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Log Grade Volume Distribution Model for Tree Species in Red Oak-Sweetgum Forests in Southern Bottomlands

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LOG GRADE VOLUME DISTRIBUTION MODEL FOR TREE SPECIES IN RED OAK-SWEETGUM FORESTS IN SOUTHERN BOTTOMLANDS

By

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Southern bottomland sites are among the most productive areas for producing high quality grade hardwood, yet the ability to estimate the quantity and quality of standing grade hardwood is almost non-existent. Measurements and observed log grades were recorded on standing trees to construct volume prediction models for individual trees. Several different modeling techniques were explored and compared during development. Developed equations predict merchantable sawtimber volume and volume by grade category in trees by species group. Two separate sets of equations were developed for each species group using either total height or merchantable height. Models were chosen based on significance of variables, index of fit, RMSE, bias, ease of use, and biological trends. The models developed to predict merchantable sawtimber and grade volumes were designed to be implemented in a larger hardwood growth and yield system.

DEDICATION

To my sons, Simon Floyd and Davis Frederick. Without your love and patience for me, I would not have had the strength or desire to have pursued this degree.

To my loving wife Rebecca Nell. Your friendship and love supported me during the most frustrating times of my college career. May our sacrifices bring us closer and be the foundation for a lifetime of love and togetherness.

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Dr. Gan Li helped develop the user interface for the models and showed me the importance and usefulness of computer programming knowledge. I have enjoyed working with Dr. Li and developed a personal and professional friendship.

Mr. Paul Jeffreys and Mr. Cade Booth provided time, energy, and experience gathering stand level data making this project possible. Without the abundance of high quality data, these models could not have been developed. Thanks for your friendship.

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CHAPTER I

INTRODUCTION

Background

Southern bottomland sites are among the most productive areas for producing high quality grade hardwood in the United States. Grade hardwood is in demand for manufacturing floors, furniture, veneer, and many other high quality products. Despite the many uses for hardwoods, the ability to quantify the amount and quality of log grade volume of standing timber is not readily available.

Accurately quantifying the amount and quality of timber in a stand is very important for determining monetary value, management options, treatment applications, research opportunities, cultural practices, aesthetics and other decisions pertinent to overall stand management. Growth and yield models have long been used to determine volume, size, density, past/future growth, site index, and many other stand and tree characteristics, but few models have been developed for hardwood stands. The development of growth and yield models for hardwoods is more challenging due to factors such as greater numbers of species, less uniform growth structures, and the need to quantify log grade and tree grade volumes. The longer rotations required for hardwoods to produce marketable logs also added to the challenges of modeling as it requires longer durations to measure the growth of each tree.

It is important to distinguish the difference between the terms log grade and tree grade. Log grade refers to the grade of an individual log, which can have different lengths. The minimum standard of a hardwood log is any piece of a tree stem eight inches or greater in diameter and eight feet or greater in length (Rast et al. 1973). A log can be referred to either as a cut section of the tree or a section of the tree while still standing. For this research, all graded logs will be in the form of standing trees. Tree grade refers strictly to the grade of the first 16-ft section of the tree, also referred to as the butt log.

Log grade is an important factor in determining value and desired product for a section of a tree. U.S. Forest Service (USFS) grades are 1, 2, and 3, with grade 1 being the highest quality (Rast et al. 1973). Log grade is based on the quantity and quality of valuable wood in the length of a log as determined by the diameter and length of the log, as well as the number and spacing of visible defects. The grade of the log is determined using the grade face which is the second worst face of the log; the worst face is passed over as products will be cut out farthest from this face first. The grades assigned to the logs in the standing tree are intended to reflect the grade of the final product. For instance, a grade 1 log should yield at least 65 percent of its volume as No. 1 Common and Better lumber (Rast et al. 1973). This research will use the U.S. Forest Service grades 1, 2, and 3 plus two extension grades of 4 for tie/timber and 5 for cull. Tie and timber logs are typically used for construction class timber where there are too many defects to be used in finished products, but the wood is sound enough for construction. Cull can be defined as any section of the merchantable stem that is unsuitable for production; this can be rot or bole fork for example.

Defect types are numerous and can be caused by many different factors. A short summary of these defects include: bird peck, branch scars, butt swell, cankers, crooks/sweep, epicormic branches, forks, holes, knots, limbs, overgrowths, rot, and wounds (Carpenter et al. 1989). The location and arrangement of defects on the log is as important as how many defects are on the log. For example, it is more desirable to have defects clustered near one end of the log than to have defects evenly spaced throughout the log. Having the defects clustered as opposed to spaced farther apart results in larger contiguous areas of clear wood. How the defects are distributed across the different sides or 'faces' of the log section is also important. If all the defects are found on a single face, this leaves more clear wood throughout the rest of the log.

Stand Influences on Grade Hardwood

Many factors go into how and why a tree develops defects. It is important to understand these influences in order to determine which variables affect grade potential within a stand. Understanding stand development for the purpose of improved grade properties is important for land management, but also very useful for modeling development.

Stand density plays a large role in straightness, branching patterns, stress, canopy position/closure, stem diameter, and other factors that lead to the presence or absence of defects. Natural pruning of lower branches at an earlier age is also affected by stand density (Miller 2000), thus helping to improve the future potential of quality grade development. Diameter growth on bottomland red oak-sweetgum stands with similar site quality, species composition, and age will favor the one with reduced stand density

(Meadows et al. 2006). Diameter growth is negatively influenced as stand density increases. This can be explained through the concept that a fully stocked site will produce a maximum amount of volume over a given time and the volume will be distributed across the number of stems depending on species, light availability, and many other factors. As an example, two fully stocked stands having all things equal except number of stems will produce greater diameter growth in the stand with fewer stems. The removal of both intra-specific and inter-specific competition allows a tree to more freely uptake available nutrients for a site.

Total height can be a determining factor in branch location on the tree. Taller hardwood trees tend to have live branches in the upper half of the stem. As the lower, merchantable stem begins to drop branches, clear wood is added to the stem improving the quality of harvestable volume. The height of a tree is greatly influenced by site quality.

Site quality influences the inherent ability of a tree to develop to its full potential. A tree grown on a low quality site could often have too few available nutrients, poor soil structure, texture, drainage, pH, or other factors for optimum development. For example, insufficient minerals can cause deficiency in vital growth areas such as height growth, foliage development, volume, disease resistance, fiber quality, branching, and many others. The higher the site index, the greater chance a tree has to reach maximum potential of growth qualities. This can be explained as trees allocate nutrients in ways that will generally increase the tree's ability to survive (Smith et. al. 1997). Sites with high amounts of nutrients available allow a tree to disseminate the nutrients to all growth regions instead of prioritizing for survival due to a shortage of resources available. The species-site relationship also determines a tree's potential. A species not grown on a site for which it is adapted will generally grow poorly and be of lower grade. The bottomland sites found in this study are all similar in site quality and are high in available nutrients.

Diameter is an important variable in determining the grade of a log. The highest quality grades have stricter stipulations on diameter to be classified as that grade. If a log section does not meet the size requirements, then the log does not qualify to be in a particular grade category. In general, diameter has influence on the number of visible defects on a logs surface. While new defects can appear on large diameter logs, added diameter often reduces the number of visible defects found on the surface of a log. This is attributed to the addition of clear wood developed concealing old defects such as branch scars. As a tree grows larger, many old defects will be overgrown, thus adding clear wood to the bole of the tree.

Crown class and canopy position can influence defects on the lower logs of the tree. When a tree is suppressed, by surrounding trees in the upper canopy it must develop alternate ways to increase sunlight intake. A tree that is stressed and competing for sunlight often puts out epicormic branches which lead to small knot or pin hole defects (Meadows 2001). Epicormic branches are frequently found protruding from the base of the canopy down to the lower sections of the bole. This causes a decrease in grade quality, thus trees stressed by light restrictions are undesirable for the purposes of timber grade development.

Northeastern Grade Prediction Models

Ward (1964) developed a technique for determining potential tree grade for young red oaks (*Quercus* spp.) in Pennsylvania on good to excellent sites containing even-aged stands which ranged from 10 to 40 years old. He used a clear volume index to define the relationship between clear log faces and crown competition. Clear volume index was calculated by: (1) determining the percentage of volume occurring in the basal half of the tree, (2) dividing this basal half into four-foot sections, (3) multiplying the percent volume of the four-foot section by number of clear faces, and (4) summing the scores for each section. The clear volume index has a maximum score of 400, which could be interpreted that a tree had no live branches and no dead branches larger than 0.75 inches in diameter. A score of 300 could be interpreted that 100 percent of the volume averaged three clear faces or 75 percent of the volume had four clear faces. The higher the clear volume index score indicates less live branches on the section, and thus the greater potential for a higher grade quality.

Stayton et al. (1971) developed a method of predicting lumber grade yields for standing sugar maple (*Acer saccharum*) trees in old-growth northern hardwood stands in upper Michigan. A total of 90 trees were collected and separated into three diameter at breast height (dbh) classes and three quality classes within each dbh class. After separation, the trees were bucked into 8-foot logs, measured, and diagrammed. Eight independent variables [dbh, dbh^2, merchantable length, merchantable cubic foot volume inside bark (calculated by Smalian's rule), stem taper, percentage of defect indicators of stem length, percentage of flutes and seams of stem length, average tree diameter] were used to develop lumber grade linear regression models. Lumber grade yields obtained from published tables were used as dependent variables. Coefficients were calculated for the prediction of six grade categories of First and Seconds, Select, #1 Common, #2 Common, #3A, and #3B, and the coefficient of determination (R^2) for each was .71, .73, .89, .94, .81, and .84, respectively. Average yields by log size and grade used from the published tables eliminated much of the variation surrounding the mean during the regression analysis which is one reason for the resulting high R^2 values.

Yaussy and Brisbin (1983) predicted green board-foot lumber grade volumes of northern red oaks (*Quercus rubra*) using multivariate regression techniques for seven lumber grade yield dependent variables (Firsts and Seconds, FAS on face, Selects, #1 Common, #2 Common, #3A Common, #3B common). The four independent variables that provided the greatest contribution to yield predictions were factory log grade, scaling diameter, log length, and percent defect. The application of this model is predicting expected total lumber tally and volume by lumber grade in sawmills. Validation for the model was achieved through the calculation of coefficients using a separate data set. The two sets of coefficients from the model and validation data sets were compared statistically. It was determined that there was no significant difference between the two, thus validating the model.

Yaussy et al. (1988) developed a means of estimating volumes and number of logs by grade from hardwood cruise data. A two-stage estimation procedure was developed which utilized regression techniques and linear discriminant analysis. Volumes by log grade were estimated with multivariate multiple linear regression equations and then combined with tree dimensions and log/tree grade to predict number of logs by log grade using discriminant functions. Standing trees were measured and graded using USFS tree grades. Variables used in the model are dbh, merchantable height to an 8-inch top, and USFS tree grade (Hanks 1976). Model outputs were used to estimate the quantity and quality of logs from a timber sale in the Appalachian and northern regions for 11 hardwood species (Yaussy et al. 1988). The results of the validation data set showed that the total merchantable volume by species was predicted within 5 percent of the actual. The predicted log grade volumes were within 20 percent, with the exception of log grade 3 chestnut (*Quercus montana*) and northern red oaks. The total predicted number of logs was within 10 percent by species with the exception of grade 3 northern red oaks. The exceptions to northern red oak might be due to the low number of grade 3 logs observed during the study.

Standing yellow-poplar (*Liriodendron tulipifera*) trees were studied in eastern Kentucky to develop log grade predictions (Hilpp and Pelkki 2003). This study recorded the tree characteristics dbh, total height, USFS tree grade, USFS crown class, and crown ratio prior to felling. After gathering standing tree information the trees were felled to record USFS log grades for the entire tree. Linear discriminant analysis showed eight independent variables (dbh, merchantable height, stem length, relative merchantable height, relative stem length, indicator variable for tree grade 1, and indicator variable for tree grade 2) were significantly related to entire tree grade. Application of this system is in assigning each log within a tree a USFS log grade. As the prediction model is run for each log section, the highest value from the three log grade equations (grade 1, 2, 3) corresponds to the grade assigned to that log section. The model correctly identified 81% of the logs in the validation set but is only applicable to the prediction of log grades in standing yellow-poplar trees.

