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Lateral transshipment of slow moving critical medical items

Gozde Agirbas

LATERAL TRANSSHIPMENT OF SLOW MOVING CRITICAL MEDICAL ITEMS

By

Gozde Agirbas

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By

Gozde Agirbas

Approved:

____
Mingzhou Jin
Assistant Professor of
Industrial and Systems Engineering
(Director of Thesis)

____
Burak Eksioglu
Assistant Professor of
Industrial and Systems Engineering
(Committee Member)

____
Sandra Duni Eksioglu
Assistant Professor of
Industrial and Systems Engineering
(Committee Member)

____
John Usher
Professor of
Industrial and Systems Engineering
Director of Graduate Studies in
Industrial and Systems Engineering

____
Sarah A. Rajala
Dean of the Bagley College of
Engineering

Name: Gozde Agirbas

Date of Degree: August 9, 2008

Institution: Mississippi State University

Major Field: Industrial Engineering

Major Professor: Dr. Mingzhou Jin

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Candidate for Degree of Master of Science

This research studies lateral transshipment of critical medical items that have low demands. Due to the high prices of medical items and their limited shelf lives, the expirations contribute significantly to the current prohibitively high cost of the healthcare system. Lateral transshipment between hospitals in a medical system provides opportunities to reduce the expiration costs. This paper studies the decision rule for lateral transshipment in a two-hospital system and extends the rule for the multiple-hospital cases. The decision rule takes the myopic best action by assuming no transshipments will be performed in the future. Numerical experiments demonstrate significant cost savings and the decision rule has a small gap from the upper bound of the total saving. The savings are more considerable when the difference of demand rates at different locations is large and the life time of the medical item is not too long or too short.

DEDICATION

To my mom, dad and brother. I love you all ...

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CHAPTER I

INTRODUCTION

1. Health Care Industry: Current Situation

The high cost of health care in United States has some consequences on social life such as the rise in number of uninsured people, which has reached 47 million in the U.S. (California Health Care Foundation, 2005). Between 2000 and 2006, employment based health insurance premiums have seen an 87 percent increase while cumulative wage growth was only 20 percent and cumulative inflation was 18 percent (The Henry J. Kaiser Family Foundation, 2006). The impact of rising health care costs on social life has also been observed by different surveys. Those surveys revealed that one out of four American families had a problem in paying medical care costs in 2006 (ABC News/Kaiser Family Foundation/USA Today, 2006), while 42 percent told they were very worried because they could not afford health care services (The Henry J. Kaiser Family Foundation ,2004).

The Organization for Economic Co-operation and Development estimated that total health care spending per capita was $5,711 in 2003 (OECD, 2006) and $6,700 in 2005 (Catlin et al., 2007) in United States. When this measure is compared with other countries such as Australia, Belgium, Canada, France, Sweden, and Switzerland for years 1970, 1980, 1990 and 2003, it is observed that the total expenditure is significantly higher in United States. Additionally, during years 1970, 1980, 1990 and 2003; 7%, 8.8%, 11.9% and 15.2% of U.S. national income was spent on health care respectively (OECD, 2006). The percentage of GDP spent on health care cost in 2003 was at least three percent more than the other countries' spending on health care analyzed in the same study (OECD, 2006). It is projected for U.S. that in the next decade this percentage will account for 20 percent of GDP (Borger et al., 2006). Another way to look at the spending in healthcare is to compare it with the total spending in other industries. In 2003, health care expenditure was 4.3 times that of national defense (California Health Care Foundation, 2005).

Reducing health care systems' cost will help to alleviate the pain in this area. Among all the cost categories that contribute to the total cost, inventory cost is one of them with the largest share. Estimations indicate that inventory investment account for 10% to 18% of net revenues in health care industry (Holmgren and Wentz, 1982; Jarett, 1998). It must be kept in mind that health care industry carries a lot of items with an expiration date which makes the burden even worse. Inventory management of perishable items needs special attention because expired goods are not just downgraded but they become worthless. This makes expiration dates one of the most important issues because once it passes, the product cannot be sold. To meet the challenges and opportunities perishable products bring, health care systems put special emphasis on inventory control policies and logistics practices. As an example of an opportunity in health care industry, any cost saving in inventory investments will lead to increased profitability (Nicholson et al., 2004). Although the problem of perishable inventory management in pharmaceutical industry is addressed rarely, for medical items such as blood, numerous studies can be found in literature.

In the healthcare industry, 60% of the inventories are critical supplies (Nicholson et al. 2004). Because of their importance to save people' lives, healthcare providers have to keep a high availability for critical supplies such as pharmaceutical, surgical supplies, and blood. The items must be immediately available when they are required, so the items must be kept in the on-site inventory. At the same time, the medical items typically have a limited lifetime. When they pass their expiration dates, they must be discarded with no or little salvage value. For slow moving items, the high availability requirement and the limited lifetime cause a high percentage of expired items. This results in increased cost of medical inventory investments and affects the cost of whole healthcare system. Table 1 shows some expensive medical items along with their shelf lives and therapeutic indications.

Together with inventory management of perishables, the logistics of these systems is very complicated to manage due to high product values and the need of supplying high levels of service. Pharmaceutical industry is referred in literature as being "on or near the cutting edge of logistics practices" (Loar, 1992) along with food and chemical industries. Inventory pooling via lateral transshipments is an efficient strategy to reduce system costs or individual location costs used by the authors. In lateral transshipments, items are transferred between two facilities which are both in the same echelon.

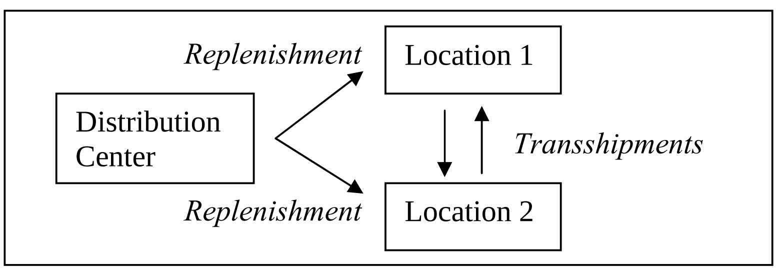

Figure 1.

