional methods or strategies best facilitated Amy's learning of number sense.

Organizing the Data

Each intervention session was audio taped using a General Electric Fast Playback Audio Voice Recorder. After each session, I delivered the audio tapes to an individual who transcribed them, saved them in MSWord documents, and returned them to me as an email attachment. Each session was also video taped, using a RCA VHS video recorder. When I received the transcriptions, I read them, added teacher comments, and added notes about Amy's and my actions. The teacher comments the included ideas such as why I chose to include particular games in an intervention session. I also noted my inferences about why Amy may have responded the way that she did. The expanded transcripts guided subsequent instruction and helped me to decide what instructional activities were the most effective and how I could revise them to make them more effective. These expanded transcripts, along with my lesson plans, provided a complete record of what occurred during each intervention session. As noted earlier in the chapter, each page of the data was numbered and included the date to help me keep the data organized.

Lesson plans for each intervention session were written on a form that that I created. The lesson plan form consisted of a table with two columns and five rows. In the left-hand column, each step of the intervention was listed in a separate row. In the right-hand column, I wrote my plans related to each step. (See Table 3.2) One form was used for each lesson, which was taught over the course of at least two days.

Building Background Knowledge	
Modeling	
Guided and Independent Practice	
Review	
Journal	

Figure 3.1 Lesson Plan Form

Notes from the interviews, expanded transcriptions, and lesson plans were organized chronologically in separate sections of a three-ring binder. The results of the *TEMA-3* pretest and posttest were filed in another section of this binder. The video and audio tapes were stored chronologically on a shelf in my bedroom closet.

Coding

Before beginning the coding process, I reviewed the study's purpose and the question to help me keep my focus as I analyzed the data. First, I assigned a color to each step of the intervention. Then using the designated color, I marked the right-hand corner of each page of the transcript. If a new step changed in the middle of a page, I drew a line where it changed and then marked the second color in the right-hand corner. For example, on every page where I modeled (Step 1), I marked the corner with a pink highlighter. This color coding made it easy for me to locate pages of transcriptions that coincided with each step of the intervention. The color-coded pages also allowed me to look at the transcriptions section by section. I could remove from the binder all the pages that were color-coded with pink and examine them together to look for patterns in the data by intervention steps.

After organizing the data, I began the inductive coding process. Bogdan and Biklen (1998) liken inductive data analysis to sorting thousands of toys in a large gymnasium. Codes come from the data rather than fitting the data into preexisting codes or categories, which is done when deductive analysis procedures are used. Bogdan and Biklen recommend that the researcher read through the data, looking for patterns in words or phrases and in the participant's language and behaviors. The researcher assigns preliminary codes and later refines those codes to describe these patterns. Oftentimes, these codes represent words taken directly from the data itself. Furthermore, they recommend keeping a list of codes (and subsequent categories) and define those codes with examples from the data. As the researcher continues the analysis process, codes may be added, deleted, or subsumed by other codes. Later in the process, similar codes may be grouped together into categories. Then these categories are assigned labels that describe the data.

I began the initial data analysis while the data collection process was taking place. After each intervention session was complete, I reflected upon how Amy responded to each lesson and if she seemed to grasp the concept that I was teaching. As the transcriptions were emailed to me, I watched the videotapes as I read the transcripts, and added teacher comments. The teacher comments contained things such as why I had chosen particular games or activities for a particular session and my inferences about why Amy may have responded the way she did. I also used the teacher comments and transcriptions to guide subsequent instruction and decide what instructional activities were effective and which ones I needed to revise.

The first code that emerged from the data was “number line”. From the very first lesson, I used the number line to show Amy the value of numbers. On the transcriptions, I highlighted in yellow every time I used the number line in the intervention sessions. As I was highlighting, I also noticed the frequency with which I used the number structure throughout the sessions. (The number structure consisted of the numbers zero through nine with manipulatives to represent the value of each numeral to the right of the numeral. See Figure 3.2) I highlighted the use of the number structure in pink.

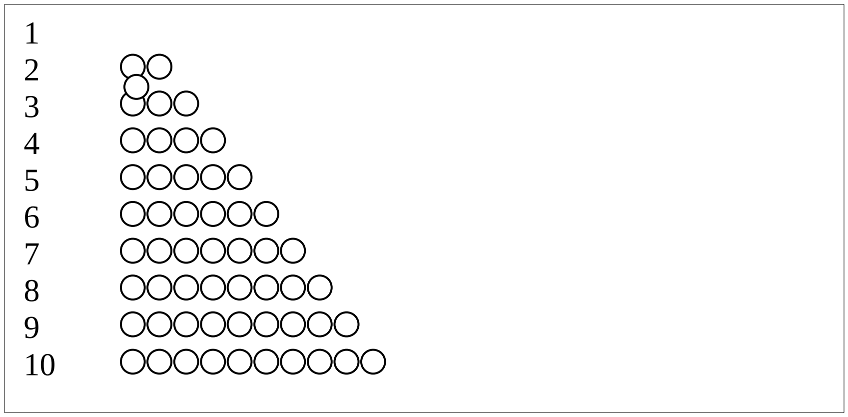

Figure 3.2 Number Structure.

When meeting with my thesis director, she commented about the different kinds of language I used during the intervention sessions. When I reread the transcripts, I noticed that most of the lessons focused on helping Amy understand the value of numbers from one to one hundred, so I circled every reference to *more* or *less*. At this stage in the data analysis phase, I also marked each instance where I used manipulatives.

I also coded every time Amy answered questions correctly “C” or incorrectly “IC”. I thought this coding might help me better understand Amy’s thinking about number sense. As I further analyzed the data, this did not seem particularly useful. What I did notice is that the coded responses in the review step provided more useful information to me about Amy’s understandings.

Refining the Codes and Categories

After extensively reviewing the expanded transcriptions, I revised the code list and identified final categories that subsumed the codes, always focusing on my research question*: How can I improve Amy's number sense*? The categorizes are described below?

Teacher Support. Under this category were the following codes: manipulatives, book, visualizing, reading, and chart. They were coded in the right-hand margin of the expanded transcriptions, using the abbreviation "TS" and then wrote the kind of support (e.g., manipulatives). Later, I shortened the code to an abbreviation to specify the type of support. For example, teacher support using manipulatives became TS-M. (See Table 3.3 for the definitions of codes that are a part of the Teacher Support category.)

Table 3.2

Teacher Support Codes and Definitions

Teacher Support	Code	Definition
Manipulatives	TS – M	Used manipulatives to help Amy see the value of a number or to add or subtract numbers
Book	TS – B	Used a trade book to help Amy remember the definition of math vocabulary. For example, in Lesson 8, I read the book, *Elevator Magic* (Murphy, 1997) where a little boy rides the elevator down the floors to finally end up on the first floor. Amy associated the word minus with the elevator going down the floors and ending up on the first floor.

Table 3.2 (continued)

Visualizing	TS – V	Asked Amy to visualize the number line in her head instead of referring to the written one in front of her. For example, in Lessons 5 and 6, Amy was becoming proficient with the number line, I would ask her to picture the number line in her head when adding and subtracting.
Reading	TS – R	Made a comparison to something Amy understood in reading to help her understand something in math. For example, in Lesson 11 Amy was having trouble remembering which number was before and after. I realized it was a vocabulary issue, so I explained the word "before" was like the beginning of the story and the word "after" was like the end of the story.
Chart	TS - Ch	Used a chart to help Amy organize her thoughts and see the mathematics concept. For example, in Lesson 11 when I associated the beginning, middle, and end of the story to the words before and after in math, I made a chart with three boxes. Above the boxes were the words *before* and *after* and under the boxes were the words *beginning*, *middle*, and *end*. I would place a number in the middle box and then Amy would state the number before the number in the middle box and the number after.

Language. Another category, language, referred to the words and phrases I used to explain *more* and *less*, the predominant concepts I taught Amy during the interventions. As mentioned above, I circled the words I used to refer to the value of numbers as they related to *more* and *less*. Later, I divided the words into two lists: one for all of the words and phrases I used to refer to *more* and the other list for the words that referred to *less*. Then I tallied the times I used them in each lesson and recorded the results on the back of each lesson plan.

Table 3.3

Language Used to Refer to More and Less.

Words referring to *More*	Words referring to *Less*
the next number	the number before
bigger	smaller
added one	take away one
one more	one less
add	subtract
going up on the number line	down on the number line
comes after	comes before
plus	minus
count up	count back
get more	take some away
next	before

Benchmarks. As described above, when I began coding, I identified Amy's correct responses with a "C" and every incorrect response with "IC." As I continued coding, I found that this analysis for answers during every step of the intervention did not paint a true picture of Amy's understandings. Coding the practice step resulted in many incorrect responses, which is understandable because she had not had a chance to practice using the strategy successfully in a variety of different situations Therefore, I decided to code correct and incorrect responses only in the review step of each lesson.

To further analyze the data in the review step of the intervention, I used deductive analysis rather than inductive analysis. I designated the Mississippi Curriculum First Grade Benchmarks as the category and then used the letter codes from the curriculum document to code the skills I taught in each lesson. I also used the MS benchmarks to code Amy's correct and incorrect responses in the review step of the lessons. The following are the benchmarks taught throughout the intervention lessons and the code assigned to them. See Table 3.4 for the codes used for Amy's answers and my instruction.

Table 3.4

Benchmark Codes

Benchmark (BM)	Code
Add and subtract basic facts	BM - A
Analyze inverse relationships	BM - C
Add and subtract two digit numbers	BM - D
Solve 1 and 2 step problems by drawing, discussing, modeling, and writing	BM - H
Identify, model, and write numbers to 1,000	BM - J
Compare numbers using before, after, and between	BM - K
Identify place value	BM - I

Finally, I developed a Transcript Analysis Record Form to summarize and display the coded data for each intervention session. At the top of the table, I recorded the lesson number and the benchmark that was the focus of the lesson. I also included the three categories on the form. In the next section of the table were two columns where I recorded the language I used to teach the value of *more* and *less* in each intervention lesson. Next on the form was the teacher support category. Under the category label, I chose to delineate the number line and number structure codes because I used the tools so often in the lessons. I recorded with a short phrase how the tools were used and the transcript lines where the use of the tools was found. Under these columns were three columns. In the left-hand column was space to list other teacher support, such as manipulatives, trade books, and so forth. The middle column provided space to record the lines in the transcripts where the teacher support was provided. In the right-hand column, was space to record the step of the intervention. At the bottom of the chart were four columns. In the first column, the benchmarks were listed. In the next column was space to record the lines of the transcript where the benchmarks were reviewed. In the last two columns was space to record the number of questions Amy answered correctly and incorrectly and were related to each benchmark. (See Figure 3.3)

Lesson No.				
Benchmark Covered				
Primary Language				
More		**Less**		
Teacher Support				
Number Line		**Number structure**		
Teacher Support				
Support	**Lines**	**Section**		
	Benchmark Reviewed	**Lines**	**Correct**	**Incorrect**
J	Identify, model, and write numbers to 1,000			
K	Compare 3 digit no. using before, after, and between			
A	Add and subtract basic facts			
C	Analyze inverse relationships			
I	Identify Place Value			
H	Solve 1- and 2-step problems by drawing, discussing, modeling, and writing			
D	Add and subt. two digit no.			

Figure 3.3 Transcript Analysis Record Sheet.

Ethical Considerations

According to Bogdan and Biklen (1998), there are two primary concerns a researcher must be cognizant of when dealing with human participants: informed consent and protecting the participant from harm. It is the responsibility of researchers to make sure individuals participate voluntarily and are not exposed to risks that outweigh the benefits of participating in the research.

Informed Consent. When seeking cooperation from the participant, Bogdan and Biklen (1998) suggest explaining to individuals of your interest in including them as research participants, describing the research project, and getting written consent from participants. Because Amy was a minor, I asked her for consent from her mother before asking Amy for consent. Amy's mother signed the IRB consent form, giving her full consent. I also asked Amy for her consent. I talked to her about the research project for a long time. I explained exactly what I would be doing with her and that she did not have to participate. She was very excited about the one-on-one time with me and seemed to want to participate, perhaps because she knew she needed help with mathematics. I also received permission from the district superintendent and the school principal to conduct the research. (See "Gaining Access to the Research Site" for additional information.)

Protection from Harm. When protecting participants from harm, Bogdan and Biklen recommend a participants' identity should be protected so that she is not embarrassed or that the research could not harm her in any way. On every document, the participant’s name was changed to a pseudonym (Amy) and all documents were kept locked at school in a filing cabinet and video tapes and audio tapes were stored at my home in a closet, where no one had access to the data. In addition, pseudonyms for the school and school system were used to insure anonymity of the research participant.

There were two other areas of concern for protecting Amy from harm that are relevant in this study. The first was my concern that other students did not think I chose to work with Amy because she was my favorite. I tried to alleviate this potential problem when I spoke to the whole class about my research project. Because I would be out of the classroom for at least 30 minutes two to three days a week, I wanted them to understand what I would be doing and that I did not choose Amy because I liked her more than any of the other students. I told the students that Amy agreed to help me and that she needed help with something. I explained it was just like when I read with them individually or helped them in the classroom. I explained to them that we had to go to another room because I was going to video tape my time with her, so I could write about what we were doing. They seemed to accept this explanation and had no questions. Later, a couple of students asked about our time together but no one else mentioned it.

Another concern was that if the other students thought Amy was my favorite because I chose her to help me with my school work, they might become jealous and exclude her from playtime or make negative comments to her. My class conversation about my research project seemed to take care of this and to my knowledge this was never an issue.

Role of the Researcher and Teacher. I was both the researcher and teacher during this study. Because of this, ethical decisions arose. First, I had to be careful to balance my responsibilities as a teacher-researcher when working with Amy one-on-one and my responsibilities as teacher of 24 second grade students. One way I resolved this dilemma was to arrange my classroom schedule to allow time for Amy and me to be out of the room during a time when my assistant could effectively teach the class. Each morning, one of the first things my students participated in was a time of math review that revolved around the calendar (i.e., calendar math). During calendar math, I went through specific questions each day to review, extend, and sometimes introduce concepts to my students. Because my research intervention did not begin until after my assistant and I had worked together for a year, I was confident she could lead this instruction as I would.

Another reason for choosing this time to work with Amy was that she would not miss critical instructional time, because it was mostly a review. I believed that the one-on-one instruction Amy would receive from me would benefit her more than whole class review. This time was also good for Amy because it was first thing in the morning, so she was fresh and not tired from being in school all day. In addition, Amy did not miss calendar math every day; I worked with her individually only two or three days each week. Similarly, the class was not denied my instruction during calendar math every day; two to three days each week I led instruction, especially when new concepts were introduced.

I chose to work with Amy outside the classroom for several reasons. First, being out of the room with Amy allowed me to focus solely on her. Second, I did not have to worry about classroom discipline problems, questions from other students, regular classroom noise, or other interruptions. Third, this setting facilitated video taping.

Working one-on-one with students who struggle was a common occurrence in my school. In fact, I was required to by the school to work one-on-one for 30 minutes each day with students who struggled in reading. I saw the work with Amy as a similar situation. She needed one-on-one intervention in math. The school principal agreed that my work was important and the arrangement to work outside the classroom was supported by her.

I also addressed this ethical dilemma by resisting the temptation to put my research and instruction with Amy before the needs of the other students. There were times that I thought it was best for us not to meet, such as the weeks before the Mississippi Curriculum Test. I believed my students all needed my completed attention during this time. There were also other times that we were not able to meet due to special activities the school was sponsoring, such as plays or presentations. To summarize, I met with Amy using a flexible arrangement, balancing both her individual needs and the needs of the entire class.

Summary

In this study, a qualitative case study design was used to study a second grade student, who struggled with mathematics, yet excelled in reading, at a rural Title One school in Northeast Mississippi. A five step teacher-researcher created intervention was used in the study. I was a teacher-researcher, teaching during the intervention sessions and functioning more as a researcher while analyzing the data. The *TEMA-3* was used as a pre- and post-test assessment of Amy’s mathematical knowledge, with other data such as video taped data, audiotaped data, fieldnotes, and artifacts from the classroom and intervention sessions. The data were analyzed by triangulating sources. Data was organized in a three ring binder, coded, and categories were formed. Finally, I discussed the ethical concerns by explaining how I received informed consent and balanced my roles as a teacher-researcher.

CHAPTER IV

RESULTS

Introduction

The purpose of this study was to help a second grader who struggled with math develop her conceptual understanding of number sense using a researcher-created intervention. In this chapter, I first report the results of the interviews and the *TEMA-3* pretest. Next, I briefly describe each intervention lesson to give the reader a sense of what the intervention looked like in action. It also shows the problems Amy experienced and how I addressed those through the different steps in the intervention. Next, I describe how different aspects of the intervention helped Amy develop number sense. Finally, I report the *TEMA-3* posttest results.

Summary of Interviews

Before starting the math intervention with Amy, I interviewed three people to find out more information about Amy's mathematical experiences. First, I spoke with her mother to find out her perception of Amy's problems with mathematics. Amy's mother told me she thought Amy felt horrible about math. She helped Amy every night with her mathematics homework, explaining that once she got her started on the problem, Amy could usually complete it. When asked if their time was stressful, she commented that it could be fun and stressful; the stress came when Amy did not understand. I then asked about any family history of persons having difficulty with math. She said no one had struggled with math in the past, but she did not like math. She added that Amy's grandmother, who took care of her after school, was unable to help Amy complete her math homework because she could not work the math problems herself. I asked Amy's mother how she encouraged Amy with her schoolwork and her response was "The more you practice, the better you will get!" Next, I asked how logical she thought Amy was. Her mother responded that she thought Amy was a logical thinker. When I asked to explain why she thought that, she could not do so. Consequently, I did not learn anything about whether Amy displayed logical thinking at home. The next question related to Amy's experience playing board games, because playing games was a part of the intervention. Although she said Amy did not play board games at home, she commented that Amy did like to play games such as *Connect Four*® and *UNO*®.