Southern Bottomland Hardwood Grade Prediction Models

Prediction equations for tree grade of Mississippi bottomland red oak-sweetgum (*Liquidambar styraciflua*) hardwood stands were developed by Belli et al. (1993). Data were obtained from 150 permanent plots in unmanaged red oak-sweetgum stands on minor stream bottoms in Central Mississippi. Model variables included dbh, relative basal area, average basal area, relative average basal area, average height of dominant/codominant trees, ratio of trees per acre to average age of dominant/codominant trees, and ratio of basal area of red oaks to the basal area of sweetgum in the plot. A linear discriminant analysis was used to predict a probable tree grade category for individual trees within plots. The validation results utilizing the discriminant functions correctly classified tree grades at 51%, 34%, 33%, and 63% of cherrybark oak (*Quercus pagoda*), water oak (*Quercus nigra)*, willow oak (*Quercus phellos*), and sweetgum (*Liquidambar styraciflua*) trees, respectively. The intended application of the grade prediction equations was to provide information for the management of hardwood stands in the minor stream bottoms of Mississippi.

Stand Level Prediction Models

Southern red oak-sweetgum stand level prediction models were developed as the foundation of a bottomland hardwood growth and yield system (Iles 2008). These stand level models were developed using the same plot locations used for this study on log grade volume distribution. Both of these projects were designed to use similar input and output variables which allows them to be integrated in a more complete system.

The stand level models predict: dominant height of the stand, site index, trees per acre, arithmetic mean diameter, and quadratic mean diameter. An individual tree model was also developed to predict total height. Models were developed for six different species groups: red oak, white oak, sweetgum, hickory (*Carya spp.*), other commercial, and non-commercial. Three different 'scenarios' are available for predicting these stand level characteristics depending on the amount of input information available to the user: bare ground, stand density, and existing inventory.

These stand level and individual tree height models will be linked to the grade distribution models developed in this thesis. Predicted total height will be used as an input variable for estimating merchantable and grade volumes. The stand level models predict trees per acre by species group and will be the basis for diameter distribution research to determine number of trees per diameter class. Diameter at breast height is another input variable to predict merchantable and grade volumes. The stand level and grade distribution models can function separately, but are designed to eventually be linked together.

Statement of Problem and Justification of Research

Making sound decisions for many forest management objectives depends on the ability to assess and predict the volume, value, and condition of a forested stand. Southern bottomland sites are among the most productive areas for producing high quality grade hardwood, yet the ability to estimate the amount and quality of standing grade hardwood is almost non-existent. Despite improved interest and understanding of these valuable resource types, there has been little development of growth and yield systems including grade prediction models for southern bottomland hardwood forest types. As a consequence, hardwood forest landowners have great difficulty evaluating management alternatives.

Stand level prediction variables have been developed for natural red oak-sweetgum forests in minor bottoms, but they do not predict volume or standing wood quality (Iles 2008). The addition of total volume and grade volume prediction to these stand level models will create a more complete understanding of the dynamics of this forest type, and in turn how those forest types are utilized. A greatly improved ability to make more accurate volume and value assessments should lead to increased investment and research, management and treatment application, and information upon which to base wildlife management, recreation, and aesthetic decisions.

Objectives

The purpose of this study was to develop volume prediction models for both total merchantable volume and volume per grade category for an individual tree. The models were developed to utilize input variables commonly collected during timber cruises. Using commonly collected cruise data will allow for ease of use for landowners, managers, and other decision makers. These log grade volume distribution models were designed to complement existing stand level models so both would be incorporated into a larger growth and yield system. The overall objective was to have a hardwood growth and yield decision support system (DSS) that predicts both total and grade volume on the stand and tree level. The DSS will be easily comprehendible and usable for all client groups, web based, and free of charge.

CHAPTER II

METHODS

Study Area

In 1981, USFS and Mississippi State University (MSU) collaboratively began a long term project to establish and model red oak-sweetgum forest types in minor steam bottoms of the southern U.S. Minor stream bottoms are classified as flood plains and terraces formed through the deposits of local soils (Hodges and Switzer 1979). One hundred and fifty permanent plots were established across northern and central Mississippi. The plots were selected in areas with naturally occurring red oak-sweetgum forest types and excluded areas where regeneration was primarily through planting or other artificial means. Measurements were taken for both stand and tree characteristics including log grades for the entire merchantable length. Plots were re-measured in 1988, 1992, and 2006 with 40 new plots added in 1993, 37 new plots in 2007 and 10 new plots in 2009. The new plots replaced plots that were lost to harvest, storm damage, or other major disturbances (Iles 2008). As of 2009, there are 168 valid plots with an expanded research area that includes western Alabama and southern Mississippi counties (Figure 1).

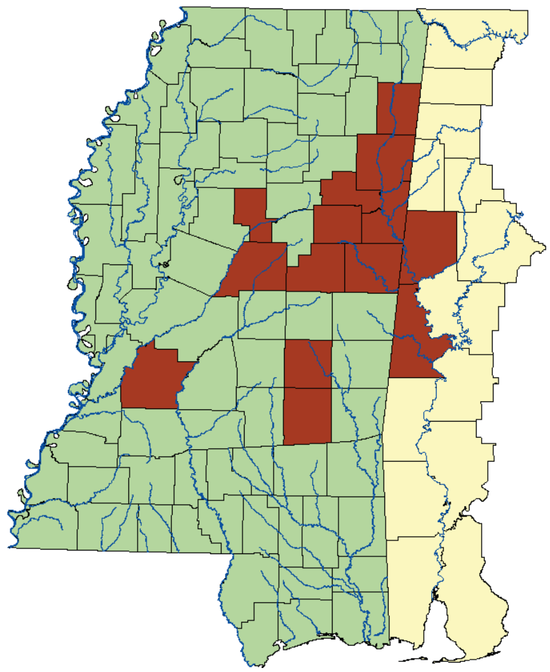

Figure 1: Mississippi and Alabama counties with permanent growth and yield plots.

All plot locations were selected in even-aged stands and include a wide range of both stand age and basal area within the plots selected. The wide range of stand age and basal area allows the models to encompass different stages of stand development. The criteria for plot establishment were:

1. Stands should occur in stream bottoms (rivers, creeks, or other streams) but not on lands occurring between the Mississippi river and the levee and not in the loess hills.
2. Mississippi and Alabama represent the general geographic area.
3. All plots must be in areas which developed as even-aged stands.
4. Stands should be essentially undisturbed from cutting and severe damage (fire, beaver, management, wind, etc.) for at least the last twenty years.
5. The minimum basal area for red oak is 30% of the total basal area.

6. The minimum age for a stand to be eligible for plot installation is twenty years; there is no maximum age.
7. Stands must have a minimum basal area of 60 square feet; all basal areas are based on 3.5” dbh and larger trees.
8. Minimum plot size acceptable is 0.1 acre.
9. Maximum plot size is 1 acre.
10. Plots and areas immediately adjacent to the plots must have the potential of remaining undisturbed for at least the next 10 years.
11. Stands should be in good condition with little disease, good crowns, and low percentage of blow-downs.

Measurements

All prior existing plots were visited to verify no major disturbance (harvest, thinning, storm damage, fire, etc.) had occurred since the last measurement period. Plot center GPS coordinates and tree and stand data were measured for 121 viable plots. Forty-seven new plots were established targeting specific age classes to make the range of the data more uniform. The information gathered for all trees on all plots was:

1. Species
2. dbh to 0.1 inch
3. Crown class
4. Log grade
5. Azimuth and distance from plot center

Data taken on a sample of ten trees across the range of dbh's was:

1. Total height
2. Merchantable height
3. Height to an 8" top
4. Height to a 4" top

The following measurements were taken on all in-growth trees:

1. All trees 4 inch dbh and above that were not recorded in the last re-measurement were tagged
2. Species
3. Azimuth and distance from plot center
4. Total height
5. dbh to 0.1 inch

Site index data taken on six dominant or co-dominant red oak trees included:

1. Age at breast height (4.5 ft)
2. Total height

Trees selected for grading within each plot had a minimum dbh of 9.6 in. All grade data for the 2005-2007 re-measurement periods were recorded by a single expert grader (Steve Meadows, USFS, Center for Bottomland Hardwoods Research, Stoneville, MS). The USFS standard for grading hardwood logs was used for grades 1, 2, and 3 (Rast et. al. 1973). Additional grade categories for tie/timber (4) and cull (5) were also used for all graded trees. Grades were taken on all logs throughout the entire merchantable length of each tree. Grades and lengths of each standing log were recorded to maximize each section's grade potential. Maximizing grade potential means a shorter section length was recorded if it improved the grade assigned to that log. Measurement variables for all gradable trees were:

1. dbh
2. Total height
3. Merchantable height
4. Height to first dead limb
5. Height to first live limb
6. Stump height
7. Gradable Section
    a. Length
    b. Grade
    c. Stopper code (attributing factor preventing a log section from being an improved grade)
8. Epicormic branches
    a. Frequency on butt log
    b. Frequency on upper logs
9. Final merchantable stopper code

Minimum Grade Specifications

Determining factors for standing tree logs to be classified as 1, 2, or 3 using the USFS log grading rules are based on position in tree, scaling diameter, length, clear cutting factors (length, number, and proportion), maximum sweep/crook allowance, and maximum scaling deduction (Rast et al. 1973). Minimum limitations of these factors for each grade are given in Table 1.

Grade 4 (tie and timber) are merchantable sawlogs that do not meet the minimum requirements to be grade 3. Major limiting factors that keep a log from being merchantable are: dead, rot, sever sweep/crook (>50%), too many limbs, too many defects, and bulges. Logs, bolts, or sections that are not merchantable and can not be classified as grade 4 are assigned grade 5 (cull).

Table 1: Reproduced, Figure 5 USFS a guide to Hardwood Log Grading describing standard minimum grade requirements for standing tree logs (Rast et al. 1973).

<table>
<tr><td colspan="2" rowspan="2">Grading Factors</td><td colspan="8">Log grades</td></tr>
<tr><td colspan="3">1</td><td colspan="4">2</td><td>3</td></tr>
<tr><td colspan="2">Position in tree</td><td>Butts only</td><td colspan="2">Butts and uppers</td><td colspan="4">Butts & uppers</td><td>Butts & uppers</td></tr>
<tr><td colspan="2">Scaling diameter, inches</td><td>13-15[a]</td><td>16-19</td><td>20+</td><td>11+[b]</td><td colspan="3">12+</td><td>8+</td></tr>
<tr><td colspan="2">Length without trim, feet</td><td colspan="3">10+</td><td>10+</td><td>8-9</td><td>10-11</td><td>12+</td><td>8+</td></tr>
<tr><td rowspan="3">Required clear cuttings[c] of each of 3 best faces[d]</td><td>Min. length, feet</td><td>7</td><td>5</td><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td><td>2</td></tr>
<tr><td>Max. number</td><td>2</td><td>2</td><td>2</td><td>2</td><td>2</td><td>2</td><td>3</td><td>No limit</td></tr>
<tr><td>Min. proportion of log length required in clear cutting</td><td>5/6</td><td>5/6</td><td>5/6</td><td>2/3</td><td>3/4</td><td>2/3</td><td>2/3</td><td>1/2</td></tr>
<tr><td rowspan="2">Maximum sweep & crook allowance</td><td>For logs with less than 1/4 of end in sound defects</td><td colspan="3">15%</td><td colspan="4">30%</td><td>50%</td></tr>
<tr><td>Min. proportion of log length required in clear cutting</td><td colspan="3">10%</td><td colspan="4">20%</td><td>35%</td></tr>
<tr><td colspan="2">Maximum scaling deduction</td><td colspan="3">40%[e]</td><td colspan="4">50%[f]</td><td>50%</td></tr>
</table>

a Ash and basswood butts can be 12 inches if they otherwise meet requirements for small #1's
b Ten-inch logs of all species can be #2 if they otherwise meet requirements for small #1's
c A clear cutting is a portion of a face, extending the width of the face, that is free of defects
d A face is ¼ of the surface of the log as divided lengthwise
e Otherwise #1 logs with 41-60% deductions can be #2
f Otherwise #2 logs with 51-60% deductions can be #3

Calculations

Total volume and volume by grade category were calculated for each graded tree using observed data. Profile functions developed by Souter (2003) for hardwood tree species in the southeastern United States were used to determine taper rates of each species group using total tree height and dbh as inputs. These profile functions were applied to the grade data for each tree for calculating volumes. Five volume types (Doyle, International ¼ inch rule, Scribner log rule, merchantable sawlog cubic foot outside bark, and merchantable sawlog cubic foot inside bark) were calculated for both total volume and volume by grade category. Doyle, International ¼ inch rule, and Scribner log rule are expressed in board feet, and cubic foot volume inside/outside bark are expressed in cubic feet.