Lateral Transshipments

Table 1

Examples of Expensive Medical Items

Item	Selling Unit	Unit Cost	Shelf Life	Therapeutic Indications
Zyprexa 15 Mg	1000 EA	$16,362	2 years	-treatment of schizophrenia and bipolar disorder
Lipitor Tab 10 Mg	5000 EA	$12,743	3 years	-lowers total cholesterol, LDL cholesterol, and triglycerides
Sutent Cap 50 Mg	1000 EA	$6,661	2 years	-treatment of gastrointestinal stromal tumor in cases of resistance or intolerance
Virazole Vial 6 Mg	4 CT	$5,989	2 years	-alleviates suffering from ethylene glycol or methanol poisoning
Busulfex Vial 10 Mg	8 CT	$5,810	1 year	-treatment of various forms of cancer
Aricept Tab 10 Mg	1000 EA	$5,310	3 years	-indicated for mild to moderately severe Alzheimer's disease
Seroquel XR Tab 400 Mg	500 EA	$5,146	3 years	-treatment of schizophrenia and manic episode
Epogen SDV 40 Mu 1 Ml	10 CT	$5,116	Short shelf life due to instability	-treatment of anemia
Prevacid Cap 15 Mg	1000 EA	$4,760	2 years	-used for certain types of ulcers and reflux disease
Nexium Cap 40 Mg	1000 EA	$4,676	3 years	-treatment of erosive reflux disease and gastric ulcers
Sprycel Tab 5 Mg	60 EA	$4,537	2 years	-indicated for stem cell disorder which involves chromosome 9 and 22 (Philadelphia disease)
Zocor Tab 40 Mg	1000 EA	$4,374	1 year	-treatment of hypercholesterolemia
Prilosec Cap 20 Mg	6 CT	$4,154	2 years	-decreases acid produced in stomach to fight ulcers

In most of the cases, lateral transshipments are required due to emergency situations such as stock-outs (Axsater, 1990). For example, in case of a stock-out a retailer may request a lateral transshipment from another retailer. However, if the retailer prefers to order items from the distribution center, this will be a replenishment since goods will be transferred between two different echelons.

Lateral transshipment of the critical medical items that have low demand rate and limited lifetime can result in cost savings by reducing the chance of expiration. Assume one location has two near-to-expiration items. Another location has one new item and needs one more to replenish its inventory. Laterally transferring an old item from the second location to the first one and replenish the inventory of the first location with a brand-new item can reduce the chance of expiration compared to the case of replenishing the second location's inventory directly with a new item.

2. Problem Statement

This study considers the inventory problem of an expensive and slow moving critical medical item in a healthcare system with two locations ($i=1, 2$). The demand at each location is reasonably assumed to follow a Poisson process with the rate of λ_i. The three conditions to validate a Poisson process is well met in this problem setting: large population size, low arrival rate from each source (patient), and independent arrivals from sources. In addition, memoryless property is satisfied via the first two conditions. Since the population size is large and each patient has a low arrival rate, an arrival from a specific patient does not have any effect on the overall system.

To keep a high availability at each location as discussed in Introduction, each location is assumed to keep an inventory of two units of this item. A new item is assumed to expire and have no salvage value in T days if it is not used. When one unit is consumed or expired, a new order for one unit is immediately placed and a brand new unit, which costs v, arrives. Lead time is assumed to be zero because replenishment lead time is negligible compared to T and $1/\lambda_i$. At any moment, the inventory status at location i can be described by the ages of the two units, a_i^1 and a_i^2, where $0 \leq a_i^1 \leq a_i^2 < T$.

3. Objectives of Research

For general critical medical items, very little study has been conducted to reduce the waste caused by expired items. This paper will study how to reduce the number of expired items in a medical inventory system with two locations by using the concept of lateral transshipment. The work can be easily extended to the system with more than two locations. In the literature, transshipment is typically used to reduce lead time and shortage (Minner et al. 2001, Grohovac and Chakravarty 2001) because the transportation lead time from another demand point could be shorter than the lead time from the central distribution center. In this paper, we will derive and evaluate a decision rule to transship medical items from one demand point to another demand point in order to reduce the possibility of expiration.

CHAPTER II

LITERATURE REVIEW

1. Perishable Inventory Management

This topic is mainly related to two research areas in the literature: perishable inventory management and inventory pooling via transshipments. The inventory research of perishable items, with fixed or stochastic lifetime, goes back to early 70's (e.g., Frankfurther 1974). Under periodic review, when the shelf life of the perishables is one period, the problem can be simplified into generic newsboy problem. Van Zyl (1964) pioneered the research on perishable items with multiple period shelf lives. His formulations are generalized for *m*-period lifetimes by Nahmias (1975) and Fries (1975) under zero lead time assumption. Optimal order quantity is also provided when the shelf life is two periods and the lead time is *L* periods (Williams and Patuwa, 1999). Later, Williams and Patuwa (2004) investigate the same model to find out the impact of ordering, holding, shortage and outdating costs on the optimal ordering quantity. As the unit shortage cost becomes larger, more items are ordered. For the other three cost drivers, an inverse relationship is identified. Among all four cost drivers, the unit ordering and the shortage costs are the most effective factors influencing the reordering point and order quantity. Nandakumar and Morton (1993) derive a myopic heuristic for perishables with finite known lifetimes while considering lost sales.

When perishable inventory systems are under continuous review policy, lead times are assumed to be either zero, constant or exponential. Weiss (1980) determines the optimal ordering policy for a model with zero lead time and limited shelf life when continuous *(s,S)* policy is implemented. Schmidt and Nahmias (1985) consider positive lead times for an item with constant shelf life. The demand is generated according to Poisson process and sensitivity analyses are performed for different system parameters such as constant lead time period and shortage cost. Tekin et. al (2001) suggest a new ordering policy which considers the remaining shelf lives of the perishables in addition to (*Q, r*) values for inventory systems with fixed lifetimes and positive lead times, *L*. For models with exponential lead times and exponential shelf lives, Kalpakam and Sapna (1994) investigate a system with Poisson demand where ordering policy is (*s,S*) and outstanding replenishment orders cannot exceed one at any point in time. Liu and Yang (1999) relaxed this restriction for the same inventory model and allow backordering.

In a perishable inventory system, the optimal reorder point depends not only on the inventory level of the system but also on the age of the perishable products (Nahmias 1982). The total cost of the system typically includes fixed ordering, inventory holding, shortage and outdating costs (Ravichandran 1995). Chiu (1995) presented a method to find a continuous *(Q,r)* ordering policy by minimizing the total expected cost per unit time with an approximation for the expected decay of the current order at the size of *Q*. The ordering policy may also be influenced by customer types (Katagiri and Ishii 2002). Some customers are only interested in the newest items and are given a higher priority while others are assigned a lower priority because they are willing to buy older items in return for a price discount.

Among all the critical inventories, the blood inventory, which has a lifetime of about 21 days, has been intensively studied in the literature. Jennings (1973) proposed inventory control policies for the blood inventory management at the levels of individual hospitals and regional blood banks. Goh et al. (1993) suggested a two-stage model for the blood inventory management. First stage holds the fresh items while the second stage has relatively older items. If the units in the first stage are not demanded until a predefined time, they will be transferred to the second stage. In the second stage, these items will become useless if they are not requested until another specified time. On the other hand, Brodheim et al. (1975) worked on the optimal delivery policies for scheduled blood deliveries to hospital blood banks. Although the research subject is blood inventories, the model can also be extended to other types of perishable items such as food inventories.

2. Inventory Pooling via Transshipments

Similar to medicines in the healthcare industry, spare parts or repairable items could also be expensive and slow moving (Cohen et al. 1986 and Axsater 1990) in the manufacturing setting. The concept of inventory sharing via lateral transshipment is introduced to reduce lead time under emergency situations such as stock-outs. In a single echelon, N-location continuous-review (S-1, S) inventory system with a single slow moving expensive item, inventory pooling with a least cost transshipment rule can result in 19% to 80% cost saving for individual locations and 68% cost improvement for the overall system (Kukreja et al. 2001). In a supply chain with one distribution center and N retailers, retailers are eager to pool their inventory since they can support each other to be less vulnerable in cases of stock-outs. However, the DC is reluctant to have retailers to pool inventory in this system because its own inventory becomes less important (Grahovac and Chakravarty 2001). Lee (1987) proposes three sourcing rules for inventory sharing: random sourcing, comparison of stock levels and choosing the location with the maximum stock level or considering the location which has fewest outstanding orders and largest on-hand stock.