To help me understand how Amy's mother helped her with mathematics at home, I gave her a sample problem and asked her to show me how she would help Amy solve the problem. The problem was: "Jack has nine apples. Johnny has five. How many do they have in all?" Amy's mother said she would ask Amy to draw nine apples and then five more apples and then count them. Then Amy's mother would write 9 + 5. This is similar to how addition was presented in the classroom and it told me she tried to help illustrate the problems and explain them.

The last question I asked was if Amy had always struggled with math. Her mother said that Amy's kindergarten teacher never really said anything. Her mother mentioned that she first noticed that Amy had difficulty with math in first grade. However, when

Amy's mother showed me Amy's report card from kindergarten, she had several N's (needs improvement) in mathematics. Also, Amy was in a different teacher's classroom at the beginning of first grade and Amy's grades were N's (needs improvement) and U's (unsatisfactory) on her Fall, mid-semester progress report. It was at this time that Amy's mother requested Amy be moved to a different teacher's classroom, which I began teaching in January.

The second person I interviewed informally was Amy's kindergarten teacher. She remembered Amy but could not remember a lot of details about her mathematics achievement except to say she recalled that Amy may have demonstrated a slight problem with mathematics. To find out the kind of mathematics instruction Amy had in kindergarten, I asked her kindergarten teacher about her teaching style, and she explained that the students visited a math center every day. At that center they might find pictures of objects in magazines or pictures that represented a certain number. Their task was to glue the appropriate number of objects on the numeral. The teacher also indicated that she used manipulatives in almost every lesson. This information helped me know more about Amy's experiences using manipulatives for developing mathematics concepts. A person I conferred with often was my teaching assistant, who had been working with Amy since she moved into the former first grade teacher's classroom. My assistant said that in first grade they used manipulatives some but not on a daily basis. She also stated that Amy struggled with mathematics the first semester of first grade, crying often and complaining of a stomach ache. My own observations of the former first grade teacher did not provide any evidence of activities in which manipulatives were used. However, I only observed for two days, before becoming the teacher in that classroom.

I also interviewed Amy to help me better understand her feelings about math and determine Amy's interests so I could make lessons more meaningful and interesting for her. The first question I asked was: "What is your favorite subject?" She answered, "reading", which confirmed what I thought. The next question I asked her was, "How do you feel when I give you a math worksheet?" She answered, "Sad, because I just do not like math." I asked about her feelings when she took a math test. She told me she was happy when she was taking a test, which did not correlate with her actions. During mathematics tests, Amy often wrote random numbers for answers, put her head down, or cried. I also asked Amy what made math fun and she said, "I like to do tubs [activities with manipulatives] because it is fun. You get to stay for five minutes and stuff."

Next, I asked Amy questions about things she liked to ascertain her interests, so I could build on these during the intervention sessions. Her favorite characters were *Lil' Bratz*®, *Dora*®, and *My Little Pony*®. Her favorite games were racing games and fighting games on the Nintendo. The candy she liked best were lollypops, *Reese's Pieces* ®and *Oreos*®. Her favorite foods were chicken, carrots, and peas. At home, she liked to play teacher, cashier, *Barbie*® dolls, and baby dolls. If she could go anywhere, she said she would go to the movies. When she watches television, she likes to watch *Bratz*® and *Dora*®.

TEMA-3 Pretest

The *TEMA– 3* (Ginsburg & Baroody, 2003) is an instrument designed to measure the mathematical performance of children between the ages of three to eight years old. The test manual states that the test can be used for several purposes: to screen for readiness, to discover why students are not excelling in math, to measure a students' progress, and to identify students who are gifted in math. It can also be used to guide instructional interventions. The authors write that it is especially useful with older children (ages six to eight), who are struggling in math, because it can help a teacher determine specific strengths and weaknesses (Ginsburg & Baroody, 2003).

The *TEMA-3* has two parallel forms with 72 questions each. It assesses students' numbering skills, number-comparison facility, numeral literacy, mastery of number facts, calculation skills, and understanding of concepts. The *TEMA-3* has two main sections: informal mathematics and formal mathematics. The informal mathematics section has four subsections: numbering, number comparisons, calculation, and concepts. The formal mathematics section also has four subsections: numeral literacy, number facts, calculation and concepts. According to the manual, the internal reliability of the instrument is above .92.

Although I had some ideas about the mathematics concepts with which Amy experienced difficulties, I wanted to confirm my suspicions using a formal instrument and make sure I had identified all the areas with which Amy struggled. I administered Form A in January 2006, following the directions in the manual.

In the informal mathematics section, Amy missed six out of twenty-three questions in the numbering subsection, which consisted of problems such as counting to twenty-one, identifying a large number of objects, and verbally counting backwards. In the number comparisons subsection, which consisted of choosing the larger number and using a mental number line to judge which number was closer to another number, Amy missed four out of six. In the calculation subsection, Amy missed four out of seven questions, which included skills such as counting on from a larger addend and mentally subtracting a number. In the concepts subcategory, she missed two of the four questions, which asked her to figure out a missing addend and equally distribute a quantity of counters.

In the formal mathematics category of the test, the items that tested number facts assessed the ability of students to quickly answer basic number facts. The calculation questions simply tests a child’s ability to add and subtract. In the numeral literacy subsection, she missed only one question out of eight. These questions asked her to simply name numerals. In the number facts subsection, she missed seven of the nine questions when asked to give an answer to a mathematics fact from memory. The next subcategory was calculation. Amy missed nine of the ten questions when she could use pencil and paper to find the answer to a math fact. Finally, out of five concept questions, Amy missed four when asked to select a written statement that correctly represented a word problem that was read orally to her.

Table 4.1

Results of the TEMA-3 Pretest

	Incorrect Problems	Total Problems
Informal Mathematics		
Numbering	6	26
Number Comparisons	4	6
Calculation	4	7
Concepts	2	4
Formal Mathematics		
Numeral Literacy	1	8
Number Facts	7	9
Calculation	9	10
Concepts	4	5

At the time of the pretest, Amy's was in the second grade, sixth month and her chronological age was seven years and eight months (7.8). Amy's overall score was first grade, fourth month (over one year below her current grade level) and her age equivalent was six years, six months (6.6), one year and two months below her chronological age. The two areas she missed the least amount of questions in were numbering and numeral literacy. All the other areas she seemed to have difficulty in.

The Intervention Summaries

The instructional intervention began on February 14, 2006, and ended May 15, 2006. I met 22 times with Amy, usually two to three times each week. During this time, we completed thirteen lessons. Although I tried to meet at least twice each week with Amy, there were days we did not meet, such as Spring Break and the week of the Mississippi Curriculum Test. A calendar follows (See Table 4.2).

Table 4.2

Intervention Calendar

Week 1	Feb. 13 -17	Lesson 1 Session 1 (2/14) Session 2 (2/15) Lesson 2 Session 3 (2/16) Lesson 3 Session 4 (2/17)
Week 2	Feb. 20 - 24	Lesson 4 Session 5 (2/20) Session 6 (2/21) Lesson 5 Session 7 (2/22)
Week 3	Feb. 27 – March 3	Lesson 5 Session 8 (2/27) Lesson 6 Session 9 (2/28)
Week 4	March 6 - 10	Lesson 7 Session 10 (3/6) Session 11 (3/7) Lesson 8 Session 12 (3/9)
Week 5	March 13 - 17	Spring Break - No Work with Amy

Table 4.2 (continued)

Week 6	March 20 - 24	Lesson 9 Session 13 (3/20) Session 14 (3/21)
Week 7	March 27 - 30	Lesson 10 Session 15 (3/27) Session 16 (3/30)
Week 8	April 3 - 7	Lesson 11 Session 17 (4/5) Session 18 (4/6)
Week 9	April 10 - 14	Easter Break
Week 10	April 17 - 21	Lesson 12 Session 19 (4/18) Session 20 (4/19)
Week 11	April 24 - 28	Test Prep – No work with Amy
Week 12	May 1 - 5	Testing – No work with Amy
Week 13	May 8 - 12	Lesson 13 Session 21 (5/8) Session 22 (5/9)
Week 14	May 15 - 19	*TEMA -3* Assessment

Session 1: Lesson 1 (February 14, 2006). After analyzing the *TEMA-3*, I identified the areas in which the intervention needed to focus. I started the first intervention on February 14, 2006. The assessment tool showed that Amy could recognize numbers, but did not know the value of them. To build her knowledge of number value and help provide an anchor for the concept, *more*, I read the book, *Ten Black Dots* (Crews, 1986). In this context, an anchor was an experience (i.e., a story) that Amy would remember and relate to the concept of *more* in future lessons. The book *Ten*

Black Dots starts with the question, “What can you do with ten dots?” The illustrations show the values of the values zero through 10 using black dots. After I read the book once, Amy then read the book. She read in a chipper voice and seemed excited to begin working with me one-on-one.

After reading the book, we revisited the illustrations and counted the dots that corresponded with the numeral on each page. In the back of the book were numerals one through five on the left side of the page and matching dots drawn horizontally on the right side of the page, showing the value of the corresponding digits. Hereafter, I refer to this arrangement of numerals and dots as a number structure. (See Figure 4.1 for an example.) We used the number structure to compare the value of numbers one through five, verbally stating which number was *more* or *less* than another by looking at which row was longer. As we turned the page and worked on the same skill for numbers six to 10, I realized Amy did not know which number was greater in value, so I used manipulatives to model how to decide what numbers had more or less value. Using Unifix© cubes, I showed Amy how to build two towers by stacking cubes on top of each other which showed the value of the numbers. Then I placed the towers next to each other to show her which tower had more cubes. Together, Amy and I compared the cubes, counting the cubes to discover which tower had one more cube. I also used a number line to show her how the larger number always comes last. (See Figure 4.4 for an example of the number line used in the lessons.) We practiced comparing values by playing *Hi Ho Cherrio™*, comparing who had more cherries in their basket. I made two cards for Amy’s Fact Pack; I wrote *more* on one index card and drew a number line on another. I showed both of these index cards to her. We discussed the meaning of the word *more* saying that more refers to the number that is greater in value and a number line can be used to help remember which number is more, because as numbers increased on the number line, they also get larger in value.

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1	○
2	○○
3	○○○
4	○○○○
5	○○○○○
6	○○○○○○
7	○○○○○○○
8	○○○○○○○○
9	○○○○○○○○○
10	○○○○○○○○○○

Figure 4.1 Number Structure

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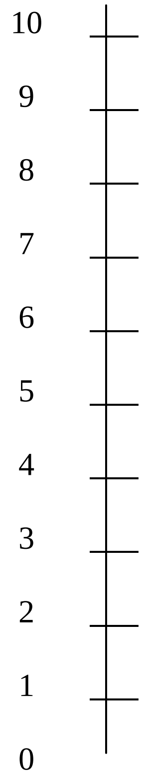

Figure 4.2 Number Line

Session 2: Lesson 1 continued (February 15, 2006). The next day, February 15th, we continued to work on Lesson One, focusing on comparing the values of numbers and the concept of *more*. I started the lesson by rereading each page of *Ten Black Dots* and counted the dots on each page. The last page of the book is where I spent most of our time. I chose two numbers with corresponding dots in a row and asked, “Which was more?” Again, Amy showed knowledge of the numbers one through five, but when I extended the question and asked, “How many more?” she could not answer the questions. I showed her how I arrived at the answer, by covering up the dots that lined up, leaving the extra dots to show how many more. I modeled my thinking aloud saying, “I know that four is one more than three, because we have one dot left in the line by the number four.” I drew one-to-one lines to connect each dot and asked Amy to circle the dot that did not have a matching line. Next, we played a game where I drew a row of dots on a paper and wrote the numeral that represented each dot. I then ask her to add a dot. She experienced success as she drew another dot and counted all of the dots. I then ask her to write the numeral that represented the number of dots on the paper. Then I verbalized what the drawing represented: “Your three dots are one more than my two dots.” I thought she was beginning to understand this concept when she began to say, “My six dots are more than your five dots.” In Step 3, we practiced identifying which number was greater in value by comparing values of numbers using Unifix© cubes. To do this activity, we made two towers; one tower had one more cube in its stack. Amy continued to make comments that demonstrated she was beginning to understand which numeral was *more* than another. She made comments such as, “My tower is taller and it has one more.” I was careful to make sure that she counted the cubes in each tower and wrote the appropriate numeral on paper. Independently, Amy began to correctly identify which number was larger in value. At this point, I knew it was time to move to Step 4 of the intervention. To review the word *more,* Amy stated the definition for *more* in her own words. Together we located the number line in her Fact Pack. I asked her to draw her own number line using the one from her Fact Pack as a guide. I then asked her to use the number line to answer the question, “Which number is more?” As she compared two numbers, I demonstrated how the number that had the larger value always came after the other number on the number line. When working on this, I noticed Amy did not understand the meaning of the word *after*. For example, when I covered up the numeral six on the line and asked the question, “What number comes after six?”, Amy had trouble telling me. I made a note to work on this concept later. We continued to review the vocabulary word *more* by playing *Hi, Ho Cherrio™* and comparing the number of cherries in our baskets. To play the game, Amy and I used the words *more*, *less*, and *same* repeatedly in a context that Amy enjoyed. Each time we worked with the cherries in the baskets, we took them out and lined them up so Amy could see a visual display of numbers getting bigger and smaller in value. She and I each wrote the numeral that corresponded to the number of cherries in our cups and Amy circled the one numeral that represented more cherries. Lastly, I asked Amy to write in her journal what the word *more* meant. Her journal entry said, “More means that a number is bigger. It comes after any number on the number line.”

Amy demonstrated that she now understood which number had a larger value, but it was clear that she needed instruction about on what the “next number” meant. It seemed that mathematical language gave Amy problems more than I realized. This concept is generally taught in kindergarten and in the first semester of first grade so I had not expected that Amy would have difficulty with these simple concepts.

Session 3: Lesson 2 (February 17, 2006). Lesson Two focused on the concept of *less*. I chose to introduce the word *less* using a book titled, *Just Enough Carrots* (Murphy & Remkiewicz, 1997). The main characters are two rabbits, a mother, and a child, shopping for a dinner party they are having with other animals. As the two shop for groceries the child compares the amounts of food the mother is buying and says they have less carrots and she wants more.

In this lesson, Amy read the book and then I used the illustrations to discuss who had *more* grocery items in their grocery cart and who had *less* items in their cart. I used small plastic grocery items, such as the carrots, as manipulatives to create concrete representations of the problems. An example of a problem we discussed was, "Amy has four carrots and Miss Mathews has eight carrots. Who has more, Miss Mathews or Amy?" I then introduced the term *less* with the question: "Who has less?" We practiced determining who had less by using grocery items as manipulatives. If Amy had difficulty answering a question, we went back to matching the items one-to-one. Each time, we wrote the numeral that corresponded to the number of items and Amy circled the numeral that represented more. I then highlighted the remaining number and said, "Six is more that five, so five is less than six." After practicing with manipulatives that resembled items that might be added to a grocery cart, we began solving problems with Unifix© cubes. Manipulating Unifix© cubes made the practice a bit more abstract than using items that closely represented those in the book. I gave Amy problems such as, "Miss Mathews will snap nine cubes together while Amy snaps eight together. Who has more? Now who has less? Numerous times we repeated this activity, using different numbers of cubes. When Amy had difficulty, we went back to the one-to-one correspondence visual and wrote the numerals. We always circled the number that represented more and highlighted the numeral that was less. Next, we played *Hi, Ho Cherrio*™ to review the value of numbers and then reviewed the two cards in the Fact Pack. Finally, Amy wrote what *less* means: "Fewer means it is a less number."

Amy did surprisingly well on this lesson. She seemed to understand which numbers represented *more* and began to make the connection between the two words:

more and *less*. To strengthen her understanding of the concept *less*, I planned Lesson Four to involve using the word *less* repeatedly.

*Session 4: Lesson 3 (February 20, 2006).*We rewrote *Ten Black Dots* so that the story began with ten and decreased on each page, instead of beginning with one and progressing to ten. Page one of our book asked a question similar to the actual book, "What can you make with ten black dots? 10 dots can make __________." Amy completed the page by drawing ten suns (ten circles on the page, colored yellow). The following pages included the text, "One ________ is _______!" She filled in the blanks with, "One less is nine!" (See Appendix B for example pages from the book.) On each page, Amy filled in the first blank with *less* and determined the number that was one less. To help her do this task, she moved her finger down one number on her vertical number line. With each number, she changed the round dots into objects. For example, she turned the nine round dots into nine ladybugs. At the end of the book, we made a chart of dots similar to the last two pages of *Ten Black Dots*, but we started at ten and worked backwards, using the number line to assist us. This activity helped Amy to see that the lines of dots got smaller in value. Because the lesson involved so much writing and illustrating to represent the value of numbers, I did not see the need to include the modeling or games steps of the intervention.

Amy's reaction to this lesson was just as I had hoped. The lesson involved things I knew she enjoyed, such as books, writing, and drawing. In addition, Amy practiced using the number line, math vocabulary, and solved problems using the word *less* in a context that was meaningful and built on her literacy strengths. She was actively involved with an eager attitude and seemed to enjoy participating in the lesson. Her response to the lesson was very different than her response to math activities in the classroom, which she tended to avoid, if possible.