Species Groups

Trees included in model development were categorized into five species groups:

1. cherrybark oak
2. other red oak
3. all red oak
4. sweetgum
5. other commercial

These groups were selected by commercial importance, species regularity, and compatibility with the prior stand level research by Iles (2008). The cherrybark oak category consists of only cherrybark oak (*Quercus pagoda*). The primary species comprising the other red oak group are water oak (*Quercus nigra*) and willow oak

(*Quercus phellos*). The all red oak category consists of all trees from both the cherrybark oak and other red oak groups. Sweetgum (*Liquidambar styraciflua*) is the only species in the sweetgum group. Tree species with commercial value that had too few observations for a separate group make up the other commercial species group. The most common species in the other commercial species group are swamp chestnut oak (*Quercus michauxii*), white oak (*Quercus alba*), overcup oak (*Quercus lyrata*), yellow-poplar (*Liriodendron tulipifera*), green ash (*Fraxinus pennsylvanica*), sugarberry (*Celtis laevigata*), elm (*Ulmus* spp.) and hickory (*Carya* spp.).

Regression Analysis

Regression analysis was used to identify the best predictor variables for estimating volume. Fit statistics and the correlation between predictor variables were calculated. Variables found to have low significance were eliminated from the model. Highly correlated variables were compared to determine which variable had the greater significance as a predictor of volume. Both linear and non-linear regression techniques were attempted, and models were evaluated using coefficient of determination (R^2) and index of fit (I^2), respectively. R^2 was calculated as one minus the quantity of the error sum of squares divided by the corrected total sum of squares.

$$R^2 = 1 - SSE / SSTC \quad (1)$$

where:

$0 \le R^2 \le 1$

R^2 = coefficient of determination

SSE = error sum of squares

SSTC = corrected total sum of squares

Index of Fit (I^2) was calculated for the nonlinear models as one minus the quantity of error sum of squares divided by the corrected total sum of squares.

$$I^2 = 1 - SSE / SSTC \quad (2)$$

where:

$-\infty \le I^2 \le 1$

I^2 = index of fit

SSE = error sum of squares

SSTC = corrected total sum of squares

Once initial predictor models were selected, probability least squares adjustments were utilized to account for over prediction and ensure the system made biological sense. Final system predictions were then tested against the observed data using fit statistics. Index of fit, root mean squared error (RMSE), and bias were used in determining the acceptance of final models. RMSE was calculated using the formula:

$$RMSE = \sqrt{\frac{\sum\left(V_i - \hat{V}_i\right)^2}{n-4}} \qquad (3)$$

where:

RMSE = square root of the mean squared error

V_i = observed volume

$\hat{V}_i$ = predicted volume

n = number of observations

Bias was calculated using equation 4. A negative bias value indicates an over-estimation from the observed. A positive bias value indicates under-estimation from the observed. Bias values nearer to zero signify a closer overall estimate to observe compared with bias values further from zero.

$$Bias = \frac{\sum\left(V_i - \hat{V}_i\right)}{n} \qquad (4)$$

where:

Bias = mean difference of observed minus predicted

V_i = observed volume

$\hat{V}_i$ = predicted volume

n = number of observations

Observed Data

Study plots included wide ranges of data for both stand level and individual tree variables (Table 2). These data ranges allow the resulting models to be applied to a broad scope of applications.

Table 2: Range of observed data recorded from all measurement periods.

Variable	Minimum	Lower Quartile	Mean	Upper Quartile	Maximum
Age (years)	15	44	57	68	92
Diameter at breast height (inches)	9.2	12.5	17.0	20.2	48.7
Merchantable height (feet)	9	25	36	47	83
Total Height (feet)	38	89	100	111	163
Basal Area (square feet)	44	124	141	158	245
Site index base age 50	67	99	105	111	133
Trees ≥3.5 in. dbh (per acre)	88	150	229	280	742

There are a total of 2,149 graded trees used for modeling, of which 614 are cherry bark oaks, 763 are other red oaks, 1377 are all red oaks, 590 are sweetgum, 61 are white oaks species, 48 are hickory species, and 73 are other commercial species. As the white oaks, hickories, and other commercial species did not have enough observations to create individual species grade prediction models, the three groups were compiled into a single group of 182 other commercial species. A complete observation break up of species groups by grade is shown in Table 3.

Table 3: Number of tree observations used for modeling by species group and grade category.

	Grade					
Species Group	**T[1]**	**1[1]**	**2[1]**	**3[1]**	**4[1]**	**5[1]**
Cherry Bark Oak	614	323	327	507	116	223
Other Red Oak	763	245	348	647	218	292
All Red Oak	1377	568	675	1154	334	515
Sweetgum	590	115	234	524	219	180
Other Commercial Spp.	182	92	92	185	185	47

1) Grade: T = total merchantable volume, 1-3 = USFS grades 1-3 volumes, 4 = tie and timber volume, 5 = cull volume

Modeling Approaches

Several different but similar approaches to developing a system for predicting grade volume of a tree were explored.

1. Fit separate weighted non-linear regression equations to predict the volume of grades 1-5 for a tree and summing the predicted grade volumes to obtain an estimate of the total volume. In this approach, grade volume zero must be included in the regressions which inflate the standard error of prediction for each regression. Grade volumes of zero occur as not all trees have portions of the merchantable volume containing all five grades. Also, because total tree volume is calculated by summing the predicted grade volume, the estimates while being unbiased will probably result in a very inflated standard error of prediction. However, it is possible that the standard error and prediction inflation of the total will be lessoned by the covariance between the grade volumes. The variance of the sum of volume is

$$Var\left(\sum_i \hat{V}_i\right) = \sum_i Var\left(\hat{V}_i\right) + 2\sum_i\sum_j Cov\left(\hat{V}_i, \hat{V}_j\right) \quad (5)$$

with negative covariance possibly somewhat abating the variance inflation due to using a sum to estimate the total.

2. Vary the procedure described in 1, by fitting a separate volume equation to independently estimate the total volume and use a weighted least square procedure to adjust the sum of the graded volumes to sum the total. The total volume estimate tends to have much lower relative standard error of prediction than any of the grade volume predictions. By using weighted least squares the error can be distributed so the grade volume with the least precision will receive most of the error by judicious choice of the weights.
3. Procedure 1 and 2 must include grade volumes of zero in the regression. These zero values appear as outliers and seriously inflate the standard error of prediction. Alternatively, the zero volumes can be excluded from the grade volume regressions and a binary logistic regression can be developed to predict the probability or proportion that a tree has a volume of a specific grade given the total merchantable volume of the tree. The grade volume estimates can be multiplied by the probability to obtain a true estimate of grade volume in a tree. These probability adjusted volumes are summed to obtain a total volume estimate.
4. Procedure 4 is like procedure 2 except zero observations were excluded during weighted non-linear regression modeling of merchantable sawlog and grade volume. Binary logistic regression is performed to find probability of $grade_i$. Probability and volume estimations for $grade_i$ are multiplied, followed by a weighted lease square procedure to adjust the sum of the graded volumes to equal

estimated merchantable volume. This procedure is described in more detail in the following section, Volume Prediction Equations.

5. Another viable method was proposed by Parresol (2001) for developing additive biomass equations. In this method, coefficients for the grade volume are estimated so the sum of errors for the grade and total volume are simultaneously minimized. A weighted version of this approach was investigated.

Volume Prediction Equations

A weighted non-linear equation (equation 6) was developed to predict total sawtimber merchantable volume and volume by grade category within a tree. SAS® 9.1 (2002) non-linear regression procedure PROC NLIN was used to construct prediction equations and obtain coefficients. Observations where $grade_i$ was zero or not observed were excluded. Coefficients were developed for two different sets of input variables (dbh, total height) and (dbh, merchantable height). A weight of $\left[\left(dbh_i^2 * h_{ik}\right)^2\right]^{-1}$ was applied to each tree to satisfy the homogeneous error variance requirements for making the parameters the best linear unbiased estimates. The weight (W_{ik}) is chosen so that the variance of $\varepsilon_{ijk} \big/ \sqrt{W_{ik}} = \sigma_{jk}^2$ is a constant for each regression.

$$V_{ij} = a_{jk} + b_{jk} dbh_i^{c_{jk}} H_{ik}^{d_{jk}} + \frac{\varepsilon_{ijk}}{\sqrt{W_{ik}}} \tag{6}$$

where:

V_{ij} = The jth volume category for the ith tree. Volume is merchantable sawtimber when j=0, is grade indexed volume for j=1, 2, 3, 4, and 5

dbh_i = Diameter at breast height

H_{ik} = Height of the ith tree. If k=1, the height is total tree height, and merchantable height when k=2

W_{ik} = Weight assigned to each tree chosen so $Var\left(\frac{\varepsilon_{ijk}}{\sqrt{\omega ik}}\right) = \sigma_{jk}^2$ a constant variance

The weighted non-linear equation applied to predicting merchantable sawtimber volume within a tree has a high precision with very little deviation from the actual. The sum of the predicted grade volumes following the application of equation 6 tends to have a large deviation from the actual merchantable sawtimber volume as observations of zero were removed. This large deviation is greatly reduced with an additional binary logistic regression procedure.

Binary logistic regression equations (equation 7) were constructed to predict the probability of finding a particular grade within a tree. Merchantable sawtimber volume, total height, merchantable height, and dbh were used as predictor variables to determine coefficients for the logistic function (equation 8). Binary refers to a number system which contains only 1 or 0. In this case 1 represented $grade_i$ was observed in $tree_i$ and 0 represented $grade_i$ was not observed in $tree_i$ during the modeling procedure.

$$P(V_{ij}) = \frac{1}{1 - e^{-S_{jk}}} \qquad (7)$$

where:

i = 1, 2,..., n ; j = 1, 2,…,5 ; k = 1, 2

S_{jk} = the logistic function

The equation for the logistic function is:

$$S_{jk} = a_{jk} + b_{jk} dbh_i + c_{jk} \left(\frac{V_i}{H_{ik}} \right) \qquad (8)$$

where:

$P(V_{ij})$ = The probability of i^th tree having grade volume j

a_{jk}, b_{jk}, and c_{jk} are parameters to the estimate for each j and k

i, j, and k are defined in equation 6

V_i = Total merchantable sawtimber volume in the i^th tree

H_{ik} = k^th height of the i^th tree

The probability calculated is then applied to equation 9 along with the weighted non-linear predicted volume for each grade_i to determine an estimated volume.

$$\hat{V}_{ij}^{est} = \left[\hat{P}(\hat{V}_{ij}) \right] \hat{V}_{ij} \qquad (9)$$

where:

$\hat{V}_{ij}^{est}$ = The estimated volume of the i^th tree having grade volume j

$P(V_{ij})$ = The probability of the i^th tree having grade volume j

$\hat{V}_{ij}$ = The weighted non-linear predicted volume of the i^th tree having grade volume j

Great improvements of the variation are found between the sum of the estimated grade$_i$ volumes from the actual merchantable sawtimber volume, but forcing the sum of the grade$_i$ volumes to equate to the best estimate of volume (merchantable sawtimber volume) is needed for consistent behavior in application and presentation. Weighted constrained least squares adjustment was implemented to accomplish the adjustments to the grade$_i$ volumes. This procedure is a flexible way to produce proportional adjustments depending on the choice of weight assigned to each grade$_i$ volume. For our volume prediction systems, the least squares base equation for generating weighted adjustments equations is:

$$Min(\hat{V}_i^{adj}, \lambda)\ SS = \sum_i W_i(\hat{V}_i^{adj} - \hat{V}_i^{est})^2 + \lambda(\sum_i \hat{V}_i^{adj} - \hat{V}_T) \qquad (10)$$

where:

$\hat{V}_i^{est}$ = predicted volume of grade$_i$

$\hat{V}_T$ = predicted merchantable sawtimber volume

$\hat{V}_i^{adj}$ = adjusted grade$_i$ volume

λ = Lograngian multiplier for imposing the constraint

W_i = weight assigned to grade volume$_i$

The results from the adjustments are the sum of the grade$_i$ volumes equates merchantable sawtimber volume.

$$\sum_i \hat{V}_i^{adj} = \hat{V}_T \qquad (11)$$

Solving equation 10 results in the adjustment equation:

$$\hat{V}_i^{adj} = \hat{V}_i^{est} - \frac{(\sum_j \hat{V}_i^{est} - \hat{V}_T) W_i^{-1}}{\sum_j W_j^{-1}} \tag{12}$$

where:

$\hat{V}_i^{adj}$ = adjusted grade$_i$ volume

$\hat{V}_i^{est}$ = predicted volume of grade$_i$

$\hat{V}_T$ = predicted merchantable sawtimber volume

W_i = weight assigned to grade volume$_i$

The weight, W_i, determines how much adjustment is applied to each $\hat{V}_i^{est}$. The weights assigned to each $\hat{V}_i^{est}$ were such that the lowest weight is allocated to the highest relative precision. That is, more error is applied to the grades with the worst estimates. As the relative variance of each grade volume tends to be inversely proportional to $\hat{V}_i^{est}$, W_i was replaced with $\hat{V}_i^{est}$ for application. The final adjustment weighted scheme adopted is:

$$\hat{V}_i^{adj} = \hat{V}_i - \frac{(\sum_j V_j^{est} - \hat{V}_T)\hat{V}_i^{est^{-1}}}{\sum_j \hat{V}_i^{est^{-1}}} \tag{13}$$

where:

$\hat{V}_i^{adj}$ = adjusted grade$_i$ volume

$\hat{V}_i^{est}$ = predicted volume of grade$_i$

$\hat{V}_T$ = predicted merchantable sawtimber volume

While having a volume of zero for a particular grade category is likely to be observed, negative observed volume is impossible, thus adjusting negative predicted volumes to zero is required. When a $\hat{V}_i^{adj}$ was < 0, that value was set to zero and equation 13 was re-calculated for the remaining $\hat{V}_i^{adj}$. The procedure was repeated until no $\hat{V}_i^{adj}$ were < 0, and $\sum_i \hat{V}_i^{adj} = \hat{V}_T$. The resulting $\hat{V}_i^{adj}$ is the final predicted volume for a particular grade$_i$ within a tree.