Evers (2001) investigated the “all or nothing” rule for transshipments in a two-location network and proposed a simple but efficient heuristics considering the service levels required by customers. The minimization of overall expected costs provides critical values which will determine transshipment usage. Minner et al. (2003) extended Evers’ research by allowing partial transshipments and including stock-out cost instead of explicitly including a service level and demonstrated a better performance. The original and the improved heuristics were compared by a simulation. Considering more complicated transshipment policy and allowing a retailer to reject a transshipment request from other retailers, Zhao et al. (2006) examined the efficiency of transshipment on a very large decentralized network consisting of infinite number of retailers, in which the decisions of a particular dealer has no effect on others’ decision making, and on a two-retailer network in which the decisions of a retailer may have an impact on the other’s actions. Hu et al. (2005) also utilizes dynamic programming as Zhao et al.(2006), but their focus is to show the effect of transshipments on overall inventory cost in a periodic review (S,s) system. An exhaustive review of inventory pooling can found in Wee and Dada (2005). They analyze a two echelon stochastic optimization problem under five different transshipment rules. Retailer only (RO) rule allows only retailers to be the source of the transshipment and retailer first (RF) rule relaxes this assumption: When all retailers are out of stock, transshipments can be sent from warehouse. On the contrary, warehouse first (WF) rule identifies warehouse as the source as long as it has positive inventory. Warehouse only (WO) rule does not permit transshipments in the retailer level and nopooling rule (NP) does not share inventory within the system. Given the complexity of finding the optimal transshipment policy for multi location and multi echelon inventory systems, research has shifted towards development of heuristics. Luo and Wang (2005) examine *N*-location network and suggest a genetic algorithm based heuristics for maximizing total profit for all locations.

Except Grahovac and Chakravarty (2001), above cited works consider transshipments to satisfy actual customer demand on a specific location after a demand occurs. There is another stream of research focusing on the use of transshipments before demands are materialized. This kind of transshipments helps to reallocate the inventory throughout the supply chain at the beginning of each period (Agrawal et al. 2004).

Latest research on the transshipment theory tends to explore decentralized networks in which each member is an independent decision maker trying to maximize its own objectives other than the entire system’s goals. Dong and Rudi (2004) studied the impact of transshipments on the resulting profits of the members of a supply chain with a manufacturer and *N* retailers. Zhang (2005) extended the work by relaxing the assumption of retailers facing normally distributed demands and supposing general demand distributions. Sosic (2006) considered a decentralized system in which transshipments among the retailers are permitted and examined regarding the effect of retailers’ decisions on the stability of the grand coalition in the long term. Although numerous related work can be listed for centralized supply chains pooling the whole inventory, such as the multi-location model proposed by Kukreja and Schmidt (2005) motivated by a real case, the research on the inventory transshipment in the healthcare industry is very limited.

In this research, transshipment opportunities are assumed to be available when a demand happens or one medical item expires. Due to the finite lifetime of medical items, remaining lifetimes should be taken into consideration to make transshipment decisions. This paper will focus on a centralized system and not allow a location to reject a transshipment request.

CHAPTER III

MODELLING A CRITICAL MEDICAL INVENTORY SYSTEM WITH TWO LOCATIONS

1. A Critical Medical Inventory System with Two Locations Without Transshipments

1.1 Long-Run Analysis

When transshipment is not allowed, the inventory at each location evolves independently. Therefore, we just need to analyze the inventory at one location *i*. In the long-run analysis, one replenishment (0^{th} replenishment) is assumed to happen at t=0 (i.e., $a_i^1 = 0$). Define X_i^n as the duration between the $(n\text{-}1)^{\text{th}}$ and n^{th} replenishments at location i and $F_i^n(x)$ as its cumulative distribution function . The replenishment can be caused by a patient demand or an expiration of one existing unit. Random number X_i^1, which is in the range of $[0, T - a_i^2]$, has a cumulative distribution function $F_i^1(x)$ as

$$F_i^1(x) = \begin{cases} 1 - e^{-\lambda_i x} & 0 \le x < T\text{-}a_i^2 \\ 1 & T\text{-}a_i^2 \le x \le T. \end{cases} \tag{1}$$

If the first patient request arrives before $T\text{-}a_i^2$, its arrival time is the inventory replenishment time. If the first patient request arrives after $T\text{-}a_i^2$, one item will perish at $T\text{-}a_i^2$, triggering a replenishment. Because $F_i^1(x)$ is not a continuous function at $T\text{-}a_i^2$, In order to conduct the long-run analysis, we first establish the following lemma.

Lemma 1: *If the cumulative distribution function of* X_i^{n-1} *is* $F_i^{n-1}(x)$, *the cumulative distribution function of* X_i^n *is*

$F_i^n(x) = 1 - e^{-\lambda_i x} F_i^{n-1}(T-x)$	(2)

Proof: 1- $F_i^n(x)$= Prob{X_n>x}

= Prob{The time between the next demand request after the *(n-1)*[th] replenishment

and the *(n-1)*[th] replenishment time > x} × Prob{ $X_i^{n-1} + x < T$ }

$= e^{-\lambda_i x} F_i^{n-1}(T-x)$.

Please note the time between the next patient request after the *(n*-1*)*[th] replenishment and the *(n*-1*)*[th] replenishment moment is independent from the random variables $X_i^{n-1},\ldots,X_i^1$. □

$F_i^1(x)$ in (1) can be derived by using (2) and the fact of $F_i^0(x) = \begin{cases} 0 & x < a_i^2 \\ 1 & x \geq a_i^2 \end{cases}$.

By using (2), we have Lemma 2 for the distribution function of X_i^n for given initial states.

Lemma 2: *For a given initial state of* a_i^2 *and* $a_i^1 = 0$*,*

If n is an even number, the cumulative distribution function of X_i^n *is*

$$F_i^n(x)=\begin{cases}1-e^{-(n/2-1)\lambda_i T}+(e^{-\lambda_i T}-e^{-\lambda_i x})\sum_{k=0}^{n/2-1}e^{-k\lambda_i T} & 0\le x\le a_i^2\\ 1+(e^{-\lambda_i T}-e^{-\lambda_i x})\sum_{k=0}^{n/2-1}e^{-k\lambda_i T} & a_i^2<x\le T\end{cases}\qquad (3)$$

If n is an odd number, the cumulative distribution function of X_i^n *is*

$$F_i^n(x)=\begin{cases}1+(e^{-\lambda_i T}-e^{-\lambda_i x})\sum_{k=0}^{(n-1)/2-1}e^{-k\lambda_i T} & T-a_i^2\le x\le T\\ 1-e^{-\frac{(n-1)\lambda_i T}{2}-\lambda_i x}+(e^{-\lambda_i T}-e^{-\lambda_i x})\sum_{k=0}^{(n-1)/2-1}e^{-k\lambda_i T} & 0\le x<T-a_i^2\end{cases}\qquad (4)$$

Therefore, we have the following asymptotic property of $F_i^n(x)$:

Theorem 1: *When $n\to\infty$, $F_i^n(x)\to\frac{1-e^{-\lambda_i x}}{1-e^{-\lambda_i T}}$ in the range of [0, T] with probability 1.*

Proof: When $n\to\infty$, $e^{-(n/2-1)\lambda_i T}$, the second term in (3) for $0\le x\le a_i^2$, and $e^{-\frac{(n-1)\lambda_i T}{2}-\lambda_i x}$, the second term in (4) for $0\le x<T-a_i^2$ approach zero. The theorem can be obtained directly following $\lim_{n\to\infty}\sum_{k=0}^{n}e^{-k\lambda_i T}=\frac{1}{1-e^{-\lambda_i T}}$. □

In the long-run analysis, the cumulative distribution function of the time between two replenishments asymptotically approaches a continuous function, which is independent from the initial state a_i^2.