*Session 5: Lesson 4 (February 20, 2006).*The next week, February 20, 2006, we continued to work on the vocabulary words *more* and *less*. As we worked on mathematics vocabulary through the use of trade books, Amy began to understand and use the words *more* and *less*, associating the terminology with larger and smaller values of numbers. In the classroom, as I helped Amy with her work, she regularly asked me how to use her number line when adding and subtracting. I determined she did not understand that addition was putting values of objects together, so the amounts would get larger and subtraction was taking objects away, making the values less.

To help Amy make the connection between the words *addition* and *more*, I used a book in which the value of numbers increased by one as you turn each page (i.e., the number of animals increased by one on each page). I hoped the book, *Over in the Meadow* (Keates, 1999) would show Amy how the words *more* and *addition* were related. It was a simple book that Amy could read easily. Because it involved nature and real world items, I hoped the book would help Amy see how mathematics is in the world around her.

I started the lesson by asking Amy to read the book once. Then, as I read the book a second time, I created a number structure similar to the one in *Ten Black Dots*, but this time we used animals to create the number structure. We started by placing one animal beside the numeral one and then beside the numeral two, we placed two animals. This arrangement showed that the line of animals increased by one in each row. (See Figure 4.3.)

After building the number structure with animals, I modeled adding "one more", without using the word "addition". I asked Amy to build rows of animals and then add another animal, using phrases similar to the book. She did this repeatedly. I pointed out that one more was always the number that came after the number when counting.

We continued to practice using manipulatives to determine which number was one more than another number. We reviewed using *Hi, Ho Cherrio*™, constantly comparing the number of cherries in our baskets to decide who had "more" cherries. We also reviewed the cards in her Fact Pack. She correctly explained in her own words what the words *more* and *less* meant. I introduced the word *addition* at this time, purposely waiting until Amy had successfully practiced adding "one more" and seemed to understand that as one more is added, numbers become larger in value. I explained that mathematicians use the word *addition* to talk about what we had done that day. I showed her the sign for addition (+) and we added it to the Fact Pack. Amy looked at me as if a light bulb had just been lit and said, "Uh, I didn't know that!" I laughed at her excitement and encouraged her to tell her mother what she learned.

*Session 6: Lesson 4 continued. (February 21, 2006).*The second part of Lesson Four took place the next day on February 21, 2006. Because Amy did not write in her journal during the previous lesson, I decided to provide Amy with more practice using the concept of "adding one more". We began building the number structure by placing one manipulative beside the numeral one. Beside the numeral two I placed one manipulative right under it, asking her if I placed one more beside this one how many would I have. She answered two. Then I placed two manipulatives under the previous two, added one more and asked her how many I had. She answered three. We continued this process until we got to 10.

To review *more* and *less*, I introduced a game I played as a child, called Battle. Originally, I played with playing cards, but with Amy, I chose to make white note cards with the numerals one through ten written on them. First I arranged the cards in numerical order from one to ten horizontally on the floor, similar to a number line. After Amy reviewed the number line and the number order from one to ten, I mixed up the cards. To play, we both turned over a card at the same time and then built towers of Unifix© cubes to correspond with the numerals on our cards. We compared the towers, placing them side by side vertically to see whose tower had more cubes. The tower with more Unifix© cubes (the taller tower) won that round. As we continued to play, I referred her to the numbers on the number line (stored in her Fact Pack) that was laying on the floor in front of her. We reviewed how the larger number in value came after the smaller number on the line.

I also showed her a picture of the number line and asked her to name it, to practice math vocabulary we had worked on previously. We then practiced counting forward and backward, using the number line. She told me the meaning of *more* and the meaning of *less*, and we added a card to the Fact Pack with an addition problem on it. It said, “Five frogs were sitting on a log. Two more came up. How many frogs were on the log?” Then I asked her how a mathematician writes *more*. The correct answer was *plus*. Amy responded by saying, “Five plus two is seven.”

To conclude the lesson, I asked Amy to write in her journal about what *plus* meant. She wrote, “Plus means more. We count up on the number line.”

Based on her responses in the lesson and her journal entry, I knew that she was starting to associate the word *more* with addition. I was very surprised at how quickly she caught on to adding one number. Previously, Amy was completely confused in class when asked to add or subtract even the simplest numbers. She sometimes drew sticks or circles, but even when she used drawings to help her solve a problem, she often solved problems incorrectly.

Sessions 7: Lesson 5 (February 22, 2006). After completing Lesson Four when Amy demonstrated an understanding of addition, I was anxious to teach her about subtraction. I started this lesson by asking Amy to read *The Oreo® Counting Book* (Lukas & Raymond, 2000). This book starts with ten *Oreos®* and as you turn each page, one less *Oreo®* is pictured. As Amy read the book the first time, we used the number line to figure out how many *Oreos®* would be on the next page. She used her number line and moved backwards down the number line to figure out the number before the number on the page. Once she got to the number three, she could recall the number before from memory. Then, without looking at the number line, she told me that the number two would be "one less."

To model subtraction for Amy, I made up word problems such as "Amy has five Oreos®. She gives one to her brother. How many does she have now?" and I used real miniature *Oreos®* as manipulatives to take away one *Oreo®* and solve the problem. Amy and I completed many of these problems together and as she began to understand I allowed her to complete them on her own for practice, manipulating the mini-Oreos® to solve the problems. At times, Amy even made up word problems to solve.

To review numbers that are *more* and less, I used the *Oreos*® to ask Amy questions in different ways than I had before. I told her how many *Oreos*® I had and then asked her to get more Oreos® and less Oreos® than I had. In past lessons, each problem had only one answer. In this lesson, she had to think a bit differently about the problems because there was more than one right answer to questions. This task was very difficult for Amy and seemed to challenge her more than I expected. We discontinued the activity, because I realized she still needed to work on the concept *more* and *less*.

Session 8: Lesson 5 continued. (February 27, 2006). I started the lesson by reviewing *The Oreo® Counting Book* and counting backwards. One of the first questions I asked Amy was, “What happened to the numbers as they go down on the number line?” She answered with the word “smaller,” which showed me we were making progress. After reading the book a second time, I asked her to close her eyes and visualize the number line in her head. Amy was familiar with visualizing because we had visualized while reading in the classroom. I asked her to tell me the number at the top of the number line and then I asked what number was before that number. We continued with this routine and she answered every question correctly. She also told me that as the numbers went up, they got bigger and as the numbers went down, as they got smaller. I then asked her to use the number line to compare numbers and tell me if they were bigger or smaller. This was very challenging for her and she was incorrect about half of the time, even though I suggested that she look at the number line when she struggled. In the previous lesson, Amy could tell me the numbers that were bigger when she used the Unifix©

cubes, but this week when using the number line, she did not understand how to use it to identify numbers that were bigger or smaller.

I concluded that Amy needed more practice with manipulatives, so we played "Battle" again and built towers of Unifix© cubes to compare the numbers. This time though, we also found the numbers on the number line and I pointed out to her each time that the larger number came after the smaller number in value. During this exercise, she only missed one question.

Before Amy wrote in her journal, we reviewed the following cards in her Fact Pack: number line, counting forward and backward using the number line, the words *more* and *less*, and the symbol a mathematician uses to write *more*. In her journal, I wrote this problem: "Miss Mathews has nine *Oreos*®. Amy has ten. Who has more?" Amy wrote that I had more, which was incorrect. I told her to look at the numbers 9 and 10 on her number line and tell me which one came last on the number line. She found her mistake and corrected it in her journal.

At this point, I felt discouraged. We had been meeting for about three weeks and had finished five complete lessons. She seemed to understand the definition of *more* and *less* and could use manipulatives to answer questions, but she could not express what number is *more* using a number line or by visualizing the number line in her head.

Session 9: Lesson 6 (February 28, 2006). After reviewing the transcriptions of the first five lessons and watching the videos of the intervention sessions, it appeared that Amy was most successful when she used the number structure to help her solve problems. Consequently, I decided to use the number structure to help reinforce the word *addition* and help her practice adding one object to a set of other objects. Because the lesson using the book, *Over in the Meadow,* went so well, I chose to read a very similar book in this lesson. Amy read the book titled, *1 Hunter* (Hutchins, 1982). As she read the book, she and I both built the same number structure we had used in the previous lessons, but this time we used animal manipulatives to build it. As we built the number structure, I asked Amy a series of questions about which number was *more* and *less* in value, and she answered almost every question correctly. This was a big improvement from the last lesson. As Amy looked at the number structure, she commented that the numbers were getting larger as they went down. This was one of the first times she noticed a pattern on her own, without my prompting.

After reviewing which number was larger in value, I modeled for her how to add one more. I explained that when adding one, the answer was the number after. I wanted her to say the larger number and then use the number line in her head to move up to the next number, a strategy called *counting on*. Previously, when I asked her similar questions, she guessed a number or needed to count manipulatives to answer the question. Today, however, she seemed to catch on very quickly.

To help her practice, we made flash cards with the following addition problems: 2+1 =, 3+1 =, 4+1 = and so forth through 9 + 1= . She wrote the answers on the back of each card. We used these math facts flash cards to practice adding one.

Next, we reviewed the Fact Pack cards. First, I asked her what a number line was (which she answered correctly) and then she answered 29 out of 31 questions correctly. For example, she used the number line to answer questions such as “What number comes after 6?” or “Which number is bigger than 6?” We also compared values by playing

“Battle”. Before this lesson she made towers of Unifix© cubes to answer questions about which number is larger, but today I asked her similar questions, which she had to answer without the use of the number line or the Unifix© cubes. She answered every question correctly.

In her journal, I wrote “7 + 1 = 3.” I then asked her why this answer was incorrect. (This is an example of an answer Amy would have given before the intervention, so I wanted to see if she could verbalize why this was incorrect.) Amy wrote, “It is wrong because you forgot to count the number after seven.” This told me she understood that she needed to count just one more number after the number she was adding to, rather than using manipulatives to count out the larger number then adding one. In other words, she demonstrated that she was beginning to understand how to *count on*.

After Session 9, I was very encouraged. Amy seemed to be grasping the concepts *more* and *less*. She was using the number line correctly. It seemed that the number structure really helped her understand the value of numbers. It provided more support than a number line alone, because it showed objects that corresponded to the value of the numbers. She could see that the values of numbers were getting larger.

Session 10: Lesson 7 (March 6, 2006). Because Amy did so well adding one in the previous session, I wanted to build on that knowledge and introduce adding two, by starting with the larger number and going up two in her head or on a number line. I chose to read the counting book, *The Button Box* (Reid & Chamberlain, 1995). This book is similar to the other counting books, except that the items used to represent increasing values were buttons instead of other objects. It started with one button and progressed to ten buttons on the last page. This book also provided a familiar context within which Amy could solve problems.

After reading the book, I counted out ten buttons and then sorted them by how many holes they had. Amy found there were more buttons with four holes than with two holes. I then started adding one to the four buttons with holes and asked her how many buttons there were in all. She answered correctly each time, stating the next number when I added one. She did not have to count all the buttons as she had done previously but demonstrated counting on by saying, “4, 5,” instead of counting all the buttons from one to five. Then I showed Amy how to add two, using the buttons as manipulatives. She told me how many buttons were on the table, and then when I placed two more buttons beside them. I modeled for her how to count on by touching each button and saying the next two numbers as I counted up. For example, if there were four buttons and we wanted to add two, I counted, “4, 5, 6”. Together, we did this over and over as I called out various word problems related to the buttons. Amy grasped counting up two just as quickly as she understood counting up one. Due to the time, we stopped the lesson at this point.

Session 11: Lesson 7 continued (March 7, 2006). The next day we continued to practice adding two. Amy picked up right where we left off the day before. She took five buttons out of the jar and added two, pointing at each button and saying the next number as she counted. She added different quantities to the five buttons and got each one correct. To continue practicing, she made ten flashcards with each number, like in the previous lesson; this time, however, the problems all added 2 (e.g., 5 + 2 =). Amy also wrote the answer on the back of each card.

To review concepts from the previous lessons, we first reviewed the addition facts that added one. She went through each flash card and answered every one correctly, without using a number line or manipulatives. Next, we played *Hi, Ho Cherrio*™. I constantly used mathematical terms, modeling how to verbalize problems such as, "I added two cherries, so four plus two is…four, five, six." I hoped this review would help reinforce the concepts of adding and the value of numbers 1-10. Next, she reviewed all the cards in her Fact Pack as she did in previous lessons and concluded with a journal entry.

In her journal I wrote, "How do you solve 2 + 5?" I wanted to see if she would start counting with the larger number and count up two. She wrote, "First you start with the bigger number and count the numbers after the bigger number [to] 7." This told me she understood five was larger in value than two, and she demonstrated how she used a mental number line to solve the equation.

I taught the concept of *counting on* in the classroom previously, but Amy did not understand the concept then. After working with Amy one-on-one, she demonstrated that she now understood *counting on*.

Session 12: Lesson 8 (March 9, 2006). Because Amy did so well adding the digits one and two to larger numbers between one and ten, I introduced subtracting one. To introduce this concept, I read the book, *Elevator Magic* (Murphy & Karas, 1987). In this story, a little boy rides the elevator down ten floors, stopping on different floors to run errands with his mother. The elevator key pad looked just like a number line with the numeral one on the bottom going to the number 9 at the top, so I discussed that with Amy, showing her how the main character in the book went down the floors. I asked Amy what a mathematician calls going up on a number line and she replied "addition". I then asked her what a mathematician calls going down on a number line, but she was unsure. I explained that mathematicians call that procedure subtracting.

To model subtracting one for Amy, I created problems using the elevator key pad and modeled how to use it to solve various story problems. After she watched me several times, I continued to tell stories involving an elevator, but this time, she moved her finger down floors to answer the questions. She moved quickly on the elevator key pad and answered all of the questions correctly. Next, Amy used Unifix© cubes to help her solve word problems that focused on subtraction.

Amy made subtraction flashcards with the numbers one through ten and subtracted one from each number (e.g. 10 – 1 =, 9 – 1 =). She used the number line to make sure each answer she wrote on the back of the flash card was correct. I then showed her each flash card in no particular order, and she answered each question correctly. Because she did so well taking one away, I introduced subtracting two. We made flash cards and she used her number line to identify the number that had the larger value and then moved backwards two numbers to find the answers.

Before journaling, we reviewed the addition facts in her Fact Pack. She answered every question correctly, without the support of a number line or manipulatives. To review the value of numbers, we played a new game. We rolled a die and then spun a spinner with more or less on it to determine if we had to name a number that was more or less than a number on the die. Amy did pretty well with this game, but she did not answer every question correctly. We also reviewed the vocabulary cards, number line, more, less, and addition facts in her Fact Pack, helping her commit the math vocabulary to memory.

In her journal I wrote, “How do you solve 8 – 2?” She answered by writing, “First you start with eight and go down two. The answer is 6.” She answered correctly and even added a down arrow to illustrate going down on the number line.

Session 13: Lesson 9 (March 20, 2006). By March 20, 2006, Amy had learned the values of numbers without using manipulatives. She understood the meaning of the words *addition* and *subtraction* and seemed to understand that adding was combining numbers, while subtracting meant the numbers got smaller. She understood how to add one and two to a number and subtracting one and two from a number.

I chose to read the *Icky Bug Counting Book* (Pallotta & Masiello, 1992) because it was a nonfiction counting book that started with one and continued to 20. On each page the author shared facts about the bugs. I chose the book because Amy enjoyed reading nonfiction and it addressed numbers from 1 to 20, rather than only 1 to 10, like the other books I had used.

Amy read the book and built the same number structure we had built in an earlier lesson, but used bug counters instead of animals or buttons. When we got to the row of ten bugs, I explained that we group the ten items together and move them to what we call the ten’s place. We put the ten bugs in a cup and then I moved the cup down the left side of the number structure. I showed her how to write a group of ten and a one. I recorded the number on a chart with two columns labeled ten’s and one’s at the top, and asked her what that numeral was. She recognized it was an eleven. I continued to move the cup down the left side of the number structure, asking her what the numbers would be and recorded them on a ten's and one's chart. (See Figure 4.3.)

Ten's	One's
1	1
1	2
1	3
1	4
1	5

Figure 4.3 Ten's and One's Chart.

Session 14: Lesson 9 continued (March 21, 2006). Because it took so long to read the *Icky Bug Counting Book*, I continued this lesson the next day. On March 21, 2006, I reviewed what Amy had learned in the previous lessons by rebuilding the number structure and re-explaining that when we got to ten, we grouped the bugs. Then I asked Amy to count from 1 to 20 and reviewed what a group of ten and a one was. She did not remember at first, but once she looked at the chart we had made, she remembered. We built the same number structure with bugs that we used in the previous session. We moved the cup of ten bugs down the left side of the number structure and Amy reviewed the numbers from 10 to 20. As we moved the group of ten bugs, I explained the value of the numbers from 10 to 20 and asked Amy questions about the value of those numbers. She answered every question correctly. She continued to group 10 bugs into cups and moved them down the number structure, stopping in front of each row of bugs and saying the number illustrated by the number structure (11, 12, 13, and so on).

To review the concepts Amy had already learned, we played a new game, *Chutes and Ladders*™. Amy was familiar with the game; she and her mother had played it at home. As we played, I asked Amy to identify the number she landed on and if the numbers were getting larger or smaller in value. I explained that the ladders were like adding and the chutes were like subtracting. We also compared whether she or I had the higher number, as we moved around the game board.

After playing *Chutes and Ladders*™, we also reviewed the value of numbers greater than 10 by showing the value of the numbers using a cup of bugs to represent the ten's place and then one-to-one correspondence to show the one's place. We also reviewed the words *more, less, addition*, and *subtraction*. Amy reviewed the addition facts (adding one and two), and the subtraction facts (subtracting one and two).