CHAPTER III

RESULTS

Parameter Estimates and Fit Statistics

Parameter estimates and fit statistics from equations developed for procedure method 4 for total merchantable sawtimber volume and volume by grade for tree species and volume type using dbh and total height as input variables are found in Table 4. The parameter estimates and fit statistics from equations developed for procedure method 4 for total merchantable sawtimber volume and volume by grade for tree species and volume type using dbh and merchantable height as input variables are found in Table 5.

The weighted non-linear regression estimates resulted in a range of acceptable to excellent fits. Among species groups, cherrybark oak gave the best fit statistics. Equations for merchantable sawlog cubic foot volume inside bark (Cvib) and merchantable sawlog cubic foot volume outside bark (Cvob) had the closest bias and RMSE to the actual among the five different volume types. Concordant percentage (percent of observed grade correctly identified by the model) was used to determine acceptability of the binary logistic regression models. Concordant percentages ranged from 45.4 to 94.8 for all models and all species groups (Tables 4 and 5). In general, grade 1 had the highest concordant percentage across all models, and grade 4 had the lowest concordant percentages.

Table 4: Parameter estimates and fit statistics for individual tree merchantable sawtimber and grade volume models using dbh and total height as predictor variables.

			Non-Linear Model (Equation 5)							Binary Logistic Regression (Equation 9)				
			Coefficients				No. Obs.	Fit Statistics		Coefficients			No. Obs.	Concordant
Species[1]	Grade[2]	Unit[3]	a	b	c	d	n	RMSE	Bias	a	b	c	n	%
CBO	T	Doyle	-37.25	0.00463	2.468	0.800	614	63.35	-0.21					
	1		-16.46	0.00064	3.067	0.693	323	112.10	-28.50	-11.06	0.72500	-0.901	614	91.2
	2		-68.28	1.62710	1.270	0.194	327	94.46	-38.42	-4.78	0.34694	-0.607	614	69.7
	3		-53.39	0.98390	1.114	0.368	507	88.65	-25.10	9.59	-0.50881	0.788	614	80.0
	4		-89.02	25.00000	0.469	0.078	116	55.51	-45.80	0.55	-0.13474	0.184	614	61.6
	5		5.49	1.13970	3.063	-1.296	223	32.76	-15.39	-2.90	0.06866	0.344	614	79.8
	T	Int 1/4	-66.91	0.07400	2.075	0.533	614	74.20	0.37					
	1		-3.14	0.00229	2.912	0.558	323	131.82	-42.97	-7.99	0.47440	-0.167	614	91.5
	2		-141.40	15.54720	0.870	0.070	327	113.06	-51.04	-3.26	0.23705	-0.318	614	69.0
	3		-164.90	25.00000	0.639	0.129	507	109.86	-32.13	9.67	-0.55984	0.867	614	80.1
	4		-109.00	25.00000	0.542	0.083	116	75.65	-59.70	2.29	-0.30406	0.555	614	65.0
	5		14.50	51.99680	3.501	-2.438	223	37.01	-18.65	-4.39	0.19113	0.017	614	79.5
	T	Scrib	-59.38	0.03820	2.073	0.653	614	69.41	-0.22					
	1		-5.18	0.00120	2.911	0.675	323	119.63	-37.85	-8.00	0.47490	-0.184	614	91.5
	2		-141.90	12.98950	0.831	0.123	327	102.21	-45.72	-3.45	0.25310	-0.386	614	69.0
	3		-177.30	28.19760	0.572	0.145	507	99.49	-28.54	10.06	-0.58700	1.002	614	80.5
	4		-112.20	25.00000	0.470	0.123	116	67.25	-53.36	2.07	-0.28170	0.549	614	64.4
	5		11.73	6.55650	3.078	-1.698	223	34.98	-18.27	-3.33	0.09400	0.274	614	79.6
	T	Cvob	-15.05	0.05760	1.802	0.400	614	12.19	0.19					
	1		1.34	0.00086	2.770	0.475	323	21.59	-8.09	-7.09	0.38507	0.443	614	91.6
	2		-40.12	12.30490	0.508	0.022	327	16.67	-5.52	-1.96	0.11938	-0.287	614	70.4
	3		-62.13	25.00000	0.336	0.053	507	18.09	-5.52	9.08	-0.55056	5.701	614	79.8
	4		-48.63	25.00000	0.250	0.038	116	13.55	-11.02	3.60	-0.45842	5.831	614	68.3
	5		3.90	25.00000	3.906	-2.953	223	7.38	-3.84	-4.80	0.23108	-0.504	614	79.5

Table 4: Continued

			Non-Linear Model (Equation 5)							Binary Logistic Regression (Equation 9)				
			Coefficients				No. Obs.	Fit Statistics		Coefficients			No. Obs.	Concordant
Species[1]	Grade[2]	Unit[3]	a	b	c	d	n	RMSE	Bias	a	b	c	n	%
CBO	T	Cvib	-11.87	0.03790	1.857	0.418	614	10.59	0.20					
	1		0.95	0.00070	2.783	0.478	323	18.67	-6.90	-7.25	0.40227	0.174	614	91.6
	2		-40.82	12.75620	0.499	0.023	327	15.70	-7.57	-2.20	0.14323	-0.749	614	70.1
	3		-116.50	74.11470	0.172	0.023	507	15.24	-4.34	9.12	-0.54703	6.424	614	79.8
	4		-45.56	25.00000	0.231	0.031	116	11.40	-9.25	3.27	-0.41985	6.036	614	67.5
	5		3.22	160.20000	3.951	-3.400	223	6.47	-3.24	-4.76	0.22663	-0.498	614	79.6
OR	T	Doyle	-42.40	0.00825	2.385	0.747	763	55.02	-0.97					
	1		-4.88	0.00123	2.960	0.610	245	116.68	-57.38	-5.16	0.20096	0.271	763	86.8
	2		-71.50	0.82230	1.317	0.331	348	89.18	-41.43	-2.55	0.12281	0.085	763	74.4
	3		-36.51	0.10960	1.525	0.594	647	94.69	-21.05	9.01	-0.57325	1.332	763	70.9
	4		-43.49	0.34880	1.202	0.493	218	78.42	-51.83	2.37	-0.25933	0.546	763	62.7
	5		-9.13	0.65930	1.770	-0.196	292	37.58	-17.31	-3.14	0.12870	0.144	763	75.0
	T	Int 1/4	-75.71	0.12080	2.003	0.484	763	65.95	-0.66					
	1		2.78	0.00637	2.815	0.389	245	137.90	-75.02	-4.26	0.09770	0.526	763	87.5
	2		-171.60	13.48310	0.836	0.151	348	105.55	-54.06	-1.12	-0.02299	0.463	763	76.5
	3		-113.20	6.50900	0.978	0.186	647	112.74	-25.46	7.66	-0.50423	1.078	763	69.9
	4		-178.20	25.00000	0.584	0.156	218	100.93	-71.13	4.42	-0.48069	1.032	763	67.3
	5		-30.88	25.00000	1.076	-0.435	292	44.58	-21.15	-3.63	0.17063	0.024	763	74.9
	T	Scrib	-67.29	0.06040	1.990	0.622	763	61.58	-0.82					
	1		4.63	0.00248	2.781	0.592	245	127.38	-69.56	-4.23	0.09712	0.561	763	87.5
	2		-138.50	6.70830	0.892	0.234	348	98.47	-50.18	-1.06	-0.02783	0.508	763	76.6
	3		-85.55	2.38300	1.056	0.319	647	105.81	-24.22	8.12	-0.54494	1.261	763	70.8
	4		-136.30	11.76430	0.634	0.244	218	93.21	-65.55	4.75	-0.51023	1.180	763	67.7
	5		-72.47	49.44280	0.655	-0.226	292	41.84	-19.97	-3.11	0.11937	0.167	763	74.9

Table 4: Continued

			Non-Linear Model (Equation 5)							Binary Logistic Regression (Equation 9)				
			Coefficients				No. Obs.	Fit Statistics		Coefficients			No. Obs.	Concordant
Species[1]	Grade[2]	Unit[3]	a	b	c	d	n	RMSE	Bias	a	b	c	n	%
OR	T	Cvob	-17.44	0.09930	1.689	0.364	763	11.03	0.00					
	1		2.34	0.00204	2.671	0.336	245	22.89	-13.53	-4.38	0.07752	3.893	763	87.6
	2		-72.22	22.84210	0.386	0.069	348	17.45	-9.65	-0.88	-0.07535	3.988	763	76.6
	3		-52.72	15.70780	0.475	0.053	647	18.39	-4.17	6.55	-0.44100	6.151	763	68.8
	4		-66.37	25.00000	0.308	0.072	218	17.65	-12.85	5.19	-0.61139	8.762	763	71.7
	5		-16.64	25.00000	0.516	-0.321	292	8.21	-4.18	-3.58	0.16321	0.286	763	74.9
	T	Cvib	-13.82	0.06500	1.755	0.375	763	9.38	-0.05					
	1		1.68	0.00170	2.697	0.328	245	19.67	-11.48	-4.34	0.08031	4.391	763	87.6
	2		-50.91	13.14210	0.458	0.078	348	14.95	-8.17	-0.90	-0.06668	4.395	763	76.6
	3		-35.05	8.02510	0.573	0.067	647	15.75	-3.57	6.72	-0.45001	7.203	763	68.9
	4		-61.57	25.00000	0.288	0.063	218	14.84	-10.77	5.06	-0.58608	9.621	763	70.8
	5		-18.36	25.00000	0.442	-0.271	292	7.03	-3.51	-3.59	0.16503	0.293	763	74.9
ARO	T	Doyle	-40.08	0.00753	2.424	0.734	1378	55.16	-0.59					
	1		-5.26	0.00061	3.131	0.649	568	176.99	-80.93	-7.68	0.43136	-0.271	1378	88.2
	2		-80.47	1.80340	1.218	0.228	675	104.78	-43.77	-4.07	0.27351	-0.388	1378	71.3
	3		-47.85	0.50200	1.312	0.401	1155	144.92	17.11	8.30	-0.46215	0.833	1378	73.7
	4		-79.91	7.08800	0.782	0.153	334	79.13	-46.27	1.51	-0.19449	0.343	1378	62.4
	5		-7.64	1.17490	1.782	-0.350	515	45.07	-19.37	-2.80	0.08384	0.274	1378	76.8
	T	Int 1/4	-69.56	0.10090	2.054	0.484	1378	68.94	-0.17					
	1		6.74	0.00244	2.978	0.487	568	203.76	-100.73	-5.58	0.24165	0.243	1378	88.8
	2		-175.10	18.71670	0.801	0.102	675	125.31	-58.94	-2.17	0.10927	0.043	1378	73.1
	3		-156.70	19.65900	0.755	0.112	1155	172.12	17.57	7.97	-0.47342	0.818	1378	74.0
	4		-143.50	25.00000	0.618	0.096	334	103.72	-70.10	3.40	-0.38755	0.762	1378	66.5
	5		-18.76	25.00000	1.217	-0.574	515	52.07	-22.37	-3.79	0.16713	0.042	1378	76.6