Theorem 2: *In the long-run, the expected duration between two consecutive replenishments at location i is* $\lim_{n\to\infty}E[X_i^n]=\frac{1}{\lambda_i}-\frac{Te^{-\lambda_i T}}{1-e^{-\lambda_i T}}$.

Theorem 2 can be easily proven by using Theorem 1. Theorem 2 implies the following three facts:

1) The long-run expected duration between two consecutive replenishments is independent from the initial state a_i^2.

2) The long-run expected duration between two consecutive replenishments is less than the expected interarrival time of patient requests.

3) The long-run expected duration between two consecutive replenishments is an increasing function in the lifetime of the medicine, T, when $T > 0$ because

$$\frac{dE_i}{dT} = \frac{e^{-2\lambda_i T} - e^{-\lambda_i T} + \lambda_i T e^{-\lambda_i T}}{(1 - e^{-\lambda_i T})^2} > 0 \text{ for } T > 0.$$

1.2 The Impact of the Initial State (The Ages of the Two Items)

In order to analyze the system with two locations and derive a transshipment rule, the impact of the initial state (i.e., the ages of the two units) at any moment (not just at a replenishment moment) on the total long-run cost of one location has to be evaluated.

Using relationship (2) in Lemma 1 and considering the ages of both units, we can have the following Lemma 3:

Lemma 3: *For a given initial state of* a_i^2 *and* a_i^1*, the cumulative distribution of* X_i^n *for n=1 is*

$G_i^1(x) = \begin{cases} 1 - e^{-\lambda_i x} & 0 \le x < T - a_i^2 \\ 1 & T - a_i^2 \le x \le T \end{cases}$	(5)

If n is an even number>1, *the cumulative distribution function of* X_i^n *is*

$$G_i^n(x) = \begin{cases} 1-e^{-(n/2-1)\lambda_i T-\lambda_i x} + (e^{-\lambda_i T} - e^{-\lambda_i x})\sum_{k=0}^{n/2-2} e^{-k\lambda_i T} & 0 \le x \le a_i^2 - a_i^1 \\ 1+e^{-(n/2)\lambda_i T+\lambda_i a_i^2} - e^{-(n/2-1)\lambda_i T-\lambda_i x} + (e^{-\lambda_i T} - e^{-\lambda_i x})\sum_{k=0}^{n/2-2} e^{-k\lambda_i T} & a_i^2 - a_i^1 < x \le T\text{-}a_i^1 \\ 1+(e^{-\lambda_i T} - e^{-\lambda_i x})\sum_{k=0}^{n/2-2} e^{-k\lambda_i T} & T\text{-}a_i^1 \le x < T \end{cases} \quad (6)$$

*If n is an odd number>*1*, the cumulative distribution function of* X_i^n *is*

$$G_i^n(x) = \begin{cases} 1+(e^{-\lambda_i T} - e^{-\lambda_i x})\sum_{k=0}^{(n-1)/2-1} e^{-k\lambda_i T} & 0 \le x \le a_i^1 \\ 1-e^{-[(n-1)/2-1]\lambda_i T-\lambda_i (x-a_i^2)} + (e^{-\lambda_i T} - e^{-\lambda_i x})\sum_{k=0}^{(n-1)/2-1} e^{-k\lambda_i T} & a_i^1 < x \le T\text{-}a_i^2 + a_i^1 \\ 1-e^{-\lambda_i [(n-1)/2-1]T} + (e^{-\lambda_i T} - e^{-\lambda_i x})\sum_{k=0}^{(n-1)/2-1} e^{-k\lambda_i T} & T\text{-}a_i^2 + a_i^1 \le x < T \end{cases} \quad (7)$$

It is easy to verify that both (6) and (7) will approach $\frac{1-e^{-\lambda_i x}}{1-e^{-\lambda_i T}}$ when $n \to \infty$, which is consistent with Theorem 1. Please also note $G_i^n(x)$ is continuous at $T\text{-}a_i^1$ when n is an even number in (6) and at $T\text{-}a_i^2 + a_i^1$ when n is an odd number in (7). Based on distribution functions $G_i^n(x)$, we can furthermore calculate the expected value for X_i^n and obtain the following theorem for $\lim_{n\to\infty}\sum_{j=1}^{n} E[X_i^j]$ for a given initial state (a_i^1, a_i^2).

Theorem 3: *The long term impact of the initial state* (a_i^1, a_i^2) *on the performance of location i without transshipment is:*

$$\lim_{n\to\infty}(\sum_{j=1}^{n} E[X_i^j] - FT(n,\lambda_i)) = [a_i^1 + (a_i^2 - T - \frac{1}{\lambda_i})e^{a_i^1\lambda_i} - \frac{1}{\lambda_i}e^{a_i^2\lambda_i}]\frac{e^{-\lambda_i T}}{1-e^{-\lambda_i T}}, \quad (8$$

Where $FT(n,\lambda_i)$ is the sum of terms that are independent from the inital state (a_i^1, a_i^2).)

The detailed proof of Theorem 3 can be founded in the Appendix A.

Because each item costs v and has zero salvage value after expiration, Theorem 2 and Theorem 3 lead to the following theorem to evaluate the impact of the initial state (a_i^1, a_i^2) on the total cost in long-run when there is no transshipment.

Theorem 4: *The long-run impact of the initial state (a_i^1, a_i^2) on the total cost is featured by*

$$\lim_{t\to\infty}(TC(t,\lambda_i,a_i^1,a_i^2) - FC(t,\lambda_i)) = -v[a_i^1\lambda_i + (a_i^2\lambda_i - T\lambda_i - 1)e^{a_i^1\lambda_i} - e^{a_i^2\lambda_i}]\frac{e^{-\lambda_i T}}{1-e^{-\lambda_i T} - \lambda_i T e^{-\lambda_i T}}, \quad (9)$$

Where $TC(t,a_i^1,a_i^2)$ is the expected total cost up to time t with the inital state (a_i^1, a_i^2), $FC(t,\lambda_i)$ is the sum of terms that are independent from the inital state (a_i^1, a_i^2).