Finally, in her journal, I asked Amy "How are 14 or 17 different?" She wrote, "Seventeen is bigger than fourteen because seventeen comes after fourteen. It is bigger. Both of them have one ten, but one has four ones and the other number has seven ones."

Session 15: Lesson 10 (March 27, 2006). I began the lesson by reading *Chicka, Chicka, 1, 2, 3,* (Martin, 2004), a favorite book of my students because of its rhyme and rhythm. In the story, the numerals 1 through 20 run up the tree and then the book continues counting by ten's to 100.

I reviewed the ten’s and one’s chart we had made in our last lesson and then started a lesson developed by Marilyn Burns that I had observed another teacher teach. It involved counting cubes and grouping them in ten’s to make them easier to count. I placed a pile of cubes on the floor and asked Amy to start counting the cubes. Each time she counted about half of them, I interrupted her and she had to start counting all over again by one’s. After I interrupted her several times, I suggested that she group the cubes into groups of ten and then count them. She put the cubes in groups of ten and successfully counted the groups, even though I interrupted her as she counted. This was something so did not do accurately when assessed with the *TEMA-3*.

To help Amy master the concept of place value, I asked her to make several sets of numbers, such as “Show me thirty-four.” She correctly showed me four different numbers. We concluded the lesson for the day and continued the lesson two days later.

Session 16: Lesson 10 continued (March 30, 2006). We picked up right where we left off, reviewing how numbers are grouped. We then constructed a number structure with wooden cubes. I asked Amy various questions, such as which number comes before (and after) as well as which number is bigger in value. She answered all these questions correctly. We then looked at the place value chart that represented numbers from 11 to 20 and labeled ten’s and one’s. I then put to bundles of 10 sticks in front of the row with one cube. She said “21” and then moved the ten sticks down the number structure in front of the number two and made 22. We then worked on what numbers came before and after numbers from 20 to 50 and the value of those numbers. She also identified numbers written in words, such as four ten’s and six one’s.

As we worked with the number structure, Amy started to get confused with the language I was using. On the number line, numbers get larger as they go up, but on the number structure they got larger as the numbers go down. I realized this was confusing her and changed the number structure to look like the number line, starting with one at the bottom. (See figure 4.4.) This seemed to clarify the language of mathematics for her, helping her see that numbers increase in value as they go "up" in value and up on the number structure and number line.

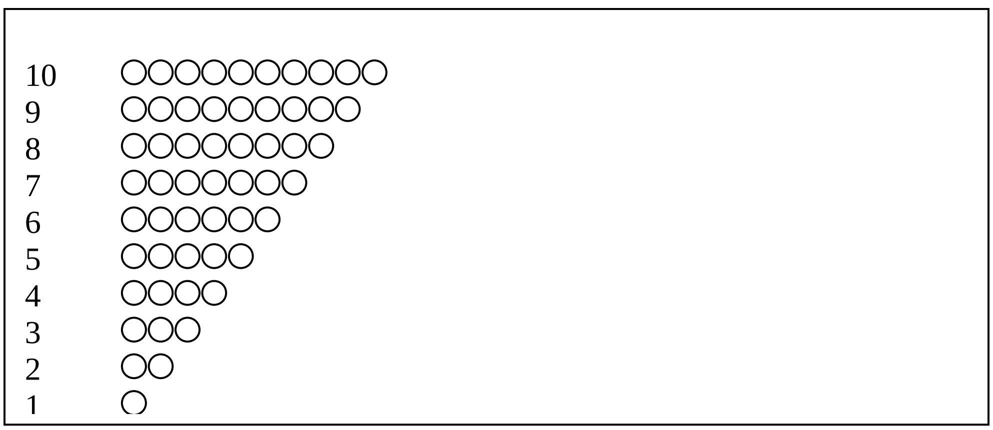

Figure 4.4 Number Structure.

Before ending the lesson, we reviewed the words in the Fact Pack words: *less*, *more*, *addition*, and *subtraction*. We also added a card stating: "1 ten and 5 ones." "15" was on the back. When I showed her this card, I asked her what number this was and to make the number using ten's and one's sticks. She answered correctly, showing me one ten stick and five one's cubes. We also reviewed her addition facts and subtraction facts. To review the numerical order of numbers and the value of numbers, we played *Chutes and Ladders*™, identifying how many ten's and one's were in each number. Amy really enjoyed playing this game. As she landed on numbers, we decided what number came before and after as well. Occasionally, she made the number using ten's and one's sticks.

To conclude the lesson and assessed the knowledge she had acquired. I asked Amy to tell me in her journal, "How are 42 and 32 the same and different?" She wrote, "They both have a two in it. They are different because one of [them] have [sic] a four in it and the other has a three in it." Amy experienced difficulty with this journal entry; I had to really talk her through it. I had hoped she would tell me they had the same amount of ones but had a different number of tens. I also hoped she would tell me 42 was greater in value than 32. I could tell she still needed to work with the concept of place value and how to verbalize it.

Sessions 17: Lesson 11 (April 5, 2006). In order to continue discussing the concept of place value and sequencing numbers from 1 to 100, I chose to read the book, *A Chair for My Mother* (Williams, 1984). This book is about a young girl who saves money to buy a chair for her mother. I used the idea of buying a chair as the context for the lesson. Amy chose the price for the chair: $79.

Then, I showed Amy an interactive hundred's board with the digits 10, 20, 30, 40, 50, 60, 70, 80, and 90 placed in it. (See Figure 4.5.) I then put the digits 79 on the hundred's board and modeled how I knew where 79 was. I explained to Amy that if the number was larger in value than 79, then we would have enough money to buy the chair.

If it was less than 79, we would not have enough money to buy the chair. I randomly drew cards from a deck with digits 1 to 100 written on them and explained how I knew if the number I chose was more or less than 79 by looking at the number in the ten's place. Eventually, Amy began drawing the number cards and deciding if the numbers were more or less than 79. We continued this activity until the lesson was over.

0	1	2	3	4	5	6	7	8	9
10									
20									
30									
40									
50									
60									
70									79
80									
90									

*Figure 4.*5 Hundred's Board.

As I reviewed previous transcriptions and teacher comments, I realized that Amy did not appear to understand the vocabulary words *before* and *after*, and how they are used in mathematics. I decided to relate those words to a similar concept in books, comparing *before* and *after* to the words *beginning* and *end*. I made a chart (see Figure 4.9) with the words *beginning*, *middle*, and *end* below the squares. I wrote the word *before* above the first square that I had labeled with the word *beginning* and wrote the word *after* above the square labeled with the word *end*. I used this chart in subsequent lessons.

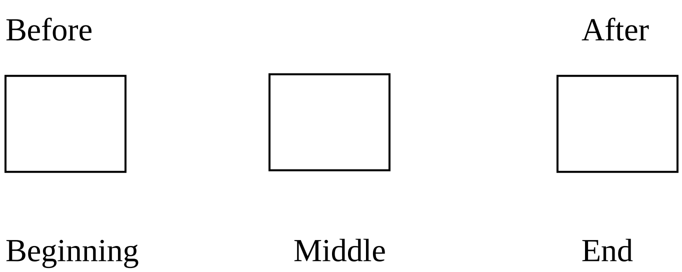

Figure 4.6 Before and After Chart.

Session 18: Lesson 11 continued (April 6, 2006). I first asked Amy if she could tell me the beginning, middle and end of the book, *A Chair for My Mother* (Williams, 1984). I then showed her the Before and After Chart. I explained the similarity between these words and then started placing number cards from the hundred’s board in the middle square on the Before and After Chart. I then asked her the number that came before and after numbers in her Fact Pack. She answered all the questions correctly when she used the chart.

We then took turns drawing a number from the remaining cards and putting each number on the hundred's board. While filling in the board with the number cards, Amy started to make connections with the numbers and see the pattern of one to ten in each number. She pointed out to me how each row had the same number of ten's and how the one's place was the same in the vertical rows. (This was a monumental moment in the intervention lessons, because Amy was discovering the patterns of numbers independently, which should allow her to complete many mathematics tasks.)

Next, we reviewed the items in her Fact Pack. Amy had trouble answering several subtraction problems and used the number line to help her find the answers. We finished reviewing by playing *Chutes and Ladders*™ and identifying the numbers before and after the space we landed on. She also told me how many ten's and one's were in each number. We also rolled a die and spun a spinner and added the two numbers; we practiced adding the numbers each time we moved our game pieces.

To assess if Amy had progressed since Lesson Nine, I posed a problem, "Amy wants to buy a bag of lollypops and they cost $4. She has $6 in her purse. Does she have enough money?" Amy wrote, "Yes. She has enough because she has six dollars and she is going to take four away." She understood that six was more than four and she would use subtraction to solve the problem.

Based on how easily she did these tasks, it appeared that she did not understand the vocabulary, *before* and *after*. Perhaps she understood the concept but not the vocabulary. By building on her understanding of the beginning, middle, and end in stories, she seemed to understand the concepts of *before* and *after*.

Session 19: Lesson 12 (April 18, 2006). I wanted to continue to work on counting and ordering numbers from 1 to 100, so I chose to read the book, *The King's Commissioners* (Friedman, 1995). The book uses the king's confusion of counting 47 commissioners to illustrate how you can count by one's, two's, five's, or ten's and get the same amount. Amy did not understand this concept when I taught it previously in the classroom.

Amy read the book to me with ease and then I used one's cubes to model the grouping of 47 by two's, five's and ten's. She practiced counting and I modeled how 47 could be grouped in different ways using one's cubes and showed her how it did not change the value of the number and the number did not change.

I introduced a game called "Race to 100". I had observed Marilyn Burns demonstrate this game in a video taped lesson. In the game, students practice regrouping and identifying the value of numbers. I laid a mat divided into three sections labeled hundreds, tens, and ones in front of us. First, I modeled how to play the game. I rolled two dice and added them together, starting with the larger number and adding the lower number. I then showed Amy with manipulatives how that number could be represented using ten's and one's sticks. Next, I explained that the object of the game was to get ten ten's sticks, which would make 100. Then Amy had a turn, and I helped her remember how to start counting with the larger number and count up. Amy used ten's and one's sticks to represent the number on the die. When I rolled again, I gathered the ten's and one's sticks and modeled how to regroup the sticks by using ten one's. I had been discussing this concept in class as well, so I showed her on a whiteboard the numeral that was represented by the ten's and one's sticks. We continued to alternate turns, rolling the dice, adding the two numbers, and representing the numbers with ten's and one's sticks, and regrouping them when needed. I also continued to write problems determined from the rolling of dice to show her on paper what number we were forming with the ten's and one's sticks. (We had played this game in the classroom in first and second grade. During this lesson was the first time Amy played this game with ease, gathering the ten's and one's cubes without much hesitation or thought, which demonstrated to me that she understood the concept.

Due to the time it took to play the game, our time together ended and we continued the game later.

Session 20: Lesson 12 continued (April 19, 2006). We began the same game we were playing the last time we met. Amy had 61 represented on her ten's and one's mat and I had 58. We continued rolling the two dice and adding the numbers with our ten's and one's sticks.

Amy reached 100 first and the game ended. We reviewed the items in her Fact Pack. We also reviewed the numbers from 1 to 100; I stated numbers and she told me the number before and after, using the Before and After chart I made in the previous lesson. We also reviewed place value by making 15, using the ten's and one's cubes.

Finally, Amy solved the problem 28 + 19 in her journal without using manipulatives or a number line. She wrote, "I am going to start in the ones. I put eight in my head and went up nine. I put the seven in the one's and then one in the ten's. I added two plus one and which was three. And I added three plus one that equal[s] four."

I was very pleased with the progress Amy showed adding larger numbers and carrying the one to the ten's place. On her weekly classroom test, she did not miss any problems that involved regrouping, which was unusual. However, she missed every problem that involved borrowing, which I had not taught during the intervention lessons.

Session 21: Lesson 13 (May 8, 2006). Two weeks later, on May 8, 2006, Amy and I met again for our last lesson. The previous week, our state gave its curriculum test to all students from second to twelfth grade and the week before that I could not leave the classroom because of the time needed to review and practice for the test.

Knowing this was the last lesson and school would be out the next week, I worked with Amy on regrouping. I chose the book *Alexander Who Used to Be Rich Last Sunday* (Viorst,1987) to introduce this concept. In this story, the main character, Alexander spends a dollar on various items throughout the day, leaving him with a bus token at the end of day.

To model subtracting with regrouping, I started with a one hundred flat and took away the amounts Alexander spent, showing her how the 100 block decreased in size and value as I took 1 ten stick away and traded them for 10 one's.

To allow her to practice, I made a poster of items ranging in price from nine cents to forty-five cents. I gave her a hundred's flat and she bought various items off a poster, trading her hundred's block for 10 ten sticks, and then trading a ten stick for 10 one's. As she used the manipulatives, I wrote on paper what the subtraction problem looked like when it was written as a number sentence. For each item she bought, I showed her how to subtract the amount of the toy from the amount of money she had. She continued regrouping manipulatives until she had used all of her money.

Session 22: Lesson 13 continued. (May 9, 2006). The next day, we finished the lesson, starting with a 100's block, trading it for 10 ten sticks and trading 10 sticks for 10 one's as she bought items off the poster. We did this together twice and the second time, she used the manipulatives and worked the problem on paper without assistance.

To review all the concepts we had learned during the intervention, we looked at the word cards in the Fact Pack. We reviewed the words *more*, *less*, *addition*, and *subtraction*. We looked at the number line and she told me which numbers were larger in value. We also reviewed the Before and After Chart with the corresponding words: beginning, middle, and end. I chose numbers from the interactive hundred's board and she told me the numbers that came before and after a chosen digit. Lastly, we reviewed her addition and subtraction facts.

For her last journal entry, I asked her to write the difference between the numbers 23 and 32. She said, "23 has 2 tens in it and 32 has three tens in it."

Results of Data Analysis

The question that guided my study was "How can I help Amy develop a conceptual understanding of number sense? Based on the analysis of the expanded transcripts of the intervention, I found that the following tools that helped Amy.

Number Line. Amy's number sense improved with the use of the number line, one of the common instructional tools I used in almost all of the intervention lessons. Starting in Lesson One, I used a hortizontal number line to teach Amy the value of numbers. At first, I asked Amy to use the number line to help her as she counted from 1 to 10, pointing to each number going left to right as well as simply identifying a number line and using the word number line verses the word "thing" or other language she used. In the second part of Lesson One (Session 2), Amy drew a number line by looking at the number line that was in her Fact Pack. I had instructed Amy on how to do this in class, part of our classroom Mississippi Curriculum Test practice. On the test, students can use a number line if they drew it on their own. To help Amy understand the value of the number increased as the numbers increased, I showed her by pointing to the numbers on the number line that as the numbers went up on the vertical number line, the value when up. I modeled answering questions, such as which is bigger four or five, by putting my finger on both numbers and explaining that whichever number was on top, was the larger number. She correctly answered questions about which number was larger in value by using the number line as a visual cue.

I continued to use the number line in Session 4 by asking her to put her finger on the two numbers she was comparing and choosing the higher number on the number line as the bigger number. By Session 5 (Lesson Four), Amy made a horizontal number line by correctly ordering the digits 0 to 9, which were written on note cards to resemble a number line. She also used the number line in her Fact Pack to add one to the numbers 1 through 10. Amy could not automatically add one to a number nor could she automatically answer which number was larger without the number line. However, when she used the number line, she answered 10 out of 10 questions correctly when asked which number was more than another. For example, I asked her, “Which is more, 4 or 3?” Later, by Session 6 (Lesson Four), she could automatically count forward and backwards from 1 to 10 and 10 to 1 without the aid of a number line. She visualized the number line to complete these tasks.

In Lesson Five (Session 7), I referred to the number line 24 times. In most instances, I used the number line to support Amy as she tried to answer questions such as “Which number is after eight? Which number is before eight, which number is bigger, 4 or 5?”. However, twice during Session 7 Amy used the number line to answer a question without being prompted to do so. Like in Lesson Four, Amy demonstrated that she could count forwards and backwards (1 to 10 and 10 to 1) without using the number line, but she needed the number line to answer questions about which number was greater in value. With the number line, she answered most of the questions correctly but did not do so automatically, still having to take time to think about it. At this point, Amy continued to look at the number line, not visualizing it and still needed practice using the concrete number line to identify the greater number.

By Lesson Six (Session 9), Amy began to answer which number was bigger using the number line, and after working with the number structure, she compared the number line and the number structure, noticing the similarities and differences. She said, “Look, when you go down [on the number structure] the numbers are getting bigger, but on the number line when I got down, the numbers get smaller.” She also visualized the number line to solve problems that required her to add one more. When I asked Amy how she knew what six plus one was, she said, “I counted up in my head.”

In Lesson Seven (Sessions 10 and 11), Amy continued to use the number line to find answers to addition problems that required her to add two to numbers one through ten, starting with the largest number. By the end of the lesson (session 11), Amy could identify the larger number and count up two without the help of a number line. In addition, Amy identified the number line and, without referring to a number line, she counted forward and backward, identified which number comes after another number, and identified what number was larger than another number. She answered these questions automatically (without hesitation), which she was unable to do in the previous sessions.