Table 4: Continued

			Non-Linear Model (Equation 5)							Binary Logistic Regression (Equation 9)				
			Coefficients				No. Obs.	Fit Statistics		Coefficients			No. Obs.	Concordant
Species[1]	Grade[2]	Unit[3]	a	b	c	d	n	RMSE	Bias	a	b	C	n	%
ARO	T	Scrib	-63.20	0.05610	2.036	0.602	1378	63.09	-0.32					
	1		6.03	0.00114	2.967	0.635	568	186.79	-91.52	-5.86	0.26956	0.183	1378	88.6
	2		-158.90	13.14560	0.805	0.155	675	115.17	-53.90	-2.24	0.11658	0.027	1378	73.0
	3		-130.20	11.01700	0.795	0.183	1155	158.62	15.79	8.44	-0.51261	0.982	1378	74.6
	4		-146.10	25.00000	0.550	0.133	334	94.96	-63.20	3.52	-0.39650	0.847	1378	66.7
	5		-29.68	21.43280	0.998	-0.370	515	49.31	-21.66	-3.00	0.09209	0.248	1378	76.7
	T	Cvob	-15.29	0.07480	1.779	0.361	1378	11.69	0.08					
	1		3.04	0.00083	2.840	0.421	568	32.69	-17.24	-5.21	0.18731	2.541	1378	88.9
	2		-75.24	28.55450	0.363	0.040	675	20.67	-10.34	-1.31	0.01462	1.850	1378	74.6
	3		-96.00	49.95690	0.272	0.021	1155	28.15	2.60	7.23	-0.44149	4.997	1378	73.7
	4		-57.30	25.00000	0.313	0.039	334	17.70	-12.79	4.56	-0.53996	7.226	1378	70.3
	5		-13.41	25.00000	0.553	-0.379	515	9.43	-4.39	-3.93	0.18036	0.058	1378	76.6
	T	Cvib	-12.05	0.04790	1.840	0.380	1378	10.02	-0.02					
	1		2.30	0.00069	2.853	0.422	568	28.27	-14.79	-5.20	0.18937	2.871	1378	88.9
	2		-52.26	16.39380	0.434	0.048	675	17.78	-8.77	-1.45	0.03237	1.780	1378	74.4
	3		-53.55	20.85270	0.385	0.034	1155	24.24	2.26	7.30	-0.44180	5.696	1378	73.6
	4		-53.57	25.00000	0.289	0.034	334	14.94	-10.68	4.23	-0.49797	7.564	1378	69.2
	5		-15.68	25.00000	0.458	-0.308	515	8.05	-3.72	-3.95	0.18218	0.033	1378	76.6
SG	T	Doyle	-32.18	0.00254	2.719	0.791	590	32.79	-0.04					
	1		-13.96	0.00236	2.916	0.509	115	46.55	-25.76	-8.25	0.36480	0.837	590	94.5
	2		-8.33	0.00515	2.174	0.770	234	42.36	-22.36	-7.73	0.57930	-0.775	590	81.6
	3		-13.83	0.01410	1.946	0.696	524	37.73	-6.75	8.58	-0.56880	1.421	590	66.1
	4		-3.04	0.00010	1.988	1.603	219	33.87	-17.17	-3.32	0.22067	-0.288	590	68.1
	5		-8.49	0.19040	2.081	-0.100	180	25.82	-15.05	-2.07	0.09158	-0.043	590	57.4

Table 4: Continued

			Non-Linear Model (Equation 5)							Binary Logistic Regression (Equation 9)				
			Coefficients				No. Obs.	Fit Statistics		Coefficients			No. Obs.	Concordant
Species[1]	Grade[2]	Unit[3]	a	b	c	d	n	RMSE	Bias	a	b	c	n	%
SG	T	Int 1/4	-65.85	0.07150	2.271	0.438	590	43.82	-0.02					
	1		-30.69	0.04820	2.441	0.237	115	62.04	-37.58	-7.29	0.23650	1.034	590	94.4
	2		-16.16	0.10830	1.699	0.484	234	58.54	-35.18	-3.34	0.14520	0.528	590	83.9
	3		-36.43	0.75830	1.423	0.265	524	50.83	-9.89	7.59	-0.51030	1.045	590	61.7
	4		-7.00	0.00320	1.508	1.261	219	50.18	-29.15	3.25	-0.45030	1.502	590	77.4
	5		-2.77	0.83210	2.201	-0.479	180	33.09	-20.40	-3.16	0.20418	-0.322	590	57.8
	T	Scrib	-56.81	0.02930	2.278	0.604	590	40.51	-0.56					
	1		-18.31	0.01270	2.563	0.410	115	56.40	-34.84	-7.24	0.23470	1.122	590	94.4
	2		-12.14	0.03670	1.769	0.650	234	53.24	-31.51	-3.19	0.13190	0.612	590	84.0
	3		-29.23	0.21800	1.457	0.488	524	46.77	-9.21	8.37	-0.58420	1.331	590	63.8
	4		-5.11	0.00123	1.502	1.445	219	45.46	-26.40	2.28	-0.34500	1.328	590	77.8
	5		-5.59	0.48360	1.981	-0.221	180	30.77	-19.46	-2.32	0.11754	-0.110	590	57.3
	T	Cvob	-15.78	0.06790	1.939	0.295	590	7.83	1.23					
	1		-5.88	0.02580	2.153	0.183	115	11.25	-7.49	-7.86	0.25520	6.125	590	94.3
	2		-4.38	0.11260	1.292	0.370	234	10.80	-6.99	-2.16	-0.00481	5.649	590	84.1
	3		-12.71	1.34090	0.943	0.114	524	9.02	-1.78	6.34	-0.41010	4.933	590	59.7
	4		-1.91	0.00388	1.189	1.071	219	9.53	-5.98	7.66	-0.99190	17.577	590	79.0
	5		0.92	0.32400	2.328	-0.752	180	6.39	-4.18	-2.76	0.16965	-1.448	590	57.4
	T	Cvib	-12.60	0.04740	2.000	0.301	590	6.52	-0.01					
	1		-4.91	0.01930	2.203	0.181	115	9.63	-6.33	-7.77	0.25310	7.119	590	94.3
	2		-3.73	0.09020	1.319	0.366	234	9.17	-5.89	-2.34	0.02034	6.094	590	84.0
	3		-9.66	0.89120	0.997	0.123	524	7.59	-1.49	6.52	-0.42520	5.975	590	60.2
	4		-1.18	0.00194	1.263	1.126	219	7.88	-4.90	7.03	-0.90890	18.948	590	78.2
	5		0.57	0.28110	2.322	-0.740	180	5.54	-3.58	-2.77	0.16972	-1.680	590	57.3

Table 4: Continued

			Non-Linear Model (Equation 5)							Binary Logistic Regression (Equation 9)				
			Coefficients				No. Obs.	Fit Statistics		Coefficients			No. Obs.	Concordant
Species[1]	Grade[2]	Unit[3]	a	b	c	d	n	RMSE	Bias	a	b	c	n	%
OC	T	Doyle	-33.27	0.00637	2.430	0.751	182	76.37	-2.92					
	1		-37.44	0.09350	2.087	0.283	92	122.13	-61.86	-6.68	0.33910	-0.449	182	85.0
	2		-37.44	0.09350	2.087	0.283	92	114.99	-43.12	-3.57	0.19600	-0.156	182	75.8
	3		-22.38	0.16040	2.224	0.053	185	92.19	-14.16	11.23	-0.74550	1.974	182	77.9
	4		-22.38	0.16040	2.224	0.053	185	136.03	-85.60	-1.58	-0.04260	0.128	182	45.4
	5		0.52	0.00763	2.743	0.076	47	34.99	-15.14	-6.02	0.34330	-0.439	182	76.6
	T	Int 1/4	-49.02	0.05110	2.152	0.542	182	86.14	-3.70					
	1		-48.35	0.35420	1.821	0.218	92	140.05	-81.80	-6.11	0.30220	-0.320	182	85.6
	2		-48.35	0.35420	1.821	0.218	92	131.27	-56.89	-2.74	0.11821	0.070	182	77.1
	3		-33.40	1.19060	1.911	-0.112	185	111.57	-16.31	9.61	-0.66590	1.640	182	77.1
	4		-33.40	1.19060	1.911	-0.112	185	164.46	-115.12	-1.19	-0.08500	0.226	182	50.8
	5		2.48	0.00864	2.531	0.206	47	42.93	-18.09	-6.70	0.42180	-0.588	182	77.4
	T	Scrib	-45.55	0.02800	2.113	0.680	182	79.74	-2.97					
	1		-49.25	0.29360	1.738	0.300	92	127.75	-74.51	-5.59	0.25200	-0.207	182	85.2
	2		-49.25	0.29360	1.738	0.300	92	119.27	-51.81	-2.32	0.07690	0.198	182	77.9
	3		-32.68	0.62710	1.820	0.068	185	101.15	-14.26	9.83	-0.68180	1.800	182	76.7
	4		-32.68	0.62710	1.820	0.068	185	148.94	-103.98	-0.73	-0.13040	0.372	182	54.7
	5		-1.67	0.03740	2.150	0.156	47	38.45	-17.81	-5.87	0.34310	-0.426	182	76.7
	T	Cvob	-9.69	0.02670	1.904	0.473	182	13.54	-0.65					
	1		-8.67	0.13420	1.565	0.218	92	22.90	-14.95	-5.63	0.26579	-1.487	182	85.7
	2		-8.67	0.13420	1.565	0.218	92	21.32	-10.54	-2.29	0.06452	1.404	182	78.2
	3		-6.86	0.58540	1.611	-0.133	185	18.54	-2.79	8.05	-0.56770	8.964	182	73.9
	4		-6.86	0.58540	1.611	-0.133	185	27.53	-20.85	-0.74	-0.14400	2.416	182	62.1
	5		0.43	0.00444	1.980	0.355	47	7.97	-3.73	-6.35	0.40910	-3.584	182	77.7

Table 4: Continued

			Non-Linear Model (Equation 5)							Binary Logistic Regression (Equation 9)				
			Coefficients				No. Obs.	Fit Statistics		Coefficients			No. Obs.	Concordant
Species[1]	Grade[2]	Unit[3]	a	b	c	d	n	RMSE	Bias	a	b	c	n	%
OC	T	Cvib	-7.67	0.01820	1.965	0.475	182	11.71	-0.40					
	1		-7.01	0.09440	1.634	0.210	92	19.51	-12.47	-5.75	0.27640	-1.932	182	85.8
	2		-7.01	0.09440	1.634	0.210	92	18.22	-8.77	-2.39	0.07771	1.361	182	77.9
	3		-5.03	0.39610	1.711	-0.157	185	15.75	-2.36	8.31	-0.58560	10.842	182	74.4
	4		-5.03	0.39610	1.711	-0.157	185	23.30	-17.39	-0.92	-0.12170	2.393	182	59.9
	5		0.20	0.00483	1.987	0.299	47	6.71	-3.10	-6.42	0.41200	-4.219	182	77.7

1) Species: CBO = cherrybark oak, OR = other red oak, ARO = all red oak, SG = sweetgum, OC = other commercial
2) Grades: T = total merchantable volume, 1-3 = USFS grades 1-3 volumes, 4 = tie and timber volume, 5 = cull volume
3) Units: Int 1/4 = International 1/4, Scrib = Scribner, Cvob = cubic foot volume outside bark, Cvib = cubic foot volume inside bark

Table 5: Parameter estimates and fit statistics for individual tree merchantable sawtimber and grade volume models using dbh and merchantable height as predictor variables.