Theorem 2 and (8) directly lead to (9). To facilitate the further analysis, we define the function of $h(a_i^1, a_i^2, \lambda_i, T)$ to represent the long-run cost impact of the initial state.

$$h(a_i^1, a_i^2, \lambda_i, T) = -v[a_i^1\lambda_i + (a_i^2\lambda_i - T\lambda_i - 1)e^{a_i^1\lambda_i} - e^{a_i^2\lambda_i}]\frac{e^{-\lambda_i T}}{1-e^{-\lambda_i T} - \lambda_i T e^{-\lambda_i T}}. \quad (10)$$

It is easy to obtain the impact of the initial state a_i^1 on the long-term cost when a replenishment just happens (i.e., $a_i^1 = 0$) is that

$$\lim_{t\to\infty}(TC(t,\lambda_i,0,a_i^2) - FC(t,\lambda_i)) = h(0,a_i^2,\lambda_i,T)$$

$$= -v[a_i^2\lambda_i - T\lambda_i - 1 - e^{a_i^2\lambda_i}]\frac{e^{-\lambda_i T}}{1-e^{-\lambda_i T} - \lambda_i T e^{-\lambda_i T}},$$

2. A Decision Rule for Transshipment between Two Locations

In this section, we consider a transshipment policy in order to reduce the cost incurred by expiration. When one medical item is consumed or expires at location i, a replenishment can be done in two ways, 1) a new item is ordered and immediately received at location i, or 2) an existing item at the other location $j \neq i$ is transshipped to location i and a new medicine is ordered and immediately received at location j. The lead time of transshipment is also assumed to be negligible compared to the long lifetime T and interarrival times of demands. In this paper, we only evaluate transshipment decisions when one replenishment is required. In other words, we do not consider exchanges between two unexpired items in order to simplify the analysis and save transportation cost. We assume that each transshipment from location i to the other location incurs the cost of c_i.

At a replenishment moment, the impact of a transshipment is to change the initial states of both locations and incurs a transshipment cost. Assume a replenishment is required at location i (i.e., a demand request happens at location i or the older item at location i expires). Without a transshipment, $a_i^1 = 0$. The initial state of location i is $(0,a_i^2)$ and the initial state of location j is (a_j^1,a_j^2) at the same moment. The impact of this initial state without a transshipment on the long-run cost is

$$h(0,a_i^2,\lambda_i,T) + h(a_j^1,a_j^2,\lambda_j,T) \quad (11)$$

The transshipment policy also needs to determine which item, the older or newer one, at location j should be shipped to location i if it is decided to do a transshipment. In other words, at each replenishment epoch at location i, three possible actions comprise the action space, no transshipment, transshipment with the item of a_j^1, and transshipment with the item of a_j^2. We define the three actions as d_k ($k=1,2,$ and 3) respectively and their impact on the long-run cost as $IM(d_k)$. This paper proposes to use the myopic-best action by assuming there are no more transshipments in the future (see Tijm 2005 for the logic behind the policy based on the myopic-best actions). The myopic-best action is defined by $k^* = \arg\min_{k=1,2,3}\{IM(d_k)\}$, where

$$
\begin{aligned}
&IM(d_1) = h(0, a_i^2, \lambda_i, T) + h(a_j^1, a_j^2, \lambda_j, T),\\
&IM(d_2) = \begin{cases} h(a_i^2, a_j^1, \lambda_i, T) + h(0, a_j^2, \lambda_j, T) + c_j & a_i^2 \le a_j^1 \\ h(a_j^1, a_i^2, \lambda_i, T) + h(0, a_j^2, \lambda_j, T) + c_j & a_j^1 < a_i^2, \end{cases}\\
&\textit{and}\\
&IM(d_3) = \begin{cases} h(a_i^2, a_j^2, \lambda_i, T) + h(0, a_j^1, \lambda_j, T) + c_j & a_i^2 \le a_j^2 \\ h(a_j^2, a_i^2, \lambda_i, T) + h(0, a_j^1, \lambda_j, T) + c_j & a_j^2 < a_i^2. \end{cases}
\end{aligned}
\tag{12}
$$

CHAPTER IV

NUMERICAL EXPERIMENTS

In this section, we conduct numerical experiments to examine the performance of the transshipment rule developed in Chapter III regarding the total cost. The healthcare system consisting of two hospitals is illustrated in Figure 2. In the figure, B_i is the number of medicines bought by location i, P_i is the number of medicines perished at location i, A_i is the number of demand requests at location i, and R_i is the number of transshipped medicines from location i to the other location. For simulation purposes, for each location, we have $B_i+R_j=R_i+A_i+P_i$ at the end of a simulation run.

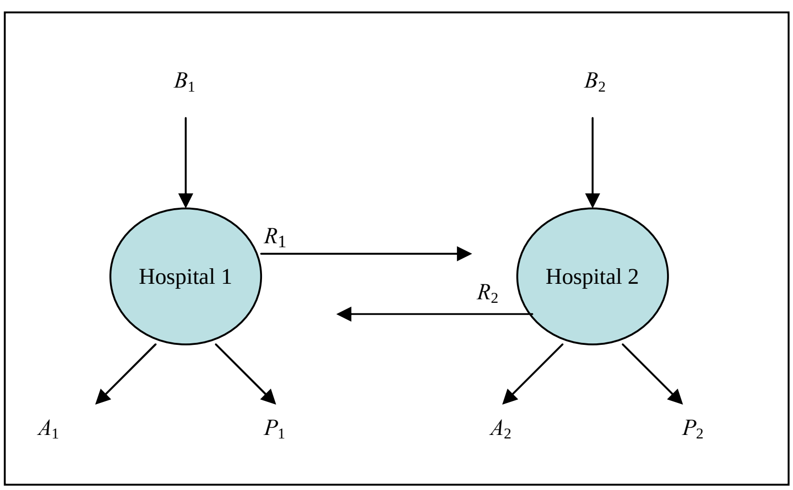

Figure 2.

A Two-location Network with Transshipments

The demand arrival process is assumed to be a Poisson process. The parameters used in the experiments are listed in Table 2. Different data sets mean different arrival rates at the two locations. In the first data set, first location's arrival rate is more than 6 times of the second location's arrival rate whereas this ratio is 5 in the second data set and 2.5 in the third set. For each data set, numerical experiments are conducted for various medicine lifetimes.

Table 2

Parameter Values for Numerical Experiments

Fixed Parameters:	
Simulation Duration in years	1000
Medicine Price (v)	$2,000
Unit Transshipment Cost from Location 1 to Location 2 (c_1)	$20
Unit Transshipment Cost from Location 2 to Location 1 (c_2)	$30
Changing Parameters:	
Data Set 1	
Arrival Rate at Location 1 (λ_1)	0.02
Arrival Rate at Location 2 (λ_2)	0.003
Medicine Lifetime in Days (T)	90 to 1260
Data Set 2	
Arrival Rate at Location 1 (λ_1)	0.02
Arrival Rate at Location 2 (λ_2)	0.004
Medicine Lifetime in Days (T)	90 to 1260
Data Set 3	
Arrival Rate at Location 1 (λ_1)	0.002
Arrival Rate at Location 2 (λ_2)	0.005
Medicine Lifetime in Days (T)	90 to 1260

Without transshipment, there is no interaction between the two locations and the total cost is computed by $v(B_1 + B_2)$ for a simulation run. The transshipment decision rule defined by (12) chooses the myopic lowest cost option among three actions. For a centralized network with transshipment as illustrated in Figure 2, total number of medicines bought by the system will be calculated by $B_i+B_j = A_i+ P_i+ A_j+P_j.$ The total cost under the transshipment policy is determined by two terms: the costs stemming from buying new medicines and the costs stemming from the transshipments. Transshipment costs are typically low compared to the medicine price and can be calculated by $\sum_{i=1}^{2} c_i R_i$. Please note that the myopic-best decision rule does not lead to the optimal transshipment policy in long-run.