Lesson Eight (Session 12) was the first intervention lesson when I focused on teaching Amy to subtract using the number line. In this lesson, I read the book *Elevator Magic* (Murphy, 1997) in which a boy goes down floors of a building, stopping on each floor until her reaches the bottom. On each page, he pushes a button that represents one number less on the elevator key pad to take him to the floor under the floor he is on. I compared the elevator pad (the numbers 1 – 10 in a vertical line) with the number line. Amy used the number line to practice subtracting one and two from different numbers by placing her finger on the first number in the number sentence and moving her finger back one to solve the number sentence. I also used it to help support her five times during the review section when she missed the answer to a question. For example, when I asked her which number was closer to three, five or eight, she answered eight. I showed her on the number line that five was closer to three. When she missed other problems, I showed her which number was correct by using the number line. By the end of the Lesson Eight (Session 12), Amy was subtracting automatically with the use of the number line. Amy learned how to subtract, using the number line, more quickly than she learned to use it to add. I spent Lessons Five and Six (Sessions 8 and 9) teaching Amy to add one and Lesson Seven (Sessions 10 and 11) teaching Amy to add two, but Amy learned how to subtract one and two after only one session. Amy's understanding of the value of numbers was improving.

In Lesson Nine (Sessions 13 and 14), I built on Amy's knowledge of the values of 1 through 10 and included problems that extended to digits 11 through 20. With the help of a number line that included digits 0 through 20, Amy correctly answered questions such as 16 is bigger than 14, 19 is bigger than 17, and other problems comparing numbers ranging from 1 to 20.

In Lesson Ten (Sessions 15 and 16), Amy began to demonstrate her understanding of the value of numbers greater than 20. I used the number line eight times to support her when she incorrectly answered questions about which number came before and after particular numbers as well as to help her add and subtract one and two from numbers 10 to 20. Following this session, Amy began manipulating and ordering the digits from 1 to 100. She could automatically count to 100 beginning at any number and tell me which number was bigger. The number 28 was written on the hundred's board and she told me that she knew where the number 26 belonged on the hundred's board because, "I counted and I have the number line in my head and I can see six going to eight."

In the classroom, I noticed that Amy had completed some addition and subtraction problems without using a number line. This showed me that Amy was applying in her classroom work what she was learning during the intervention sessions. She did not use a concrete number to solve these problems.

> Let's go over addition facts and subtraction facts. I noticed on your test today in the classroom you were not using your number line. You did such a good job. You are putting that bigger number in your head and you are counting up.

Then Amy proceeded to correctly answer 56 out of 61 addition and subtraction questions, without using a number line. When she said the wrong answer, usually it was because she added when the problem said to subtract. I supported her by showing her how to arrive at the correct answer on the number line.

In Lesson 12 (Sessions 19 and 20), my goal was to teach Amy to identify the larger of two numbers greater than 20 and use the number line in her head to count up when adding with regrouping. In the second part of the lesson (Session 20), Amy answered 2 + 0 automatically and continued to answer simple addition questions automatically, using the number line in her head. She also added 3 + 8, saying aloud, "8, 9, 10, 11." This answer demonstrated that without prompting Amy knew eight was the larger number and she should count up three. She also answered this question without using the number line. Later in the session, I asked Amy to identify the number line, which she was able to do. She identified the number it started and ended on. She also answered a series of questions, using the mathematical vocabulary of *before* and *after*, tasks she previously had trouble with and visualized the number line to answer those questions correctly. Moreover, I saw evidence that Amy was using in class what she learned in the intervention sessions. She answered every question correctly that involved regrouping on the weekly classroom assessment.

I ended the intervention sessions with Amy on May 8, 2006. The final lesson was an introduction to borrowing, which Amy had already been practicing in class. I knew Amy was not likely to master this skill in a single lesson, but I wanted her to have the opportunity to work with me one-on-one and to use manipulatives to practice solving the problems. At the end of the lesson in the review step, I asked Amy a series of questions about the number line. First, I asked, “If I wanted to know the number *before* on the number line, would I go up or down?” She answered, “up” and then when I asked if she had visualized the number line, she corrected herself. I then asked her which direction she would go on her number line if I asked her to name the number *after* another number and she answered “up.” She then correctly answered addition and subtraction facts, with no assistance from a number line.

Number Structure. I introduced the number structure in Lesson One, after reading *Ten Black Dots* (Crews, 1982). In the back of the book, the author included a chart with the numeral on the left and dots representing the value of each number to the right. Zero was placed at the top of the structure and the numbers increased as they went down. (See Figure 4.3.) The dots show that the numbers increase in value because the rows become longer by one dot. At the end of Lesson One, I thought Amy could use the number structure to answer questions, such as which number is bigger, but she could not. This surprised me because the visual depiction seemed to illustrate the problems so explicitly

In Lesson Three, I used a number structure to illustrate numerals decreasing in value. This time, the digit 10 was at the top of the chart and the digits decreased to zero. Beside the digits on the right side of the page, Amy drew dots to represent the value of each digit. Unfortunately, even after drawing the dots, she still was unable to correctly answer questions about the value of the numbers from 5 to 10.

In Lesson Four, after reading a book with frogs in it, Amy manipulated plastic frogs to create a number structure, starting with one frog, then continuing to build the number structure with frogs. Then I asked Amy to answer questions about the value of the numbers 1 through 10. She missed many of these questions when I asked her which number was bigger, even when looking at the number structure. The next time we met to finish Lesson Four, we played a game called Battle. In the game, we turned over note cards with a digit from one to 10 written on them, and then decided which number was larger in value by building a line (or tower) of Unifix® cubes (similar to a row of the number structure) and seeing which line was longer. We did this 15 times and Amy answered every question correctly. Building the number structure with manipulatives provided support for her as she solved the problems.

In Lesson Five (Session 8), I did not use number structure, but we played Battle again, making rows with manipulatives (similar to the number structure) and compared the value of the digits from one to 10. Amy answered 33 or 34 questions correctly.

In Lesson Six (Session 9), Amy built the number structure out of animals after reading *One Hunter* (Hutchins, 1982). During this lesson, I asked Amy to “show me the value of three” or asked her “how many more.” During the lesson, Amy began using mathematical language, something she had not done previously. She said, “We need one more” when I asked her to “make the next number”. Then, without being asked, Amy made the following statement. “When you go down, the numbers are getting bigger” on the number structure. She also told me that on the number line, the numbers go up (in value), which is different than the number structure. The next day, as we continued Lesson Six, I asked Amy questions about the values of numbers, using the number structure to help her. Although she did not answer quickly (automatically), she answered most questions correctly. The one she missed probably had to do with the language I used when asking the question. I asked her, “Which number is bigger, seven or eight? She answered, “seven”. When I looked at the line, the animals in line seven were bigger in size than the animals in line eight. When I clarified my question, she answered correctly.

The next time we used the number structure was when we began working on the value of numbers from 10 – 20 in Lesson Nine (Sessions 13 and 14). Amy made the number structure from 1 to 10 and answered seven of eight questions correctly. Then I used the number structure to model moving a cup of ten bugs in front of the numbers zero through nine to make the numbers 10 through19. After modeling that 10 + 1 through 10 + 9 were the numbers 10 through 19, Amy immediately answered correctly that 16 was bigger than 14, 19 was bigger than 17, and 20 was bigger than 10. The next time we met to finish this lesson, Amy rebuilt the number structure and when I placed a cup of ten bugs to the left of the rows of bugs, Amy counted from 1 to 40, using the number structure to visually represent the numbers and did not miss a number. Before the intervention lessons, Amy could not do this. Next, I asked her a series of questions about the numbers 11 through 20, such as “Which is bigger, 18 or 13?” and she answered every question correctly.

Lesson 10 (Sessions 15 and 16) involved Amy building the number structure, this time using cubes instead of bugs. After making the number structure, I put multiples of ten sticks to the left of the rows of cubes to make numbers 10 through 50, modeling how to represent 10 through 50 with ten sticks and one's cubes. Amy visually represented numbers greater than ten, using the ten sticks and cubes correctly, stating how many ten's and one's were in the numbers. When I asked questions about which number was greater in value, I noticed that Amy still struggled. At this point, I realized the contrast between the directionality of the number line and the number structure. On the number line, as you move *up*, the numbers get larger in value. However, as you move *down* the number structure, the numbers get larger. As a result, I switched direction of the number structure so that it was consistent with the direction of the number line. (See Figure 4.6). After making the number line and number structure look similar, Amy was more confident than ever when answering which number was greater in value and which number came *before* and *after* other numbers. This was the last time we built the number structure in the lessons.

Trade Books. Beginning in Lesson One, I anchored each skill I wanted Amy to master with a book that would help her recall the lesson and to use mathematics in a memorable context. Table 4.3 shows the books I used to anchor each skill and the intervention session in which it was used. (See Appendix C for a trade book reference list.)

Table 4.3

Trade Books Used in Intervention

Book	Skill	Lesson Number
Ten Black Dots	Determining which number was more	1
Just Enough Carrots	Determining which digit was less	2
Over In the Meadow	Stating what one more number is	4
The Oreo Counting Book	Practicing which digit was less in value	5
One Hunter	Adding one	6
The Button Box	Adding two and three	7
Elevator Magic	Subtracting one, two and three	8
Icky Bug Counting Book	Reading & determining the value of digits 11 – 20	9
Chicka Chicka One, Two, Three	Counting to 100	10
A Chair for My Mother	Determining the value of numbers 20 – 100	11
The King's Commissioners	Addition with carrying	12
Alexander Who Used to be Rich Last Sunday	Subtraction with borrowing	13

Another purpose for using trade books was to provide Amy with anchors for the skills I taught with each one. Amy demonstrated that, at times, she used these books to help her remember how to solve problems. For example, she made comments such as “like when read….” I also referred to the books to remind her how we had worked a similar problem. Table 4.4 provides examples of how I used books as an anchor to a skill.

Table 4.4

Trade Books as Anchors

Teacher Support	Skill	Lesson # (Session #)
I supported Amy by saying, “Remember when we read the book, *Just Enough Carrots* and we talked about numbers that were less?”	less	4 (5)
I supported Amy with the word smaller, by saying it was like when we had fewer and fewer Oreos© in *The Oreo Counting Book*.	smaller	4 (6)
I supported Amy when she was learning to add, by reminding her, “It is was like when we kept adding one more dot when reading *Ten Black Dots*.”	adding	6 (9)
I supported Amy when she was learning how to subtract, by comparing subtraction and the numbers getting lower in value to the elevator going down the floors in *Elevator Magic*.	subtraction	8 (12)
I supported Amy when she was trying to subtract by saying “it is like we are going down on an elevator [in *Elevator Magic*].”	subtraction	8 (12)
I supported Amy when she was learning how to subtract by reminding her about the book *Ten Black Dots* and how we added black dots, but when we subtract we take away black dots.	subtraction	8 (12)

Table 4.4 (continued)

I supported Amy when she was trying to recall what subtraction was by reminding her it was like an elevator going down (Referring to *Elevator Magic*). The numbers got smaller and smaller.	subtraction	8 (12)
I supported Amy when trying to help her when trying to recall how many tens a number had, by reminding her it was like when we put ten bugs in cups and counted them with the *Icky Bug Counting Book.*	ten’s in a number	9 (13)
I supported Amy when she was trying to recall what ten’s and one’s were by saying it was like when we had one cup of bugs with the *Icky Bug Counting Book.*	ten’s and one’s in a number	9 (14)
I supported Amy when she was having difficult remembering the definition of before and after by comparing the word *before* to the beginning of *A Chair for My Mother* and the word *after* to the end of the story. The number we were talking about was in the middle of the two numbers.	before, after	11 (17)
I supported Amy a second time by reminding her the words *before* and *after* were like the beginning and end of *A Chair for My Mother*.	Before , after	12 (19)

Journaling. In her first journal entry, Amy defined the word *more* by writing, “More means that a number is bigger. And it comes after any number on the number line.” Even though Amy verbalized this, she could not solve problems that asked her to identify numbers that were more than others.

The third time Amy wrote in her journal, I posed the scenario: Miss Mathews has nine Oreos®. Amy has 10. Who has more?" Amy wrote, "Miss Mathews has more than Amy." This was an incorrect answer, which showed me that Amy did not understand the concept of *more*. At this time, I chose to explain to Amy why she was incorrect and guide her to verbalize through writing, "Amy has more than Miss Mathews. We looked on our number line." After this lesson, I tried not to help Amy in any way when she was journaling. I changed how I viewed journaling. Rather than using journaling as a way to help solidify her knowledge of a concept, I used the journal for assessment purposes to see how much she understood about a concept, when she needed more assistance with a concept, and when she was ready to learn a new skill. The journaling helped me plan subsequent lessons.

Value of Numbers 6 -10. Once Amy understood the concept of ten's and one's, her ability to count and add extended to include numbers 1 to 100. Once I realized that Amy did not understand the value of the digits 6 through 10, I immediately focused my instruction on this concept.

Lesson One focused on the vocabulary word *more* and comparing groups of manipulatives, choosing which digit and group of manipulaitves was more. Lesson Two focused on the vocabulary word *less*, while Lesson Three helped Amy comprehend this concept by rewriting the book *10 Black Dots* (Crews, 1982). Lessons Four through Eight continued to focus on working with the digits 6 through 10, adding one, adding two and three, and subtracting. We continued to review the value of the digits six through 10 so that Amy could recognize the values automatically, without using manipulatives.

Lesson Nine focused on the numbers 11 through 20. I began by reading *The Icky Bug Counting Book* and used bug manipulatives to model building the number structure. I then put ten bugs in a cup and placed the cup to the left of the row of numerals. As I moved the cup in front of a different row, I modeled for Amy how a mathematician writes what was visually represented. To represent the one cup of bugs, I placed a one on the paper and for the row of zero bugs, I placed a zero. Amy immediately recognized this number and stated it was 10. I moved the cup of ten bugs beside the row of one, and again modeled for Amy how a mathematician would write this number. Amy told me the number was 11. This process continued. I then modeled for Amy how I knew which of these digits was bigger in value, by looking at the one's place. This language was familiar to Amy because she had heard about the ten's and one's place each day of school since January of her first grade year until this day, March 6, 2006, which was one year and two months later. I provided Amy with guided practice time where I asked her to compare two digits and tell me which was greater in value. I asked her many questions and , we referred to the number structure for support each time. In additioin, I explained why I knew she was correct to model the thinking process. Amy and I played *Chutes and Ladders* ™ to help her automatically identify which digit was greater in value, constantly comparing which player landed on the greater digit on the game board. Amy demonstrated knowledge of the value of numbers 11 – 20 by answering the question, "How are 14 and 17 different?" Amy wrote, "Because 17 comes after 14. It is bigger. Both of them have one 10, but one has four one's and the other number has seven one's.

After this lesson, Amy progressed rapidly, demonstrating her knowledge of the value of numbers from 20 - 100 (Lesson 10), adding with carrying (Lesson 12), and subtracting with borrowing (Lesson 13). Lessons One through Nine covered the numbers 5 through 10, but it only took four lessons to for her to identify the values of, add, and subtract the numbers 20 -100.

Value of Numbers 11 - 100. Once Amy understood the concept of ten's and one's, her ability to count and add extended to include numbers 1 to 100. Once I realized that Amy did not understand the value of the digits 6 through 10, I immediately focused instruction on this concept.

Lesson One focused on the vocabulary word *more*. In this lesson, Amy compared groups of manipulatives and which digit and group of manipulaitves was more than another. Lesson Two focused on explicitly introducing the vocabulary word *less*. Rewriting the book, *10 Black Dots* (Crews, 1982), helped reinforce the meaning of the concept. In Lessons Four through Eight, we continued to do a variety of activities to help Amy extend her knowledge of the digits 6 through 10. In these lessons, we also worked on the concepts of adding one, adding two and three, and subtracting, always focusing on the value of these numbers so Amy could recognize the values more automatically and without using manipulatives.

Lesson Nine focused on the value of numbers 11 through 20. I began by reading *The Icky Bug Counting Book* and modeled how to build the number structure, using bug manipulatives. I then put ten bugs in a cup and placed the cup to the left of the row of numbers. As I moved the cup in front of a different row, I modeled for Amy how a mathematician would write what was represented with the manipulatives. For the one cup of 10 bugs (representing the ten's place), I placed a one on the paper and for the row of zero bugs, I placed a zero. Amy immediately recognized this numeral and stated it was 10. I moved the cup of ten bugs beside the row of one, and again modeled for Amy how a mathematician would write this number. Amy told me the number was 11. I continued this process up to 20. I then modeled for Amy how I knew which of these digits was bigger in value, by looking at the one's place. This language was familiar to Amy because I had used it in the classroom for over a year. I provided Amy with guided practice, using the number structure to support her as she identified numbers and compared two digits to determine which was greater in value. For each problem she solved, I explained why she was correct. Amy and I played *Chutes and Ladders* ™ to give her more practice identifying which digit was greater in value, so she could become more automatic with this skill. In Session 14, Amy demonstrated her knowledge of the value of numbers 11 – 20 by answering the question, "How are 14 and 17 different?" in her journal. Amy wrote, "Because 17 comes after 14. It is bigger. Both of them have one 10, but one has four one's and the other number has seven one's."

After Lesson Nine (Sessions 13 and 14), Amy progressed rapidly, demonstrating her knowledge of the value of numbers from 20 - 100 (Lesson 10, Sessions 15 and 16), adding with carrying (Lesson 12, Sessions 19 and 20), and subtracting with borrowing (Lesson 13, Sessions 21 and 22). It took 14 sessions (Lessons 1 - 9) for Amy to understand the value of numbers 5 - 10, but it only took 8 sessions (four lessons) for her to identify the value of numbers 20 – 100 and to add and subtract numbers 20 - 100.

TEMA-3 Posttest

On May 15, 2006, I administered Form B of the *TEMA-3*. I began the test administration at age five, question 15, the same question I began with on the pretest.