			Non-Linear Model (Equation 6)							Binary Logistic Regression (Equation 10)				
			Coefficients				No. Obs.	Fit Statistics		Coefficients			No. Obs.	Concordant
Species[1]	Grade[2]	Unit[3]	a	b	c	d	n	RMSE	Bias	a	b	c	n	%
CBO	T	Doyle	-8.50	0.00759	2.724	0.631	614	68.79	-1.09					
	1		-7.22	0.00306	3.281	0.263	323	115.63	-29.28	-11.63	0.77179	-0.403	614	91.8
	2		-68.23	3.00980	1.278	0.070	327	92.97	-36.95	-6.21	0.46942	-0.366	614	74.1
	3		-27.88	0.93540	1.446	0.161	507	91.07	-27.33	4.29	-0.12268	-0.019	614	76.6
	4		-37.24	4.33020	1.154	-0.109	116	58.59	-43.95	-4.82	0.35618	-0.535	614	67.2
	5		-14.28	0.96410	2.030	-0.582	223	35.05	-16.24	-2.81	0.05685	0.157	614	80.3
	T	Int 1/4	-8.17	0.03200	2.268	0.690	614	78.09	-0.78					
	1		6.35	0.00697	3.077	0.255	323	134.14	-42.91	-12.15	0.87584	-0.479	614	91.9
	2		-145.70	20.29120	0.830	0.051	327	110.89	-48.28	-6.62	0.55962	-0.432	614	74.6
	3		-65.70	8.02830	0.845	0.192	507	107.87	-30.55	3.67	-0.06504	-0.069	614	77.2
	4		-125.60	43.89790	0.539	-0.021	116	75.71	-59.78	-5.61	0.51550	-0.678	614	69.7
	5		1.75	0.40520	2.776	-1.053	223	39.45	-19.23	-3.76	0.12956	0.070	614	79.7
	T	Scrib	-8.10	0.02630	2.301	0.690	614	75.40	-1.36					
	1		4.70	0.00523	3.116	0.270	323	122.32	-37.51	-11.80	0.83917	-0.483	614	91.9
	2		-134.00	17.09840	0.852	0.055	327	100.67	-43.77	-6.28	0.52307	-0.431	614	74.1
	3		-51.42	4.80560	0.972	0.193	507	100.68	-30.17	4.01	-0.09617	-0.045	614	76.8
	4		-329.00	208.60000	0.227	-0.008	116	66.42	-53.15	-5.19	0.46096	-0.666	614	68.5
	5		-11.61	1.45450	1.899	-0.573	223	37.66	-20.58	-2.66	0.02208	0.198	614	80.5
	T	Cvob	-1.27	0.01340	1.983	0.688	614	12.80	-0.13					
	1		2.61	0.00209	2.898	0.242	323	21.84	-8.07	-11.97	0.92670	-3.517	614	92.0
	2		-59.17	25.37850	0.366	0.018	327	16.43	-5.04	-6.27	0.58757	-3.069	614	74.8
	3		-24.81	7.81540	0.450	0.133	507	17.32	-4.74	3.18	-0.00475	-0.837	614	78.1
	4		-75.79	53.68060	0.173	-0.001	116	13.03	-10.56	-5.18	0.57360	-4.949	614	71.3
	5		1.79	0.05180	3.162	-1.326	223	7.79	-3.90	-4.04	0.14970	0.329	614	79.6

Table 5: Continued

			Non-Linear Model (Equation 6)							Binary Logistic Regression (Equation 10)				
			Coefficients				No. Obs.	Fit Statistics		Coefficients			No. Obs.	Concordant
Species[1]	Grade[2]	Unit[3]	a	b	c	d	n	RMSE	Bias	a	b	c	n	%
CBO	T	Cvib	-0.89	0.00994	2.029	0.688	614	11.06	-0.02					
	1		2.05	0.00175	2.911	0.241	323	18.92	-6.93	-11.93	0.91185	-3.904	614	92.0
	2		-60.23	26.22580	0.359	0.018	327	15.37	-7.09	-6.29	0.57930	-3.440	614	74.8
	3		-18.58	5.35540	0.488	0.144	507	14.90	-4.00	3.28	-0.01797	-0.860	614	78.0
	4		-28.37	14.47410	0.350	-0.006	116	11.23	-9.06	-5.15	0.55530	-5.505	614	71.1
	5		1.49	0.04370	3.174	-1.327	223	6.77	-3.33	-4.01	0.14818	0.387	614	79.6
OR	T	Doyle	-9.13	0.00801	2.667	0.685	763	30.12	-0.74					
	1		-11.77	0.00853	3.055	0.181	245	115.31	-54.14	-13.38	0.88142	-0.578	763	87.1
	2		-65.76	2.06930	1.501	0.011	348	89.74	-41.56	-9.19	0.70671	-0.579	763	77.2
	3		-27.76	0.45530	1.734	0.185	647	94.40	-22.12	1.85	0.03757	-0.118	763	64.3
	4		-20.95	0.29750	1.896	0.057	218	86.18	-55.38	-6.71	0.59601	-0.818	763	71.4
	5		-8.69	1.15650	2.721	-1.241	292	38.13	-12.10	-3.93	0.19839	-0.020	763	74.8
	T	Int 1/4	-9.42	0.03300	2.218	0.735	763	20.15	-0.85					
	1		-1.89	0.01630	2.883	0.187	245	138.20	-75.63	-13.54	0.98030	-0.651	763	87.3
	2		-164.30	20.24410	0.920	0.008	348	105.73	-53.88	-9.53	0.82244	-0.667	763	77.5
	3		-50.35	2.60040	1.213	0.226	647	111.19	-26.83	0.85	0.15473	-0.227	763	65.6
	4		-20.72	0.50310	1.547	0.263	218	107.03	-72.29	-7.99	0.85160	-1.058	763	73.7
	5		0.73	5.49330	2.917	-1.927	292	49.36	20.47	-4.71	0.28054	-0.103	763	74.7
	T	Scrib	-8.99	0.02840	2.241	0.737	763	25.77	-0.96					
	1		-2.50	0.01500	2.877	0.193	245	125.43	-65.74	-13.18	0.94207	-0.655	763	87.3
	2		-131.10	13.08000	1.008	0.014	348	98.55	-49.88	-9.19	0.78295	-0.669	763	77.3
	3		-45.76	2.12910	1.266	0.216	647	104.57	-25.32	1.41	0.09673	-0.182	763	64.5
	4		-25.30	0.75030	1.502	0.186	218	99.51	-67.50	-6.80	0.70584	-0.939	763	72.3
	5		-5.98	3.46610	2.578	-1.430	292	42.18	-13.96	-3.72	0.17933	0.001	763	74.8

Table 5: Continued

			Non-Linear Model (Equation 6)							Binary Logistic Regression (Equation 10)				
			Coefficients				No. Obs.	Fit Statistics		Coefficients			No. Obs.	Concordant
Species[1]	Grade[2]	Unit[3]	a	b	c	d	n	RMSE	Bias	a	b	c	n	%
OR	T	Cvob	-1.41	0.01540	1.902	0.724	763	2.32	-0.12					
	1		1.78	0.00385	2.741	0.198	245	22.59	-13.06	-12.34	0.96820	-4.402	763	87.4
	2		-86.63	38.65070	0.360	0.003	348	17.43	-9.56	-8.57	0.83343	-4.642	763	77.6
	3		-13.98	1.86930	0.819	0.192	647	18.08	-4.39	0.03	0.29805	-2.514	763	66.7
	4		-0.81	0.04920	1.450	0.462	218	18.12	-12.60	-7.14	0.94810	-7.919	763	75.2
	5		3.31	735.80000	3.368	-4.786	292	13.35	-2.62	-4.17	0.23474	-0.385	763	74.7
	T	Cvib	-1.18	0.01110	1.962	0.720	763	2.01	0.00					
	1		1.19	0.00315	2.765	0.194	245	19.41	-11.05	-12.49	0.96150	-4.953	763	87.4
	2		-57.99	21.30030	0.446	0.003	348	14.95	-8.12	-8.70	0.82454	-5.214	763	77.6
	3		-10.63	1.22330	0.886	0.200	647	15.48	-3.75	0.07	0.28011	-2.722	763	66.7
	4		-0.90	0.04240	1.484	0.433	218	15.43	-10.68	-7.29	0.92700	-8.863	763	75.0
	5		2.75	479.80000	3.423	-4.747	292	11.11	-2.08	-4.22	0.23824	-0.466	763	74.7
ARO	T	Doyle	-9.52	0.00870	2.672	0.649	1378	37.56	-1.36					
	1		-4.98	0.00337	3.265	0.246	568	174.32	-79.44	-12.11	0.79238	-0.467	1378	89.4
	2		-83.89	4.31890	1.263	0.020	675	104.70	-43.16	-7.53	0.57104	-0.455	1378	76.0
	3		-30.92	0.75590	1.584	0.150	1155	143.57	14.56	3.02	-0.04353	-0.066	1378	70.3
	4		-32.03	1.23790	1.504	-0.011	334	86.96	-52.53	-4.82	0.39303	-0.577	1378	67.8
	5		-11.12	0.95080	2.583	-1.052	515	50.26	-17.12	-3.00	0.09845	0.095	1378	77.1
	T	Int 1/4	-9.22	0.03380	2.233	0.711	1378	25.27	-1.09					
	1		7.78	0.00724	3.073	0.241	568	201.06	-99.27	-12.43	0.88856	-0.537	1378	89.4
	2		-195.40	33.00330	0.770	0.017	675	124.76	-57.64	-7.93	0.67394	-0.532	1378	76.4
	3		-60.26	4.71460	1.045	0.194	1155	170.33	15.78	2.24	0.03966	-0.141	1378	71.7
	4		-44.23	2.93510	1.168	0.114	334	106.89	-71.14	-5.89	0.59116	-0.765	1378	70.7
	5		-1.23	1.42850	2.940	-1.507	515	59.05	-19.31	-3.83	0.17013	0.014	1378	76.7

Table 5: Continued

			Non-Linear Model (Equation 6)							Binary Logistic Regression (Equation 10)				
			Coefficients				No. Obs.	Fit Statistics		Coefficients			No. Obs.	Concordant
Species[1]	Grade[2]	Unit[3]	a	b	c	d	n	RMSE	Bias	a	b	c	n	%
ARO	T	Scrib	-9.36	0.02970	2.254	0.706	1378	32.41	-0.79					
	1		6.32	0.00585	3.095	0.254	568	184.03	-89.89	-12.25	0.86690	-0.554	1378	89.5
	2		-169.30	25.43420	0.814	0.020	675	114.73	-52.75	-7.54	0.63168	-0.528	1378	76.1
	3		-53.86	3.67370	1.114	0.182	1155	156.76	13.50	2.73	-0.00739	-0.105	1378	70.8
	4		-56.80	5.39480	1.035	0.065	334	98.40	-65.05	-4.85	0.46584	-0.661	1378	68.5
	5		-6.88	1.56960	2.584	-1.191	515	56.00	-20.08	-2.80	0.06601	0.133	1378	77.3
	T	Cvob	-1.40	0.01480	1.935	0.705	1378	3.39	-0.08					
	1		3.12	0.00195	2.913	0.237	568	32.25	-16.95	-11.62	0.89018	-3.648	1378	89.5
	2		-133.90	79.26270	0.231	0.004	675	20.55	-10.01	-7.32	0.69321	-3.719	1378	76.6
	3		-19.37	3.99850	0.636	0.152	1155	27.91	2.44	1.57	0.14002	-1.605	1378	74.0
	4		-5.30	0.40980	1.098	0.206	334	18.08	-12.71	-5.35	0.66106	-5.673	1378	72.1
	5		0.71	0.38370	2.907	-1.599	515	10.86	-4.18	-3.78	0.16199	0.152	1378	76.7
	T	Cvib	-1.05	0.01060	1.990	0.706	1378	2.88	0.06					
	1		2.40	0.00164	2.927	0.233	568	27.88	-14.53	-11.61	0.87370	-4.037	1378	89.4
	2		-82.64	40.84800	0.311	0.006	675	17.71	-8.56	-7.39	0.68548	-4.177	1378	76.5
	3		-14.03	2.47930	0.705	0.165	1155	24.07	2.18	1.64	0.12480	-1.714	1378	73.7
	4		-3.96	0.27380	1.163	0.210	334	15.45	-10.76	-5.60	0.67002	-6.580	1378	72.4
	5		0.56	0.30530	2.935	-1.603	515	9.30	-3.53	-3.82	0.16755	0.129	1378	76.7
SG	T	Doyle	-4.36	0.00324	2.919	0.716	590	16.29	-0.27					
	1		-82.97	0.52580	2.065	-0.022	115	46.88	-12.75	-19.79	1.41750	-1.035	590	94.6
	2		-23.86	0.18740	2.110	0.092	234	42.47	-20.44	-11.20	0.94705	-0.899	590	83.8
	3		-15.58	0.27240	1.783	0.199	524	36.29	-6.81	3.28	-0.07591	-0.033	590	61.3
	4		-10.71	0.15030	2.376	-0.221	219	35.32	-17.26	-17.12	1.67320	-2.448	590	84.1
	5		-8.30	0.06740	3.044	-0.591	180	27.47	-14.97	-0.12	-0.10894	0.275	590	60.8