Simulation is used to evaluate the performance of the decision rule by comparing it to the case without transshipment and to the case providing an upper bound for maximum saving. The upper bound is achieved by assuming the two locations are consolidated into one location with arrival rate $\lambda_1 + \lambda_2$ and holding four units in inventory. In order to evaluate the performance of the decision rule defined by (12), the total cost obtained from this simplified one-location system is compared to the total cost of the two-location network under the transshipment policy without considering any transshipment costs (i.e., let $c_1=c_2=0$ in (12)). Since no transshipment cost is incurred in the upper bound case, it is fairer and consistent to define zero transshipment costs to evaluate the gap between the performances.

The numerical experiment results are listed in Table 3. For each data set, the first column displays the total cost in millions without transshipment while the second column shows the total cost in millions under the transshipment policy considering the transshipment costs (c_1=\$20 and c_2=\$30). The third column is the percentage of improvement brought by the transshipment policy. The fourth column is the upper bound percentage of improvement brought by assuming there is only one location. The fifth column is the percentage of improvement brought by the transshipment and assuming the unit transshipment costs are zero (c_1=c_2=\$0).

As shown in Table 3, the total cost under the transshipment policy is significantly lower than the cost under no transshipment policy. The relationship between saving caused by transshipment and lifetime T is illustrated for data set 1 in Figure 3. The dashed line represents the percent of improvements under different T values while the solid line shows the amount of savings. This figure visualize that both the percent of improvements and the saving amount curves have bell-like shapes. It is interesting that the benefit of the transshipment policy is small if T is too small or too large. The benefit of transshipment increases in T to a certain point and then decreases in T. Before the peak point, when the T value is very small, although the transshipment policy is able to decrease the total cost, it cannot avoid many expirations of the medicines due to short lifetime. After the peak point, the T value becomes large enough to provide sufficient time for a medicine to have a customer arrival regardless of its location. This will result in reduced total number of perished items. The same pattern in the realized improvements is observed with other two data sets. The third data set, however, has a much larger peak position (T=810) because arrival rates are low in it. Since the advantage of transshipping is not significant when the arrival rates are very low, transshipment policy in the third data set is not as effective as the other data sets. Please note that the upper bounds of saving that can be achieved for different T values are also relatively smaller in the third data set than the first two sets.

Figure 3.

Saving and Percent of Improvement of the Transshipment Policy for Data Set 1

Figures 4, 5 and 6 are plotted to compare the saving caused by the transshipment rule and the upper bound of the saving. Please note transshipment costs are not counted when we calculate upper bound. For a fair comparison, we also consider the case in which the locations are allowed to make transshipments without any transshipment cost.

Table 3

Numerical Results with the Sensitivity Analysis on *T*

	Data Set 1					Data Set 2					Data Set 3				
Life Time, *T* (days)	W/out Trans. Policy ($M)	Trans. Policy ($M)	% Imp.	Upper Bound (% Imp.)	% Imp. (c_1=0, c_2=0)	W/out Trans. Policy ($M)	Trans. Policy ($M)	% Imp.	Upper Bound (% Imp.)	% Imp. (c_1=0, c_2=0)	W/out Trans. Policy ($M)	Trans. Policy ($M)	% Imp.	Upper Bound (% Imp.)	% Imp. (c_1=0, c_2=0)
T= 90	39.20	37.60	4.08	7.09	5.19	39.44	37.96	3.75	6.97	4.80	33.82	33.63	0.56	2.11	0.70
T=120	31.79	29.84	6.13	9.23	7.26	32.03	30.26	5.53	8.98	6.60	25.84	25.65	0.74	2.73	0.94
T= 150	27.54	25.37	7.88	10.87	9.02	27.82	25.82	7.19	10.46	8.29	21.08	20.88	0.95	3.28	1.25
T=180	24.89	22.56	9.36	12.17	10.48	25.19	23.04	8.54	11.49	9.58	17.93	17.70	1.28	3.99	1.62
T= 210	23.11	20.72	10.34	12.93	11.48	23.42	21.25	9.27	12.15	10.36	15.69	15.44	1.59	4.61	2.03
T=240	21.89	19.49	10.96	13.46	12.09	22.20	20.05	9.68	12.38	10.75	14.00	13.75	1.79	5.06	2.28
T= 270	21.02	18.65	11.27	13.54	12.33	21.31	19.25	9.67	12.20	10.71	12.72	12.44	2.20	5.71	2.78
T=300	20.34	18.09	11.06	13.32	12.16	20.66	18.72	9.39	11.79	10.42	11.67	11.40	2.31	6.01	2.86
T= 330	19.82	17.67	10.85	12.82	11.95	20.16	18.34	9.03	11.17	10.05	10.86	10.56	2.76	7.07	3.65
T=360	19.41	17.39	10.41	12.26	11.48	19.76	18.08	8.50	10.45	9.54	10.17	9.86	3.05	8.18	4.70
T=540	18.07	16.86	6.70	7.97	7.80	18.49	17.57	4.98	6.13	5.98	7.74	7.38	4.65	8.81	5.43
T=720	17.42	16.80	3.84	4.94	4.88	17.96	17.51	2.51	3.40	3.35	6.64	6.27	5.57	9.22	6.24
T=810	17.30	16.78	3.01	3.98	3.93	17.79	17.50	1.63	2.51	2.47	6.32	5.95	5.85	9.31	6.57
T=1080	16.95	16.75	1.18	2.05	1.99	17.53	17.44	0.51	1.07	1.03	5.74	5.44	5.23	7.79	5.88
T=1260	16.84	16.72	0.71	1.35	1.30	17.45	17.41	0.23	0.61	0.57	5.53	5.28	4.52	6.61	5.16

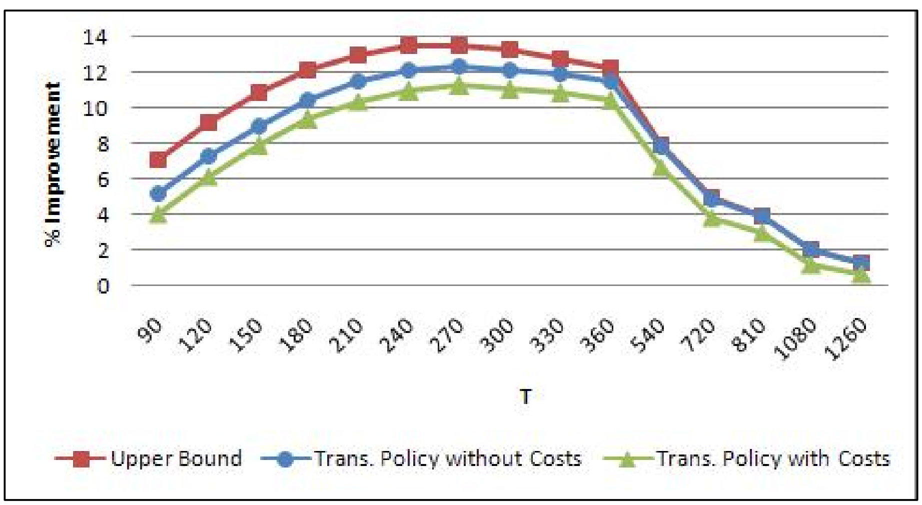

Figure 4.