The first section of Forms A and B has seven questions, with subparts, and a total of 22 questions. On the pretest, Amy experienced difficulty with question A17, which asked, "___ + 3 = 5, ___-2 = 7, ____+ 4 = 7, _____-3 = 4." Amy answered 1 out of 4 on the pretest and on the posttest, Form B, she answered 3 out of 4 questions correctly. Amy also experienced difficulties with question A20, answering 3 out of 5 correctly and on Form B answered all 5 questions correctly.

The next section of the *TEMA-3* tests concepts that six-year-old children are expected to understand. It included 10 questions and with the subparts, a total of 23 questions. Question A25 was the first question with which Amy experienced difficulty. The question asked that she equally share quantities. On Form A, she missed both questions. On Form B, she answered both questions correctly. Amy also experienced difficulty with Question A27, which asked her to use a mental number line to determine which number was closer. On Form A, she answered 4 out of 6 questions correctly and on Form B, she answered 5 out of 6 questions correctly.

The next section focused on concepts a seven-year-old should understand. It contained 11 questions, and with subparts, a total of 28 questions. Amy missed the first question on Form A, which asked her to count on from the larger addend. Of the three sub-questions, she answered no questions correctly. On Form B, she answered all three questions correctly. The next question with which Amy experienced difficulty was A33, which asked her to count by tens. Amy counted by ten's up to 60 but skipped the number

70. On Form B, Amy answered the question correctly, with ease. Question A34 was also difficult for Amy. It asked which number sentence was correct for the word problem. Amy answered all three questions incorrectly, but on Form B, she answered all three questions correctly. Amy answered questions A35 and A36 correctly, but on A37 she missed 1 of 6 questions when asked which number was closer to a specified number between 1 and 100. On Form B, Amy missed 2 out of 6 questions, which was more than she missed on the pretest. On Form A, Amy missed one additional question: counting backwards from 20. She attempted to count backwards, but in order to determine the number before 20, she had to count forward first. For example, she said, 20, then counted forward to 20, to determine 19 was before 20. On Form B, she counted backwards without having to count forward each time and answered the question correctly.

The next section tested the concepts an eight-year-old should understand. On Form A, Amy started experiencing real difficulties. After answering three questions correctly, she missed questions A46, A47, and A48. A46 asked four simple addition facts, including doubles (e.g., 2+2, 3+3, 4+4). Amy answered 2 out of 4 questions correctly. On Form B, Amy answered all 4 questions correctly. Question A47 asked how many ten's were in 100. Amy answered "11" on Form A, but on Form B, she answered correctly. When given a number greater than 100, Question A48 asked the student to add one to the number. Amy stated random numbers, which were not even close to the right answer. In contrast, on Form B, she answered the questions correctly. On the pretest, Amy answered the following question correctly, but then missed the next question (A50). It had two sub-questions, asking simple subtraction facts. Amy answered both questions incorrectly. One answer was higher than the original number, demonstrating that she did not understand the value of numbers or subtraction. On Form B, she answered one question correctly. Question A51 asked her to add doubles, and she missed both sub-questions. On Form B, she answered 1out of 2 correctly. On the pretest, Amy missed the next three questions, so I discontinued the testing. Question A52 asked Amy to add or subtract groups of ten's, and she missing all six sub-questions. On question B52, Amy answered all six sub-questions correctly. She missed the next two questions on Forms A and B. Questions A53 and B53 asked how many one hundred's were in a thousand, and Questions A54 and B54 asked four multiplication facts.

I continued with the posttest assessment. Amy correctly answered questions B55 and B56 correctly but missed question B57. At this point, I mistakenly discontinued the test. Technically, I should have continued administering the test until Amy answered five consecutive questions incorrectly, but I mistakenly thought she had to answer five questions in a section incorrectly. However, I believe Amy would not have missed the next five questions. Her work in the classroom, demonstrated that she could add with carrying, which was required in a few questions. However, she did not understand multiplication or skip counting by fours, so I believe that B67 was a fairly accurate stopping place.

Amy's raw score on Form A was 40, which was an age equivalent of 6.6 years and a grade equivalent of first grade, fourth month (1.4). Her raw score on Form B was 49, which was an age equivalent of 7.3 years and a grade equivalent of second grade, second month (2.2). Therefore, Amy made six months progress according to the *TEMA-3*, after I worked with her one-on-one for 22 sessions. This is in contrast to very little progress in the classroom after a year and a half of instruction.

Other Evidence of Amy's Progress

Amy's understanding of number sense improved during the study. She did not improve rapidly enough to catch up with what the rest of the class was learning, but she consistently scored an average of B or C on her tests. On the cumulative review tests, which she really struggled with before the intervention, showed she did learn the material after it was covered in the intervention. Also, on the mathematics portion of the second grade Mississippi Curriculum Test, Amy scored proficient, on a scale ranging from advanced (highest), proficient, or basic (minimal).

Summary

In Chapter IV, I summarized the interviews with Amy, Amy's mother, and her former kindergarten teacher. I presented the results of the *TEMA-3* pretest results, which guided my instruction for the intervention lessons. Next, I briefly described each intervention lesson. Next, I described the four aspects of the intervention that seemed to help Amy the most. I compare the results of the TEMA-3 pre-and posttest. Finally, I present other data that demonstrates the intervention helped Amy develop number sense.

CHAPTER V

RECOMMENDATIONS

Building on Students' Strengths

I recommend that teachers build on students' strengths when developing instruction on concepts with which they struggle. The findings from this study support the use of literature to teach number sense to students who excel in reading but struggle with mathematics. This practice links students' strengths and interests with something that students may otherwise avoid. Data from my observer comments revealed that Amy enjoyed reading trade books, and the stories set the stage for using mathematics in a meaningful context, whether it was counting cookies or buying items from a store. Also, linking reading-related concepts (beginning and end of a story) to math vocabulary (*before* and *after*) helped Amy understand the way the words were used in math. After I made this comparison and showed Amy how the words were related, she answered questions correctly related to the concepts of numbers that come *before* and *after* other numbers.

Based on my work with Amy, I also recommend the use of writing with mathematics for students who excel in written communication but struggle with mathematics. During the intervention, Amy did not hesitate to write answers to mathematics questions. She was confident when writing about mathematics concepts during the intervention.

As discussed in Chapter 4, I initially incorporated journal writing into the intervention as a way to solidify (or reinforce) Amy's understanding of a concept. What I expected was that if Amy could explain how to solve a problem in writing, then she would have no difficulty remembering the concept and using it later. What I found in Lessons 1, however, was that Amy correctly explained the concept of *more* in writing but could not use that concept when solving a problem. In this instance, it seemed that she could explain *what* the concept meant (declarative knowledge) but did not understand *how* to use that knowledge (procedural knowledge). Therefore, I caution teachers that if a journal prompt simply asks a student to write a definition-type explanation of a concept, even a correct definition may not provide evidence that a child can use this knowledge to solve problems.

Ultimately, I found journal writing to be very useful for assessment purposes. When Amy needed help with the journal writing, I knew she did not possess the higher order thinking skills that were necessary to analyze what she had done, and I needed to reteach (or extend my teaching of) a concept or skill. The journal writing also provided me with a record of the skills Amy had learned. In addition, she could read her journal, like a book, to review how she solved problems. I recommend the use of journal writing by classroom teachers for assessment purposes, especially when students excel in writing.

Use of Games for Review

I also found that incorporating games into the intervention not only helped Amy practice using concepts she was learning but it also provided an enjoyable context. Her desire to complete mathematics tasks, in a game context, also seemed to improve. Before the intervention, when Amy was expected to complete mathematics worksheets or participate in class, she typically put her head down, cried, asked to be excused to the restroom, and stayed gone for a while, or she would quickly complete an exercise and answer every problem incorrectly. Her attitude toward math was poor, and she stated in the interview that she did not like math. During the intervention, Amy played games such as Battle, *High Ho Cherrio*®, "Race to 100", and *Chutes and Ladders*™; she smiled and played with enthusiasm. Her confidence grew as she answered questions such as, "Who has more cherries in their basket?—especially when she had the most cherries—and she appeared excited as she played math games.

Use a Number Structure

During the intervention, I noticed that Amy began to understand the value of numbers more easily when I introduced the use of the number structure than when I used a simple number line. The reason the number structure helped Amy more than the number line is because it visually represented the value of each number (with dots or other objects), rather than simply showing numerals on a line. It is important to note, however, that the directionality of the number structure and number line should be consistent. For example, if values on a vertical number line increase as one moves up on the number line, the value of numbers in the number structure should also increase as one moves up the structure. When I first built the number structure for Amy, I began constructing it with the numeral one and one object on the first row. Then I placed the numeral two below the one and placed two objects beside the two and so forth. Consequently, the values of the numbers increased as we moved down the number structure. In contrast, when we used the number line, numbers increased in value as we moved up the number line. Moving in two different directions to represent an increase in value was confusing to Amy. When I realized this, I changed the number structure so that numbers increased as we moved up the structure. This simple change, made the language and the visual representation of the numbers consistent and less confusing to Amy.

Now that I have reflected more on how I could have used Amy's knowledge of reading to strengthen her number sense, I think I would make another change to the directionality of the number line and number structure. Amy was familiar with the left to right progression of reading. Perhaps using a horizontal number line, rather than a vertical one would have helped her understand more quickly the concepts of *before* and *after*. In reading, if you ask a child to read a word in a sentence that comes before *frogs*, for example, she would read the word to the left of *frogs*. Conversely, if you tell a child to read the word after *frogs*, she would read the word to the right of *frogs*. If the number line had been arranged horizontally, Amy would probably have understood the concepts of *before* and *after* more quickly, because the words were used in the same way as in reading. Thus, reading numbers left to right, just like reading a sentence, would capitalize on students' automatic knowledge of left-to-right progression. The meaning of the words, *before* and *after*, would have meant the same in reading and mathematics. (See Figure 5.1 for an example of the number line.)

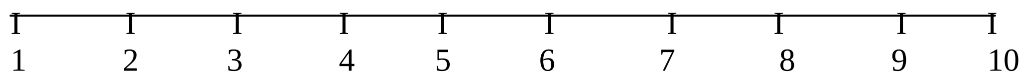

Figure 5.1 Number Line.

Likewise, I would make the number structure run horizontally and lay the objects above the numeral to show that the value increases—as does the height of the number structure—as the numbers get larger. (See Figure 5.2.) This would make the directionality of the number structure consistent with the number line, thus less likely to confuse students like Amy.

Teachers could use the number structure to organize numbers from the least to the greatest and compare the values of numbers by allowing the students to visually see the lines of the manipulatives increasing in value. This could be done in an intervention time as well as in whole group class instruction.

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		o	o	o	o	o	o	o	o
	o	o	o	o	o	o	o	o	o
o	o	o	o	o	o	o	o	o	o
1	2	3	4	5	6	7	8	9	10

Figure 5.2 Number Structure.

Use of Consistent Vocabulary

There is evidence from the data that a lack of understanding about the vocabulary in mathematics may have been a problem for Amy. As I reviewed the summary data charts and the different vocabulary I used for a single concept, I wondered if using so many different words to refer to the values of numbers may have hindered Amy's development of number sense. For example, I referred to numbers of higher values with words such as *more*, *larger*, and *bigger*. In one instance, Amy thought I was referring to the physical size of objects when I asked her which number was larger or bigger. Perhaps more consistent use of vocabulary would have helped Amy. This recommendation would also extend to teachers across grade levels, especially in the early grades when basic concepts are being learned. If teachers all used the same vocabulary to refer to the same concept, children like Amy might experience less confusion.

Use of a Fact Pack

One way I kept track of the concepts and skills Amy learned was the use of the Fact Pack. During each lesson, I wrote on note cards the vocabulary and/or sample problems that Amy was learning. Then I stored this pack of cards in a decorative tin that was appealing to Amy. (I knew that Amy liked princesses, so I used a *Disney Princess* tin to store her cards. This storage container may have made the use of the Fact Pack a little more palatable to Amy. Any storage container could be used for this purpose.) I also used the Fact Pack to review concepts and practice working problems Amy had learned previously. The repetition of different skills in different contexts may have helped make Amy's skills more automatic. In addition, keeping track of the facts Amy learned in this way made review a very quick and easy process for both me and Amy.

Implications for Future Research

First, to determine if this intervention would be useful for developing number sense of other children who struggle with understanding number sense, a quantitative study using this intervention would be important. Furthermore, determining if individual steps of the intervention would be as useful as combined steps is necessary before widespread use of this intervention would be warranted.

Another investigation worth pursuing would be the importance of consistent vocabulary instruction. After reflecting upon my teaching and viewing others teach, I wonder if students' number sense and mathematical ability would improve if mathematical vocabulary was consistent from classroom to classroom and was taught with the same diligence as reading vocabulary.

Third, research to determine the effectiveness of the design of number lines and number structures could be investigated. Does the directionality of a number line make a difference in children's understanding of basic concepts like the value of numbers? Should a number structure be used as an intermediate step between the use of manipulatives and the use of a number line to illustrate the value of numbers?

Finally, it would be interesting to analyze math and reading concepts and the way they are taught to see if there are ways to build on what students learn about reading, for example left-to-right directionality, and make ways of "reading" mathematics and alphabetic text consistent.

Summary

In this chapter, I recommend that teachers build on students' strengths when developing instruction on concepts with which they struggle. When students excel in reading and writing but struggle with mathematics, as was the case in this study, the use of trade books and writing may help them better understand mathematics concepts. Next, I recommend that games be used to provide an enjoyable and meaningful context in which to practice using mathematics concepts. Third, I recommend the use of a number structure to help students understand the value of numbers. Fourth, the use of consistent vocabulary by the teacher may be important when students struggle with basic concepts, such as number sense. Last, I recommend the use of a Fact Pack to help teachers easily keep track of the concepts students have learned and to facilitate efficient review of concepts. I conclude the chapter with implications for future research.

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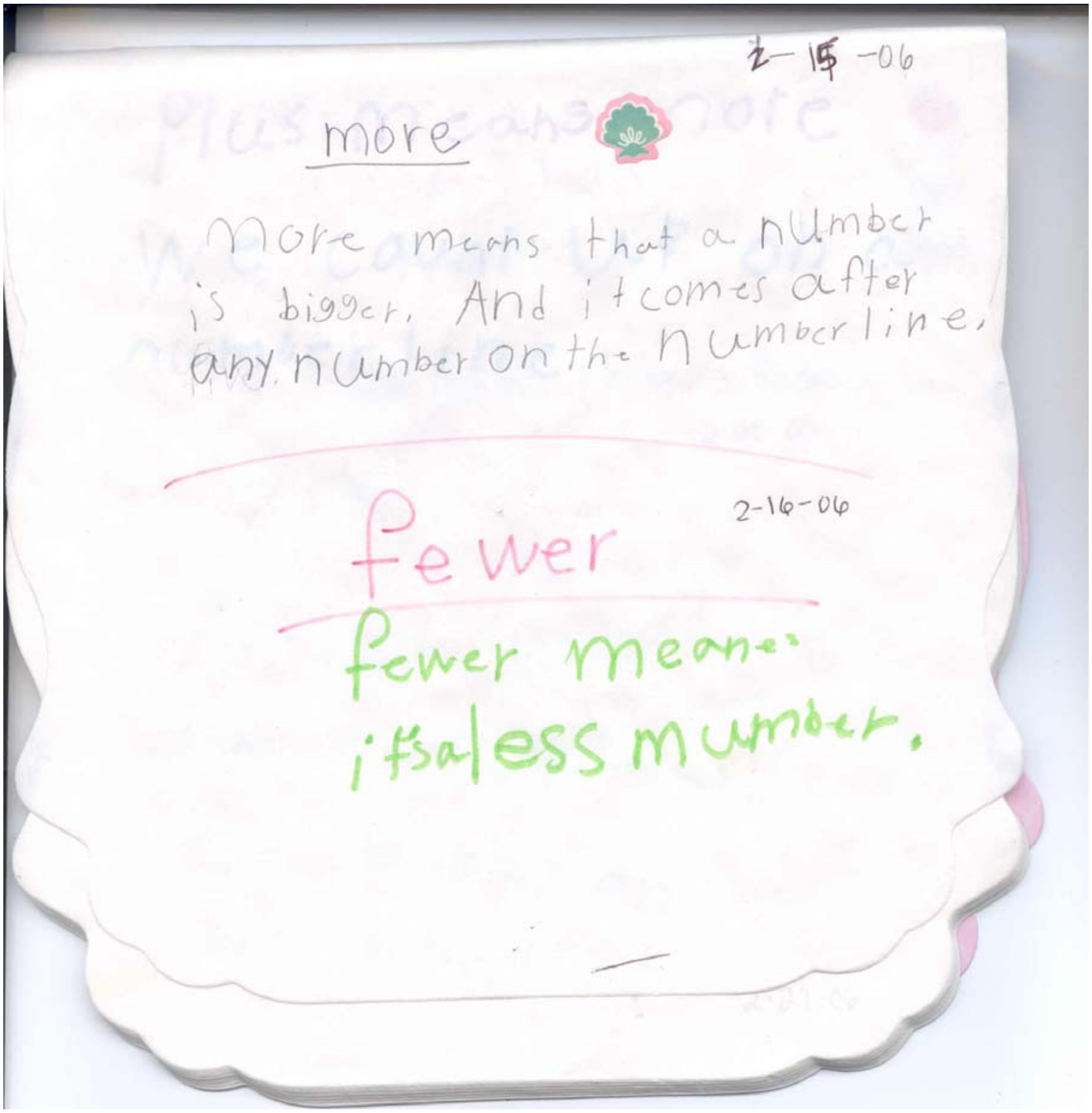
APPENDIX A