Table 5: Continued

			Non-Linear Model (Equation 6)							Binary Logistic Regression (Equation 10)				
			Coefficients				No. Obs.	Fit Statistics		Coefficients			No. Obs.	Concordant
Species[1]	Grade[2]	Unit[3]	a	b	c	d	n	RMSE	Bias	a	b	c	n	%
SG	T	Int 1/4	-6.02	0.02300	2.333	0.736	590	12.67	-0.35					
	1		-130.00	2.33260	1.676	-0.023	115	60.49	-23.56	-19.87	1.55720	-1.102	590	94.7
	2		-39.79	1.10830	1.606	0.100	234	57.47	-32.21	-11.64	1.11320	-1.016	590	84.4
	3		-86.99	16.80130	0.661	0.150	524	48.49	-7.64	2.33	0.03800	-0.165	590	62.0
	4		-12.53	0.48770	1.915	-0.080	219	67.58	-52.65	-18.63	2.17250	-2.848	590	85.8
	5		-10.05	0.19930	2.853	-0.667	180	36.17	-21.46	-0.81	-0.05742	0.175	590	58.1
	T	Scrib	-4.51	0.01630	2.388	0.759	590	15.67	-0.02					
	1		-127.10	2.46030	1.644	-0.029	115	55.15	-19.45	-19.04	1.46280	-1.073	590	94.5
	2		-33.80	0.76210	1.684	0.111	234	52.33	-28.71	-10.97	1.02020	-0.961	590	83.9
	3		-46.02	4.54240	0.966	0.191	524	44.68	-7.90	3.24	-0.06690	-0.043	590	61.2
	4		-14.43	0.57050	1.869	-0.101	219	45.74	-25.11	-16.34	1.83050	-2.502	590	84.4
	5		-8.97	0.16810	2.771	-0.573	180	33.29	-20.22	0.17	-0.17314	0.353	590	61.0
	T	Cvob	-0.85	0.01210	1.951	0.746	590	1.76	-0.10					
	1		-26.59	0.99570	1.391	-0.018	115	10.71	-5.18	-18.62	1.58810	-7.577	590	94.7
	2		-9.45	0.69340	1.207	0.091	234	10.46	-6.42	-10.54	1.15110	-7.087	590	84.5
	3		-46.41	25.00000	0.238	0.078	524	8.77	-1.45	1.73	0.15390	-2.019	590	60.6
	4		-1.33	0.12620	1.697	-0.012	219	9.44	-5.52	-15.50	2.24650	-19.401	590	86.5
	5		-1.22	0.05340	2.742	-0.684	180	7.05	-4.52	-0.79	-0.09660	1.501	590	57.6
	T	Cvib	-0.80	0.00906	2.020	0.725	590	1.52	0.00					
	1		-21.19	0.66040	1.466	-0.017	115	9.22	-4.45	-18.68	1.55830	-8.375	590	94.8
	2		-7.86	0.52820	1.251	0.086	234	8.92	-5.44	-10.60	1.12570	-7.870	590	84.5
	3		-310.50	282.20000	0.037	0.011	524	7.38	-1.03	1.70	0.14900	-2.261	590	60.6
	4		-1.37	0.11490	1.715	-0.046	219	7.85	-4.56	-15.66	2.18300	-21.665	590	86.5
	5		-1.20	0.04400	2.759	-0.680	180	6.10	-3.87	-0.78	-0.90550	1.659	590	57.6

Table 5: Continued

			Non-Linear Model (Equation 6)							Binary Logistic Regression (Equation 10)				
			Coefficients				No. Obs.	Fit Statistics		Coefficients			No. Obs.	Concordant
Species[1]	Grade[2]	Unit[3]	a	b	c	d	n	RMSE	Bias	a	b	c	n	%
OC	T	Doyle	-9.01	0.00860	2.609	0.700	182	270.87	-1.70					
	1		-13.82	0.04120	2.418	0.266	92	149.78	-62.95	-7.80	0.43620	-0.261	182	84.3
	2		-13.82	0.04120	2.418	0.266	92	118.33	-44.21	-6.42	0.46030	-0.344	182	76.6
	3		-14.63	0.10940	2.124	0.234	185	131.69	-10.07	1.91	0.07440	-0.159	182	65.0
	4		-14.63	0.10940	2.124	0.234	185	138.95	-81.51	-2.73	0.06510	-0.072	182	52.6
	5		-7.19	0.06570	2.411	-0.156	47	63.58	-15.86	-0.84	-0.14240	0.401	182	82.1
	T	Int 1/4	-8.43	0.03190	2.176	0.770	182	297.12	-1.79					
	1		-8.45	0.08890	2.204	0.280	92	170.13	-82.96	-8.92	0.58300	-0.390	182	84.2
	2		-8.45	0.08890	2.204	0.280	92	137.42	-58.05	-7.24	0.59390	-0.460	182	77.3
	3		-23.72	0.62860	1.563	0.297	185	146.83	-8.59	1.70	0.11810	-0.194	182	66.1
	4		-23.72	0.62860	1.563	0.297	185	158.14	-107.40	-3.59	0.17030	-0.181	182	58.7
	5		-8.35	0.13490	2.218	-0.154	47	71.08	-19.37	-1.86	-0.07910	0.296	182	79.1
	T	Scrib	-8.36	0.02780	2.211	0.758	182	278.50	-2.37					
	1		-8.63	0.08210	2.196	0.289	92	157.71	-76.30	-6.87	0.38180	-0.213	182	84.4
	2		-8.63	0.08210	2.196	0.289	92	127.34	-53.59	-5.99	0.46110	-0.351	182	76.4
	3		-21.10	0.49740	1.631	0.281	185	136.20	-8.47	1.32	0.15620	-0.247	182	62.8
	4		-21.10	0.49740	1.631	0.281	185	147.21	-98.19	-2.89	0.09200	-0.103	182	55.6
	5		-8.92	0.17420	2.107	-0.135	47	63.10	-19.25	-0.08	-0.27980	0.565	182	82.4
	T	Cvob	-0.91	0.01330	1.871	0.785	182	46.82	-0.14					
	1		0.41	0.02210	2.050	0.292	92	27.75	-15.21	-7.96	0.54740	-2.413	182	84.2
	2		0.41	0.02210	2.050	0.292	92	22.77	-10.80	-6.51	0.59320	-3.082	182	77.7
	3		-6.13	0.45600	1.125	0.281	185	23.02	-1.06	0.92	0.25280	-2.170	182	65.7
	4		-6.13	0.45600	1.125	0.281	185	25.49	-19.12	-3.84	0.24230	-1.738	182	62.1
	5		-1.00	0.03640	2.049	-0.122	47	12.14	-4.09	-2.18	-0.09550	2.091	182	78.8

Table 5: Continued

			Non-Linear Model (Equation 6)							Binary Logistic Regression (Equation 10)				
			Coefficients				No. Obs.	Fit Statistics		Coefficients			No. Obs.	Concordant
Species[1]	Grade[2]	Unit[3]	a	b	c	d	n	RMSE	Bias	a	b	c	n	%
OC	T	Cvib	-0.75	0.00936	1.931	0.783	182	39.79	-0.17					
	1		0.13	0.01700	2.089	0.287	92	23.58	-12.68	-8.49	0.59300	-3.127	182	84.1
	2		0.13	0.01700	2.089	0.287	92	19.28	-8.98	-6.78	0.60820	-3.687	182	77.8
	3		-4.46	0.27980	1.210	0.290	185	19.64	-0.94	0.92	0.24160	-2.431	182	64.5
	4		-4.46	0.27980	1.210	0.290	185	21.60	-15.97	-4.00	0.25370	-2.110	182	62.2
	5		-0.94	0.03040	2.066	-0.134	47	10.22	-3.38	-2.18	-0.08390	2.318	182	79.1

1) Species: CBO = cherrybark oak, OR = other red oak, ARO = all red oak, SG = sweetgum, OC = other commercial
2) Grades: T = total merchantable volume, 1-3 = USFS grades 1-3 volumes, 4 = tie and timber volume, 5 = cull volume
3) Units: Int 1/4 = International 1/4, Scrib = Scribner, Cvob = cubic foot volume outside bark, Cvib = cubic foot volume inside bark

Fit statistics for the modeling system (procedure 4) are given in Tables 6 through 15 by species group, input variable, and volume unit. Bias and RMSE were calculated to evaluate the modeling system. Bias ranged from -2.97 to 1.23 for total merchantable sawtimber volume, -42.79 to 2.07 for grade 1, -4.11 to 4.10 for grade 2, -3.75 to 47.84 for grade 3, -12.01 to 1.62 for grade 4, and -2.07 to 8.05 for grade 5. The total merchantable sawtimber volume equation typically had the lowest bias and was the best predictor of all the volume categories. Of the five grade volumes, grade 1 was generally the closest estimate to observed volumes and grade 4 was the farthest from observed volumes when summed totals were compared. Of the five species categories, cherrybark oak was generally the closest estimate to observe. Merchantable height was a better predictor variable than total height, but the difference of predictions between the two variables was marginal. Merchantable sawlog cubic foot volume inside and outside bark volume categories were generally the most accurate across the different species groups for merchantable sawtimber and grade volumes.

Table 6: Individual tree merchantable sawtimber and grade volume prediction fit statistics for cherrybark oak using dbh and total height as predictor variables.

		Grade					
Volume Unit[1]	**Fit Statistic**	**T[2]**	**1[2]**	**2[2]**	**3[2]**	**4[2]**	**5[2]**
Doyle	Mean	331.04	165.68	68.18	71.37	6.95	18.86
	Bias	-0.21	-2.59	1.27	1.27	-0.72	0.54
	RMSE	63.35	104.94	80.42	77.58	23.02	29.07
Int ¼	Mean	411.58	193.41	87.27	100.35	11.04	19.5
	Bias	0.37	-3.55	2.79	1.88	-0.87	0.08
	RMSE	74.2	124.67	97.73	99.46	32.37	37.46
Scribner	Mean	374.43	175.23	79.22	90.84	9.89	19.25
	Bias	-0.22	-3.58	2.63	1.9	-1.13	-0.05
	R MSE	69.41	113.57	87.72	89.78	28.72	35.03
Cvob	Mean	69.61	31.41	14.62	17.79	2.09	3.7
	Bias	0.19	-0.58	2.51	0.07	-1.09	-0.73
	RMSE	12.19	19.99	15.95	16.42	6.13	7.07
Cvib	Mean	60.01	27.22	12.57	15.2	1.78	3.24
	Bias	0.2	-0.41	0.57	0.44	-0.32	-0.08
	RMSE	10.59	17.33	13.64	14.21	5.04	6.16

1) Volume Unit: Int 1/4 = International 1/4, Scrib = Scribner, Cvob = cubic foot volume outside bark, Cvib = cubic foot volume inside bark
2) Grade: T = total merchantable volume, 1-3 = USFS grades 1-3 volumes, 4 = tie and timber volume, 5 = cull volume

Table 7: Individual tree merchantable sawtimber and grade volume prediction fit statistics for other red oak using dbh and total height as predictor variables.

		Grade					
Volume Unit[1]	**Fit Statistic**	**T**[2]	**1**[2]	**2**[2]	**3**[2]	**4**[2]	**5**[2]
Doyle	Mean	261.76	78.30	59.62	88.80	16.01	19.03
	Bias	-0.97	-3.73	1.00	-0.70	0.20	2.26
	RMSE	55.02	99.91	75.87	89.73	36.82	40.68
Int 1/4	Mean	328.86	91.72	73.78	118.42	23.74	21.19
	Bias	-0.66	-4.15	1.84	-0.40	1.13	0.92
	RMSE	65.95	115.14	89.02	108.09	50.68	46.34
Scribner	Mean	305.59	84.52	68.75	109.94	21.79	20.59
	Bias	-0.82	-4.06	1.64	-1.16	1.33	1.42
	RMSE	61.58	106.14	83.04	100.76	46.49	43.52
Cvob	Mean	56.49	15.00	12.43	20.83	4.40	3.82
	Bias	0.00	-0.62	0.30	-0.01	0.19	0.16
	RMSE	11.03	18.57	14.50	17.82	9.06	8.03
Cvib	Mean	48.17	12.94	10.62	17.64	3.70	3.28
	Bias	-0.05	-0.56	0.25	-0.03	0.13	0.16
	RMSE	9.38	16.05	12.46	15.24	7.64	6.94

1) Volume Unit: Int 1/4 = International 1/4, Scrib = Scribner, Cvob = cubic foot volume outside bark, Cvib = cubic foot volume inside bark
2) Grade: T = total merchantable volume, 1-3 = USFS grades 1-3 volumes, 4 = tie and timber volume, 5 = cull volume

Table 8: Individual tree merchantable sawtimber and grade volume prediction fit statistics for all red oak using dbh and total height as predictor variables.