Percent Improvements of Transshipment Policy and Upper Bounds for Data Set 1

Comparing three lines in Figures 4, 5 and 6, we can see the myopic-best decision rule work well regarding its gap from the upper bound. The optimal transshipment rule cannot improve the performance much for data set 1 and 2 compared to the decision rule (12), especially when T is large. For data set 3, the optimal transshipment rule may have more savings compared to the decision rule (12). In other words, the decision rule (12) works well when the arrival rates at the two locations are significantly different. Beyond these three data sets, we test more arrival rates and observe similar results.

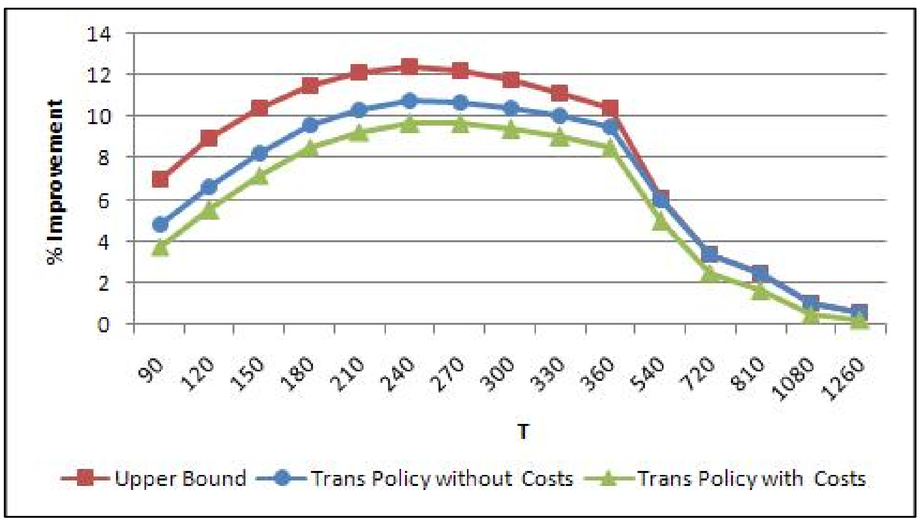

Figure 5.

Percent Improvements of Transshipment Policy and Upper Bounds for Data Set 2

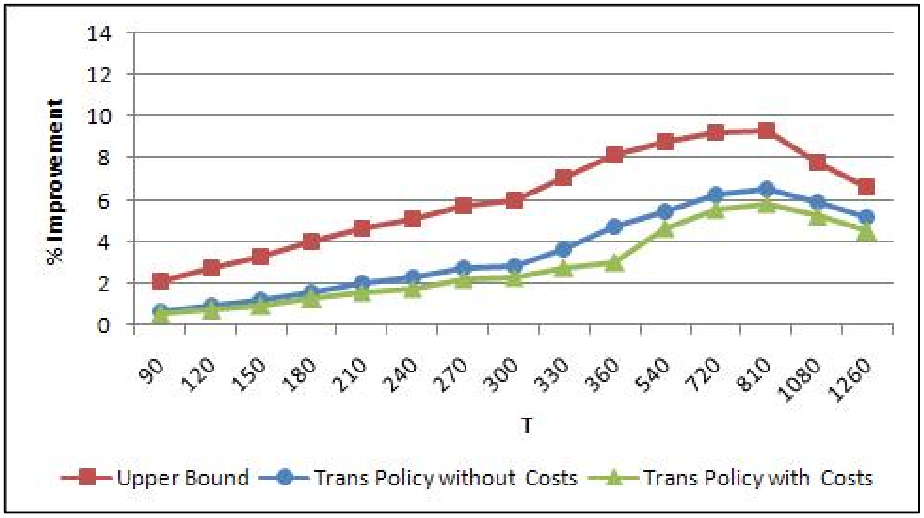

Figure 6.

Percent Improvements of Transshipment Policy and Upper Bounds for Data Set 3

We also conduct numerical experiments on different arrival rates λ_i beyond the three data sets. If the arrival rates of the locations differ from each other greatly, the benefit of the transshipment policy is large. Intuitively, in such a case, older medicines should be transshipped to the location whose arrival rate is higher. As the arrival rates become closer, the improvement achieved by the transshipment policy is expected to be smaller. Figure 7 depicts such a pattern with the same cost parameters shown in Table 2 and T = 270 days. The arrival rate for the first location is fixed at 0.01 units per day, while the second arrival rate takes values between 0.001 and 0.05 units per day. The detailed results are available in Table 4. When the two arrival rates are the same, although the percent improvement is relatively small, the transshipment still causes a significant cost saving. Once the arrival rate of the second location exceeds the rate of the first one, the improvement starts to increase because of the differentiation of the arrival rates but it never reaches the highest improvement achieved when λ_2 is small due to smaller expiration chance.

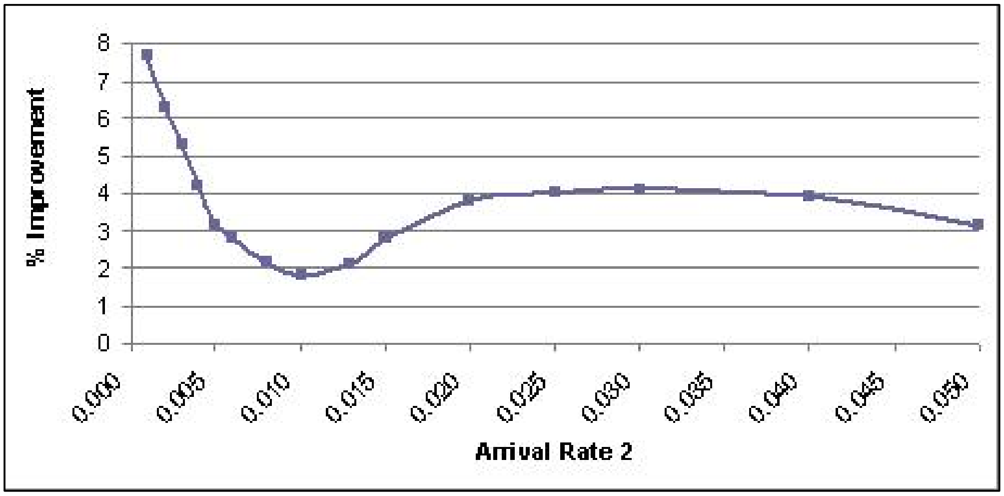

Figure 7.