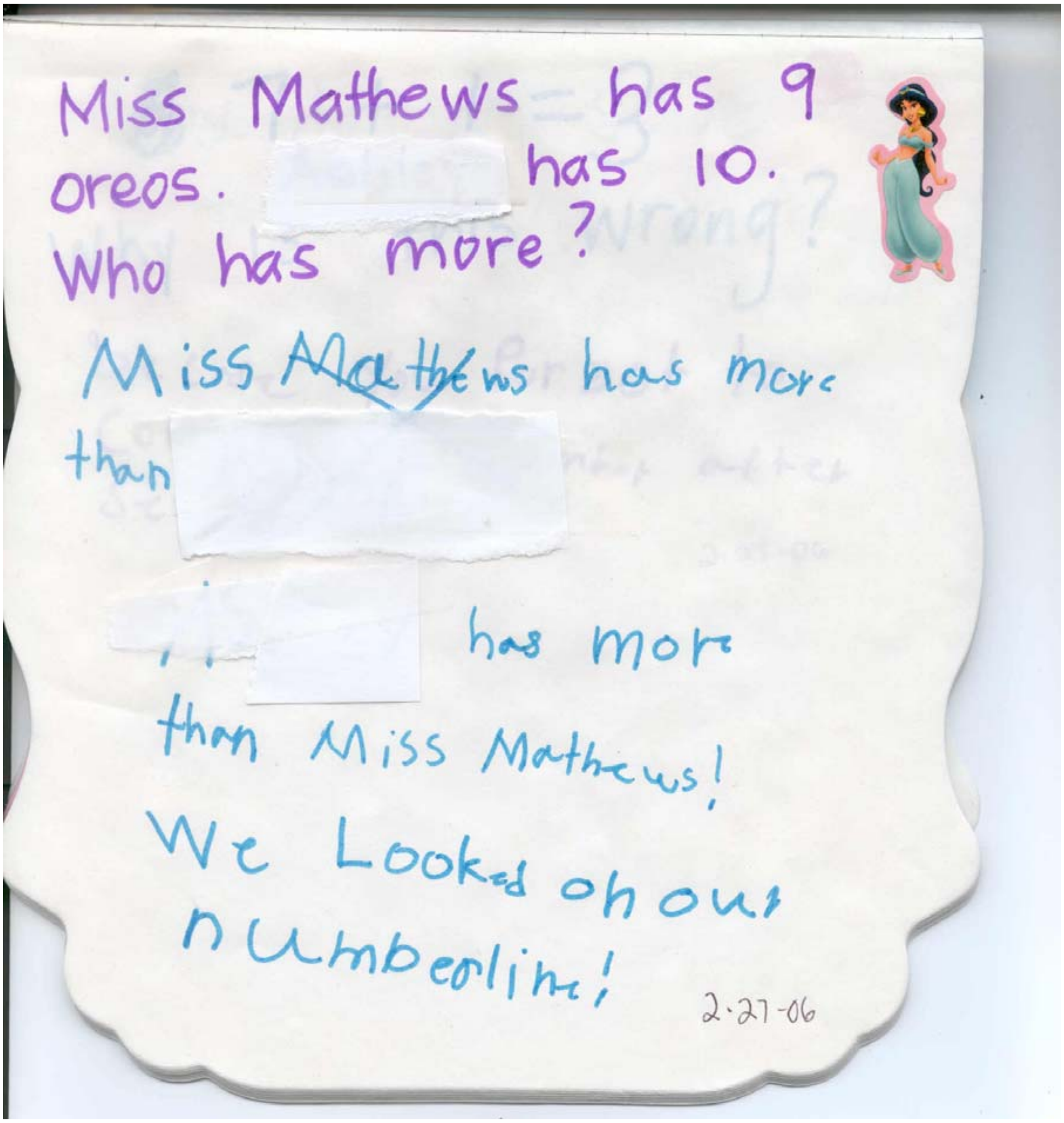
MATH JOURNAL


2-15-06
more
More means that a number is bigger. And it comes after any number on the number line.
2-16-06
fewer
fewer means
its a less number.

Plus means more

We count up on a numberline.

2-21-06



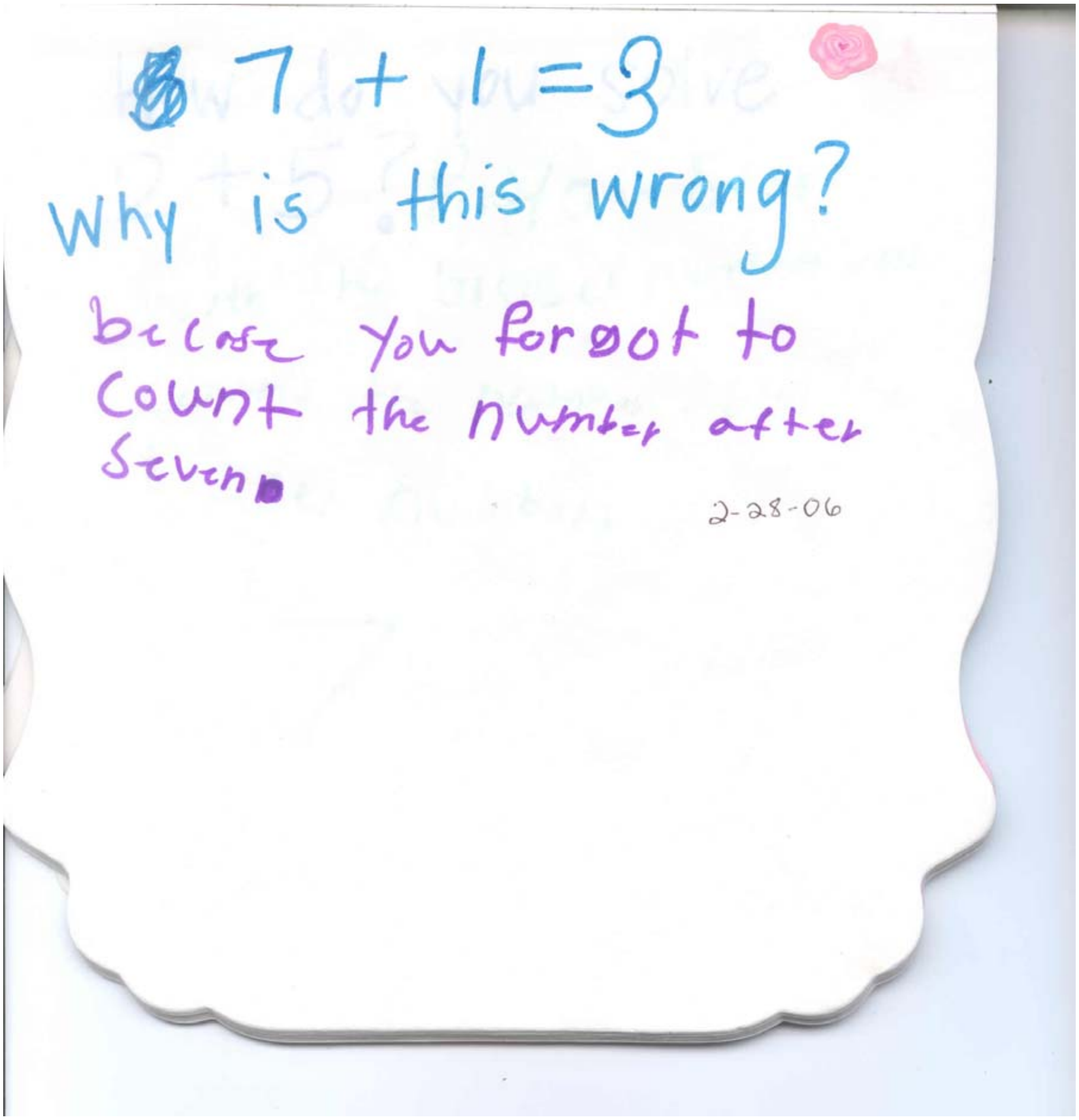
Miss Mathews has 9 oreos. has 10. Who has more?

Miss Mathews has more than

has more than Miss Mathews!

We Looked on our numberline!

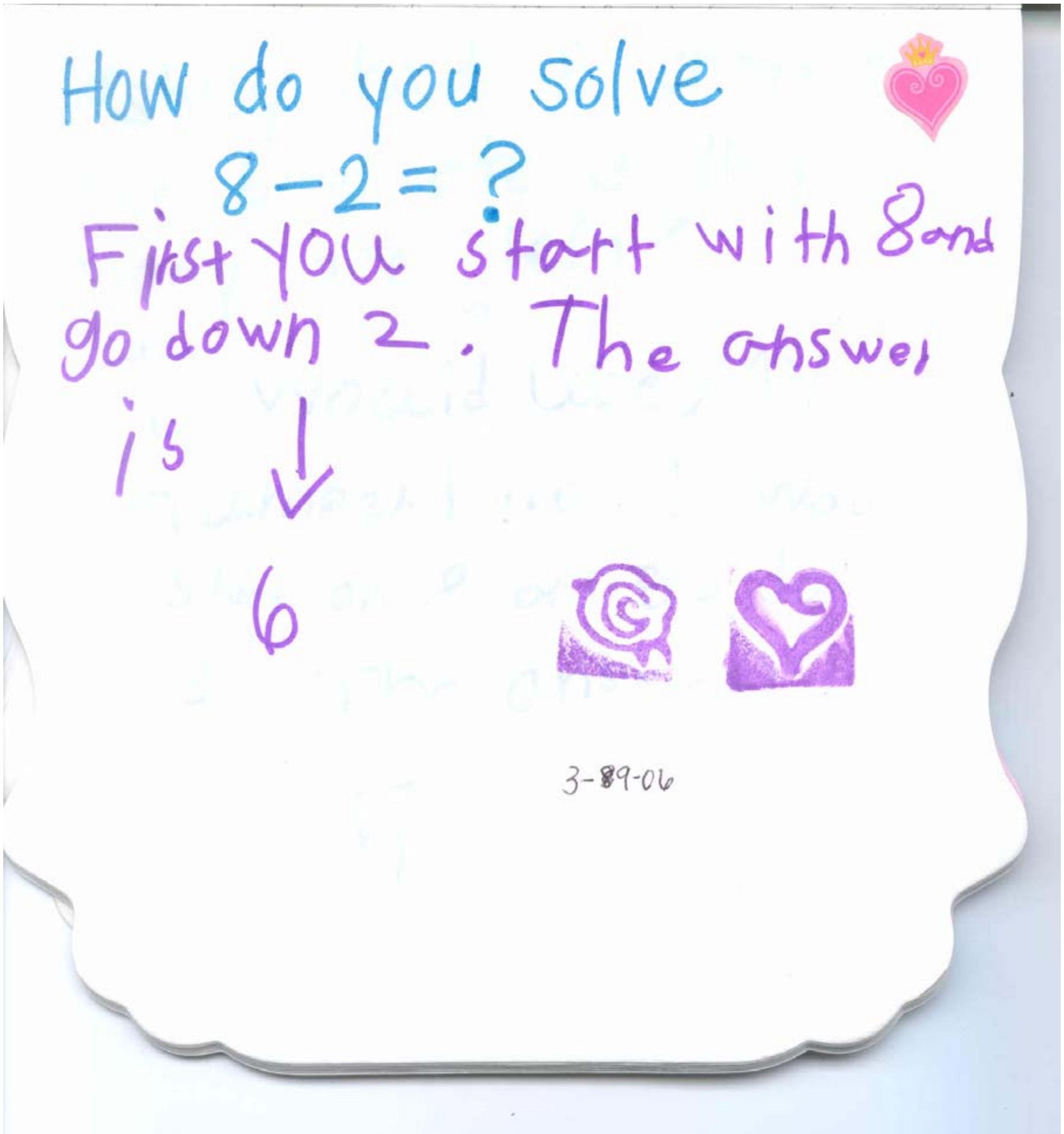
2-27-06

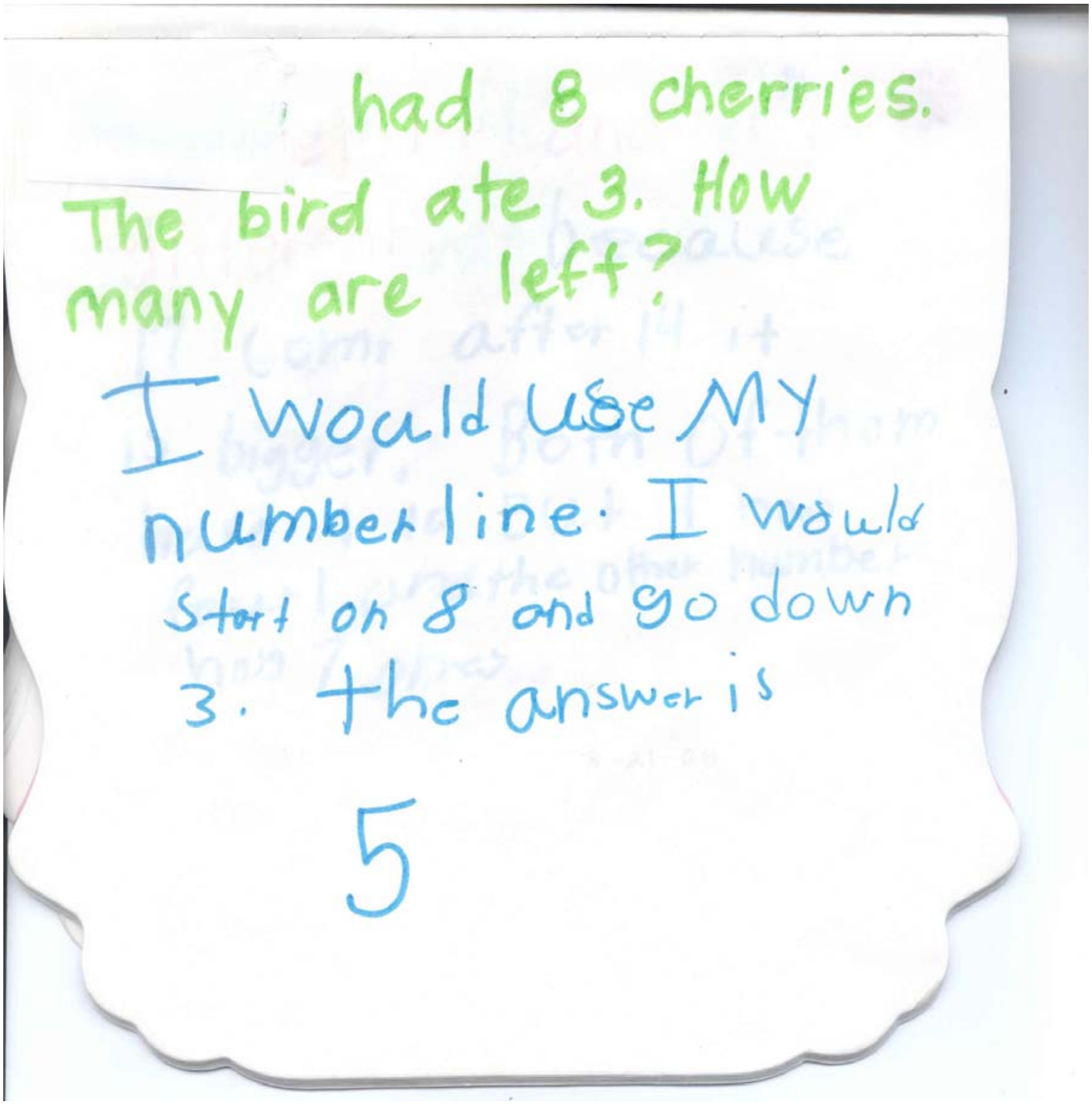

7 + 1 = 3
Why is this wrong?
becase you forgot to count the number after seven.
2-28-06

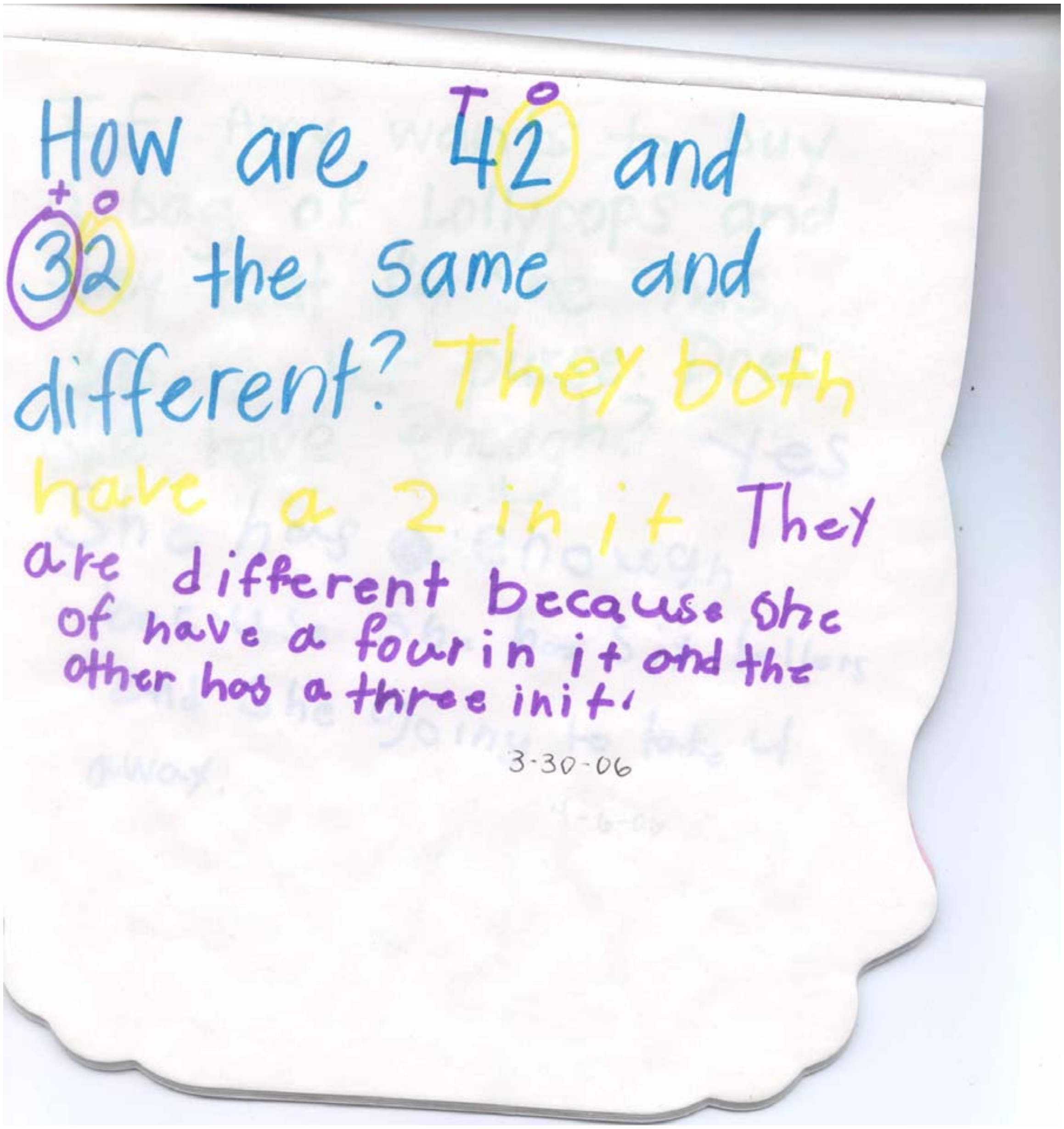
How do you solve 2 + 5? first you Start with the bigger number and count the number after the bigger number.

7

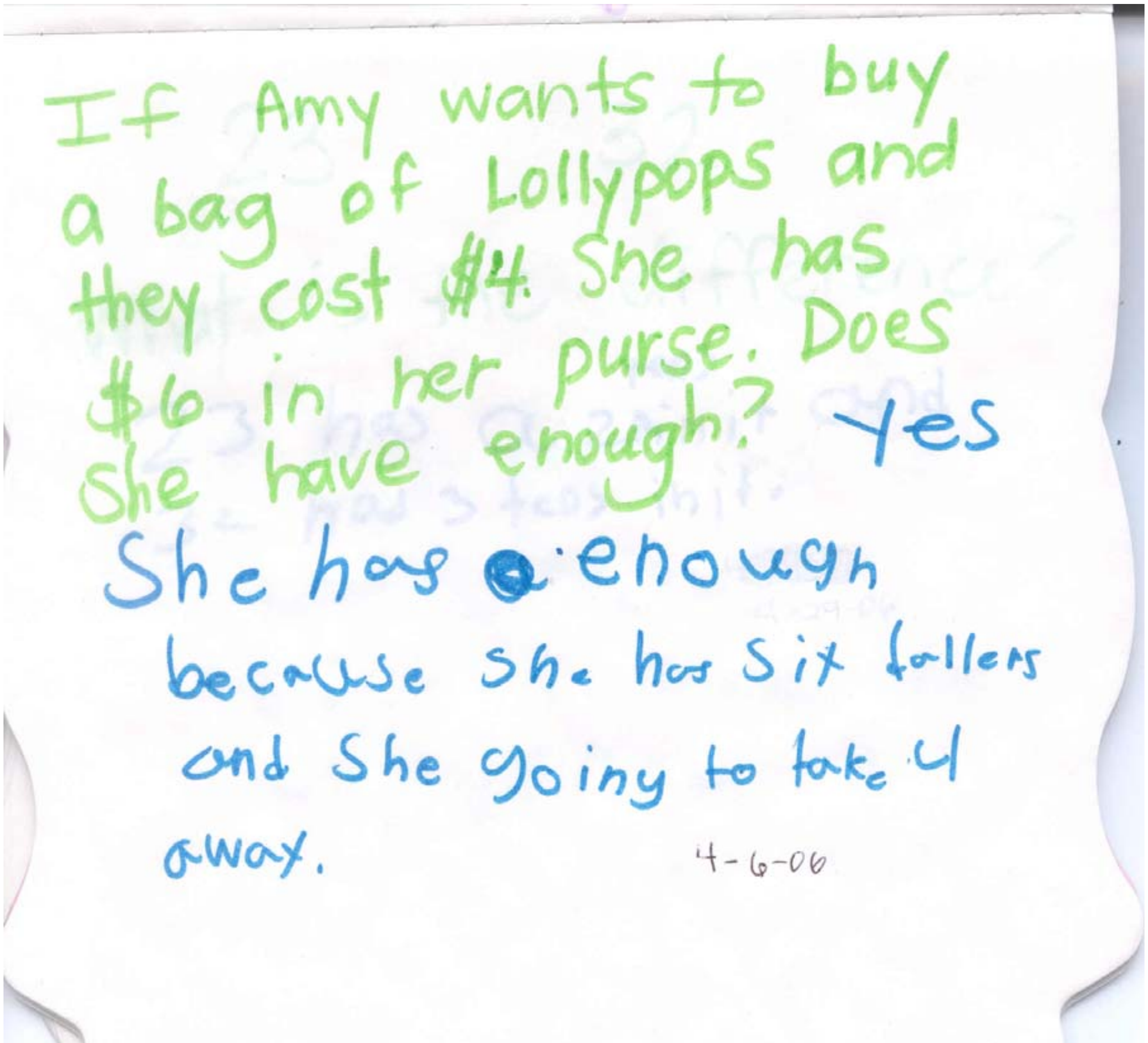
3-7-06


How do you solve
8 - 2 = ?
First you start with 8 and
go down 2. The answer
is
6
3-89-06


had 8 cherries.
The bird ate 3. How
many are left?
I would use my
numberline. I would
start on 8 and go down
3. the answer is
5


How are 42 and 32 the same and different? They both have a 2 in it They are different because one of have a four in it and the other has a three in it.

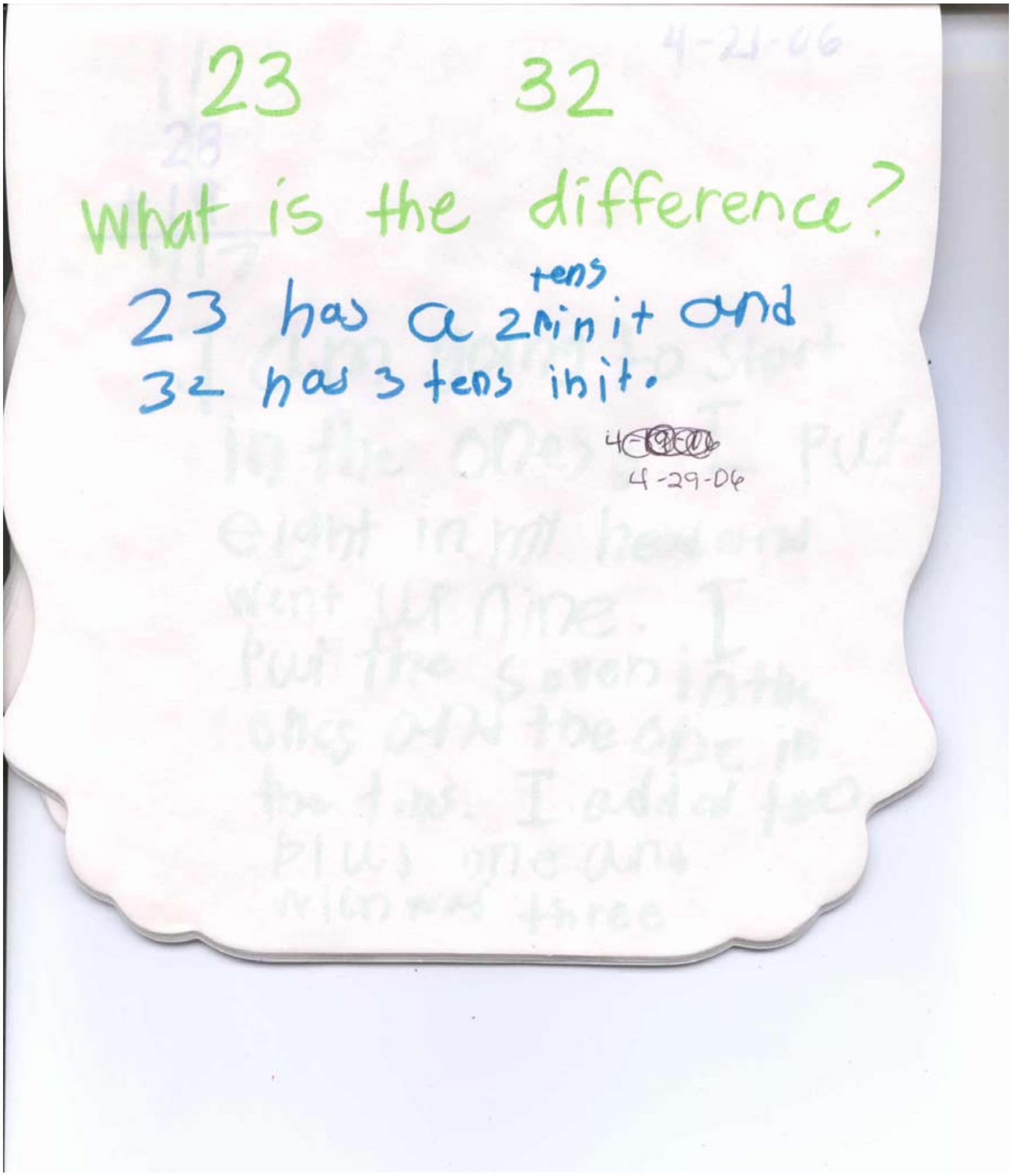
3-30-06



If Amy wants to buy a bag of Lollypops and they cost \$4. She has \$6 in her purse. Does She have enough? Yes

She has enough because She has Six follers and She going to take 4 away.

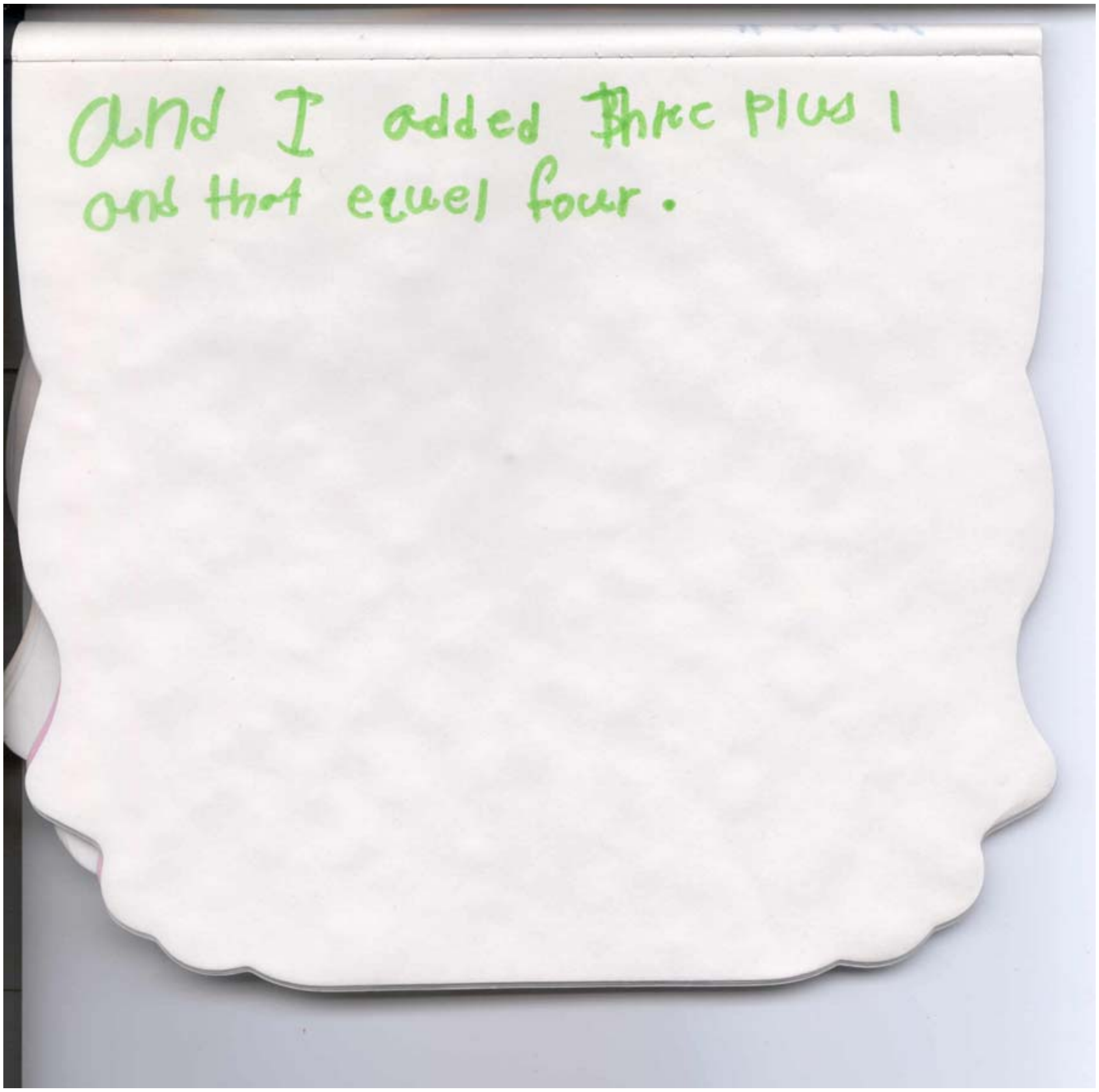
4-6-06


4-21-06
23 32
What is the difference?
tens
23 has a 2nin it and
32 has 3 tens in it.
4-29-06

4-21-06

$$\begin{array}{r} {}^{1}\\ 28 \\ +\,19 \\ \hline 47 \end{array}$$

I am going to start in the ones. I put eight in my head and went up nine. I put the seven in the ones and the one in the tens. I added two plus one and wich was three


and I added three plus 1
and that equel four.

APPENDIX B

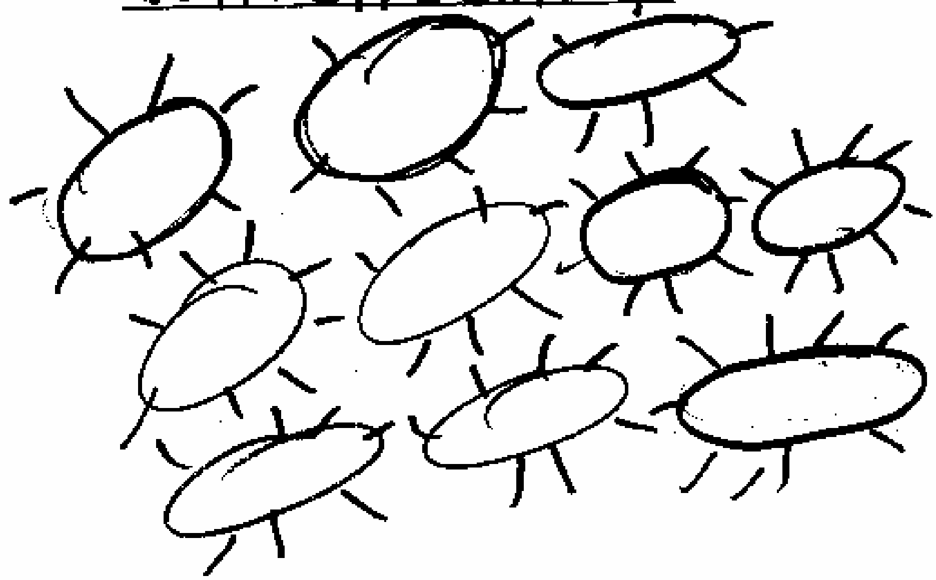
TEN BLACK DOTS BOOK BY AMY

Ten Black Dots

What can you do with ten black dots?

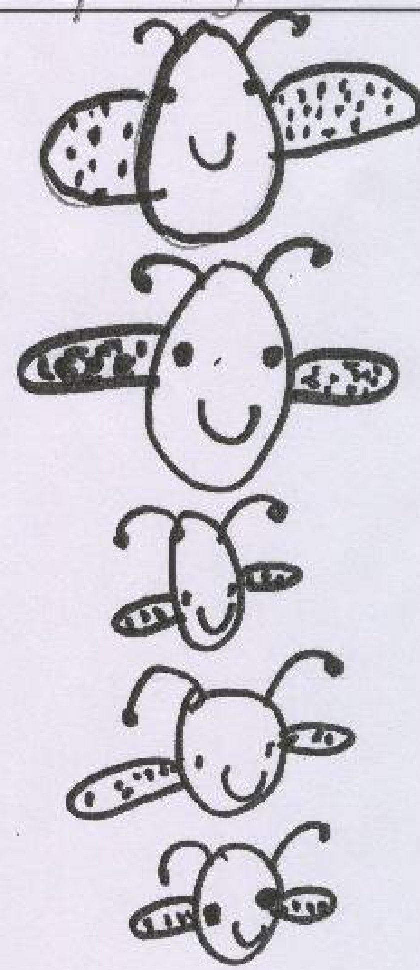
10 dots can make a

Cantensuns.



One less is nine!

Nine dots can make lady bugs.



APPENDIX C

LIST OF TRADE BOOKS USED IN INTERVENTION

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Crews, D. (1986). *Ten black dots*. China: Harper Children's Press.

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