Sensitivity Analysis on λ_2

Table 4

Results for the Sensitivity Analysis for λ_2

	W/out Transshipment Policy ($M)	Transshipment Policy with Transshipment Costs ($M)	% Imp.	% Imp. Under Transshipment Policy W/out Transshipment Costs	Upper Bound (%)
λ_2=0.001	$14.58	$13.48	7.57	8.72	11.07
λ_2=0.002	$14.86	$13.92	6.31	7.43	10.43
λ_2=0.003	$15.15	$14.34	5.36	6.37	9.79
λ_2=0.004	$15.46	$14.79	4.32	5.32	9.18
λ_2=0.005	$15.79	$15.26	3.34	4.23	8.69
λ_2=0.006	$16.18	$15.71	2.93	3.68	8.29
λ_2=0.008	$17.04	$16.67	2.21	2.74	7.87
λ_2=0.01	$17.96	$17.65	1.69	2.09	7.44
λ_2=0.013	$19.51	$19.11	2.07	2.25	6.96
λ_2=0.015	$20.68	$20.12	2.73	3.4	6.94
λ_2=0.02	$23.80	$22.93	3.67	4.34	6.56
λ_2=0.025	$27.17	$26.07	4.05	4.66	6.12
λ_2=0.03	$30.73	$29.44	4.2	4.83	5.66
λ_2=0.04	$37.96	$36.53	3.79	4.45	4.73
λ_2=0.05	$45.13	$43.70	3.19	3.86	4.01

CHAPTER V

EXTENSIONS AND CONCLUSIONS

The transshipment decision rule (12) can be easily extended to the case with more than two locations. In a three-location network, for example, there are five possible actions. Assuming that an item at location 1 is just used or expires, the long-run relative costs of the five possible actions are

$$
\begin{aligned}
IM(d_1) &= h(0,a_1^2,\lambda_1,T) + h(a_2^1,a_2^2,\lambda_2,T) + h(a_3^1,a_3^2,\lambda_3,T), \\
IM(d_2) &= \begin{cases} h(a_1^2,a_2^1,\lambda_1,T) + h(0,a_2^2,\lambda_2,T) + h(a_3^1,a_3^2,\lambda_3,T) + c_{21} & a_1^2 \le a_2^1 \\ h(a_2^1,a_1^2,\lambda_1,T) + h(0,a_2^2,\lambda_2,T) + h(a_3^1,a_3^2,\lambda_3,T) + c_{21} & a_1^1 < a_2^2, \end{cases} \\
IM(d_3) &= \begin{cases} h(a_1^2,a_2^2,\lambda_1,T) + h(0,a_2^1,\lambda_2,T) + h(a_3^1,a_3^2,\lambda_3,T) + c_{21} & a_1^2 \le a_2^2 \\ h(a_2^2,a_1^2,\lambda_1,T) + h(0,a_2^1,\lambda_2,T) + h(a_3^1,a_3^2,\lambda_3,T) + c_{21} & a_1^2 < a_2^2. \end{cases} \\
IM(d_4) &= \begin{cases} h(a_1^2,a_3^1,\lambda_1,T) + h(a_2^1,a_2^2,\lambda_2,T) + h(0,a_3^2,\lambda_3,T) + c_{31} & a_1^2 \le a_3^1 \\ h(a_3^1,a_i^2,\lambda_1,T) + h(a_2^1,a_2^2,\lambda_2,T) + h(0,a_3^2,\lambda_3,T) + c_{31} & a_j^1 < a_i^2, \end{cases} \\
IM(d_5) &= \begin{cases} h(a_1^2,a_3^2,\lambda_1,T) + h(a_2^1,a_2^2,\lambda_2,T) + h(0,a_3^1,\lambda_3,T) + c_{31} & a_1^2 \le a_3^2 \\ h(a_3^2,a_1^2,\lambda_i,T) + h(a_2^1,a_2^2,\lambda_2,T) + h(0,a_3^1,\lambda_3,T) + c_{31} & a_3^2 < a_1^2. \end{cases}
\end{aligned}
\qquad (13)
$$

Here, c_{ij} is the unit transshipment cost from location i to j. The decision is to choose the action with the least long-run relative costs. The number of actions at any decision moment for a network with N locations is $2N+1$, which is polynomial and computationally tractable.

This research studies lateral transshipment of critical medical items that are slow moving. To keep high availability, the healthcare providers are assumed to hold two units in the inventory and replenish it immediately when one unit is used or expires. The low demand and limited life time cause expensive medical items to expire frequently. Lateral transshipment between hospitals can help to reduce expiration and therefore reduce the total cost. In this study, the decision rule for lateral transshipment in a two-hospital system is investigaed. The decision rule is derived based on a myopic comparison of long-run relative costs among possible actions assuming no transshipment in the future. The numerical experiments demonstrate significant cost savings achieved by the proposed transshipment decision rule. The developed myopic-best decision rule works well regarding the gap from the upper bound of the total savings. The sensitivity analysis of the life time of the medical item and demand arrival rates shows the savings are more significant when demand rates at locations are more different and the life time of the medical item is not too long or too short. The decision rule can be easily extended to the case in which there are more than two locations.

This paper only studies the myopic-best decision rule because it is difficult to derive the long-run performance and the impact of initial states for the system implementing the developed transshipment rule. In other words, it is hard to obtain the long-run relative value of actions based on the current transshipment rule. Therefore, the developed transshipment rule is not optimal and may be improved. Numerical experiments also show that there is still some space for improvement, though not very large, for some data sets based on the gap of the current saving percentage from the upper bound percentage. One possible extension to this research is to establish a continuous-time Markovian Chain by approximating the problem with limited state space.

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APPENDIX A

PROOF OF THEOREM 3

Based on (5), (6), and (7), we have

$$E[X_i^1] = \frac{1}{\lambda_i}(1 - e^{-\lambda_i (T - a_i^2)})$$

If n is an odd number>1, the expected value of X_n is

$$E[X_i^n] = \frac{1}{\lambda_i} - T\sum_{k=1}^{(n-1)/2} e^{-k\lambda_i T} - \frac{1}{\lambda_i} e^{-\lambda_i((n+1)T/2 - a_i^2)} + a_i^1 e^{-(n-1)\lambda_i T/2}$$

If n is an even number>1, the cumulative distribution function of X_n is

$$E[X_i^n] = \frac{1}{\lambda_i} - T\sum_{k=1}^{(n-2)/2} e^{-k\lambda_i T} + (a_i^2 - T - \frac{1}{\lambda_i})e^{-\lambda_i(nT/2 - a_i^1)}$$

for n>1, the first and the second terms, no matter n is odd or even, are independent from the initial states. Therefore,

$$\lim_{n\to\infty}(\sum_{j=1}^{n} E[X_i^j] - FT(n,\lambda_i)) = -\frac{1}{\lambda_i}\sum_{k=1}^{\infty} e^{-k\lambda_i T + a_i^2\lambda} + a_i^1\sum_{k=1}^{\infty} e^{-k\lambda_i T} + (a_i^2 - T - \frac{1}{\lambda_i})\sum_{k=1}^{\infty} e^{-k\lambda_i T + a_i^1\lambda}$$

$$= (a_i^1 + (a_i^2 - T - \frac{1}{\lambda_i})e^{a_i^1\lambda_i} - \frac{1}{\lambda_i}e^{a_i^2\lambda_i})\frac{e^{-\lambda_i T}}{1 - e^{-\lambda_i T}}.$$

□