		Grade					
Volume Unit[1]	**Fit Statistic**	**T[2]**	**1[2]**	**2[2]**	**3[2]**	**4[2]**	**5[2]**
Doyle	Mean	292.45	81.14	59.44	121.95	13.51	16.41
	Bias	-0.59	-39.57	-2.96	41.35	1.62	-1.03
	RMSE	55.08	163.84	93.50	160.82	56.96	39.05
Int 1/4	Mean	365.51	97.91	73.34	156.67	19.05	18.54
	Bias	-0.17	-42.79	-4.11	47.84	0.32	-1.44
	RMSE	68.84	188.25	107.58	191.20	63.98	47.43
Scribner	Mean	336.07	88.83	67.43	144.15	17.65	18.01
	Bias	-0.32	-39.48	-3.83	43.57	0.50	-1.08
	RMSE	63.00	172.59	98.99	175.70	60.51	44.61
Cvob	Mean	62.30	16.43	12.32	26.72	3.44	3.39
	Bias	0.08	-6.48	-0.62	7.55	-0.10	-0.27
	RMSE	11.68	29.91	17.48	31.24	10.20	8.43
Cvib	Mean	53.42	14.17	10.58	22.85	2.91	2.91
	Bias	-0.02	-5.66	-0.52	6.49	-0.09	-0.24
	RMSE	10.00	25.91	15.10	26.87	8.78	7.22

1) Volume Unit: Int 1/4 = International 1/4, Scrib = Scribner, Cvob = cubic foot volume outside bark, Cvib = cubic foot volume inside bark
2) Grade: T = total merchantable volume, 1-3 = USFS grades 1-3 volumes, 4 = tie and timber volume, 5 = cull volume

Table 9: Individual tree merchantable sawtimber and grade volume prediction fit statistics for sweetgum using dbh and total height as predictor variables.

		Grade					
Volume Unit[1]	**Fit Statistic**	**T[2]**	**1[2]**	**2[2]**	**3[2]**	**4[2]**	**5[2]**
Doyle	Mean	117.50	25.99	29.35	41.65	12.48	8.02
	Bias	-0.04	-2.36	-0.15	0.65	0.48	1.35
	RMSE	32.79	37.74	35.69	36.70	22.08	21.19
Int 1/4	Mean	170.33	33.16	40.62	65.02	21.08	10.44
	Bias	-0.02	-3.59	1.14	1.09	1.20	0.14
	RMSE	43.82	47.77	47.62	49.23	37.03	27.49
Scribner	Mean	155.30	30.01	37.24	59.18	18.94	9.94
	Bias	-0.56	-3.25	0.86	0.64	0.71	0.48
	Sqrt MSE	40.51	43.41	43.66	45.01	32.85	24.90
Cvob	Mean	32.05	5.71	7.36	12.66	4.24	2.08
	Bias	1.23	-0.52	0.51	0.34	0.87	0.03
	RMSE	7.83	8.19	8.49	8.82	7.24	5.18
Cvib	Mean	27.03	4.95	6.26	10.54	3.49	1.79
	Bias	-0.01	-0.62	0.22	0.15	0.25	0.00
	RMSE	6.52	7.09	7.22	7.42	6.02	4.52

1) Volume Unit: Int 1/4 = International 1/4, Scrib = Scribner, Cvob = cubic foot volume outside bark, Cvib = cubic foot volume inside bark

2) Grade: T = total merchantable volume, 1-3 = USFS grades 1-3 volumes, 4 = tie and timber volume, 5 = cull volume

Table 10: Individual tree merchantable sawtimber and grade volume prediction fit statistics for other commercial species using dbh and total height as predictor variables.

Volume Unit[1]	Fit Statistic	Grade T[2]	1[2]	2[2]	3[2]	4[2]	5[2]
Doyle	Mean	178.93	29.42	48.17	79.07	7.64	14.64
	Bias	-2.92	-0.81	-1.75	-2.49	-0.63	2.75
	RMSE	76.37	72.32	97.81	89.22	45.23	39.22
Int 1/4	Mean	230.00	34.79	59.69	109.08	10.27	16.16
	Bias	-2.70	-1.48	-0.63	-3.75	-1.20	3.36
	RMSE	86.14	87.75	112.63	107.60	50.83	55.86
Scribner	Mean	211.39	32.28	54.98	99.29	9.57	15.27
	Bias	-2.97	-1.69	-0.45	-2.37	-1.38	2.91
	RMSE	79.74	83.34	102.18	98.35	48.61	47.68
Cvob	Mean	40.79	5.85	10.26	19.91	1.85	2.91
	Bias	-0.41	-0.19	0.02	-0.71	-0.31	0.54
	RMSE	13.54	14.54	17.92	17.85	8.21	9.34
Cvib	Mean	34.15	4.95	8.65	16.57	1.54	2.44
	Bias	-0.22	-0.16	0.03	-0.57	-0.21	0.52
	RMSE	11.71	12.30	15.37	15.17	6.94	7.90

1) Volume Unit: Int 1/4 = International 1/4 Scrib = Scribner, Cvob = cubic foot volume outside bark, Cvib = cubic foot volume inside bark
2) Grade: T = total merchantable volume, 1-3 = USFS grades 1-3 volumes, 4 = tie and timber volume, 5 = cull volume

Table 11: Individual tree merchantable sawtimber and grade volume prediction fit statistics for cherrybark oak using dbh and merchantable height as predictor variables.

		Grade					
Volume Unit[1]	**Fit Statistic**	**T[2]**	**1[2]**	**2[2]**	**3[2]**	**4[2]**	**5[2]**
Doyle	Mean	331.04	165.68	68.18	71.37	6.95	18.86
	Bias	-1.09	2.88	3.69	1.91	-11.93	2.36
	RMSE	68.79	137.24	81.12	80.42	131.66	31.13
Int 1/4	Mean	411.58	193.41	87.27	100.35	11.04	19.50
	Bias	-0.78	-0.19	4.10	3.02	-7.74	0.04
	RMSE	78.09	127.21	96.96	99.15	68.84	34.55
Scribner	Mean	374.43	175.23	79.22	90.84	9.89	19.25
	Bias	-1.36	-0.40	3.86	1.84	-6.50	-0.17
	RMSE	75.40	114.82	87.27	90.13	57.19	31.66
Cvob	Mean	69.61	31.41	14.62	17.79	2.09	3.70
	Bias	-0.13	-0.50	2.50	0.39	-1.27	-1.24
	RMSE	12.80	20.06	15.70	16.17	7.05	6.69
Cvib	Mean	60.01	27.22	12.57	15.20	1.78	3.24
	Bias	-0.02	-0.38	0.58	0.55	-0.55	-0.23
	RMSE	11.06	17.38	13.40	13.99	5.68	5.75

1) Volume Unit: Int 1/4 = International 1/4 Scrib = Scribner, Cvob = cubic foot volume outside bark, Cvib = cubic foot volume inside bark
2) Grade: T = total merchantable volume, 1-3 = USFS grades 1-3 volumes, 4 = tie and timber volume, 5 = cull volume

Table 12: Individual tree merchantable sawtimber and grade volume prediction fit statistics for other red oak using dbh and merchantable height as predictor variables.

		Grade					
Volume Unit[1]	**Fit Statistic**	**T[2]**	**1[2]**	**2[2]**	**3[2]**	**4[2]**	**5[2]**
Doyle	Mean	261.76	78.30	59.62	88.80	16.01	19.03
	Bias	-0.74	3.12	2.11	1.30	-12.01	4.74
	RMSE	30.12	107.31	76.45	88.57	94.81	36.23
Int 1/4	Mean	328.86	91.72	73.78	118.42	23.74	21.19
	Bias	-0.85	0.56	1.37	0.89	-11.73	8.05
	RMSE	20.15	117.56	88.22	106.46	98.03	45.21
Scribner	Mean	305.59	84.52	68.75	109.94	21.79	20.59
	Bias	-0.96	1.14	1.91	1.02	-8.60	3.57
	RMSE	25.77	107.76	81.96	99.25	76.53	38.49
Cvob	Mean	56.49	15.00	12.43	20.83	4.40	3.82
	Bias	-0.12	-0.22	0.20	0.12	-1.79	1.57
	RMSE	2.32	18.52	14.19	17.40	12.22	7.42
Cvib	Mean	48.17	12.94	10.62	17.64	3.70	3.28
	Bias	0.00	-0.19	0.13	0.10	-1.57	1.53
	RMSE	2.01	16.03	12.21	14.91	10.80	6.46

1) Volume Unit: Int 1/4 = International 1/4, Scrib = Scribner, Cvob = cubic foot volume outside bark, Cvib = cubic foot volume inside bark
2) Grade: T = total merchantable volume, 1-3 = USFS grades 1-3 volumes, 4 = tie and timber volume, 5 = cull volume

Table 13: Individual tree merchantable sawtimber and grade volume prediction fit statistics for all red oak using dbh and merchantable height as predictor variables.

		Grade					
Volume Unit[1]	**Fit Statistic**	**T[2]**	**1[2]**	**2[2]**	**3[2]**	**4[2]**	**5[2]**
Doyle	Mean	292.45	81.14	59.44	121.95	13.51	16.41
	Bias	-1.36	-32.61	-1.13	41.66	-9.00	-0.29
	RMSE	37.56	154.81	101.84	162.67	94.05	46.52
Int 1/4	Mean	365.51	97.91	73.34	156.67	19.05	18.54
	Bias	-0.96	-37.72	-3.58	47.47	-7.04	-0.23
	RMSE	25.27	176.78	115.63	192.25	77.41	53.67
Scribner	Mean	336.07	88.83	67.43	144.15	17.65	18.01
	Bias	-0.79	-35.16	-3.16	43.61	-4.47	-1.61
	RMSE	32.41	161.97	105.74	176.84	66.51	50.98
Cvob	Mean	62.30	16.43	12.32	26.72	3.44	3.39
	Bias	-0.08	-6.16	-0.63	7.44	-0.20	-0.54
	RMSE	3.39	28.11	18.26	31.20	9.45	9.48
Cvib	Mean	53.42	14.17	10.58	22.85	2.91	2.91
	Bias	0.06	-5.35	-0.57	6.49	-0.25	-0.25
	RMSE	2.88	24.31	15.84	26.96	7.90	8.13

1) Volume Unit: Int 1/4 = International 1/4, Scrib = Scribner, Cvob = cubic foot volume outside bark, Cvib = cubic foot volume inside bark
2) Grade: T = total merchantable volume, 1-3 = USFS grades 1-3 volumes, 4 = tie and timber volume, 5 = cull volume

Table 14: Individual tree merchantable sawtimber and grade volume prediction fit statistics for sweetgum using dbh and merchantable height as predictor variables.

Volume Unit[1]	Fit Statistic	Grade T[2]	1[2]	2[2]	3[2]	4[2]	5[2]
Doyle	Mean	117.50	25.99	29.35	41.65	12.48	8.02
	Bias	-0.27	-0.46	0.69	0.95	-1.97	0.52
	RMSE	16.29	40.86	41.14	34.56	43.09	21.20
Int 1/4	Mean	170.33	33.16	40.62	65.02	21.08	10.44
	Bias	-0.35	2.07	1.45	3.05	-9.68	2.76
	RMSE	12.67	46.23	50.77	49.03	45.36	25.83
Scribner	Mean	155.30	30.01	37.24	59.18	18.94	9.94
	Bias	-0.02	-1.06	0.57	1.73	-0.76	-0.49
	RMSE	15.67	41.97	46.44	44.40	36.26	24.53
Cvob	Mean	32.05	5.71	7.36	12.66	4.24	2.08
	Bias	-0.10	-0.32	0.07	0.33	0.12	-0.30
	RMSE	1.76	7.59	8.58	8.83	6.46	4.82
Cvib	Mean	27.03	4.95	6.26	10.54	3.49	1.79
	Bias	0.00	0.09	-0.06	0.39	-0.01	-0.42
	RMSE	1.52	6.54	7.39	7.46	5.65	5.60

1) Volume Unit: Int 1/4 = International 1/4, Scrib = Scribner, Cvob = cubic foot volume outside bark, Cvib = cubic foot volume inside bark
2) Grade: T = total merchantable volume, 1-3 = USFS grades 1-3 volumes, 4 = tie and timber volume, 5 = cull volume

Table 15: Individual tree merchantable sawtimber and grade volume prediction fit statistics for other commercial species using dbh and merchantable height as predictor variables.

		Grade					
Volume Unit[1]	**Fit Statistic**	T[2]	1[2]	2[2]	3[2]	4[2]	5[2]
Doyle	Mean	178.93	29.42	48.17	79.07	7.64	14.64
	Bias	-1.70	0.09	-0.07	5.02	-6.10	-0.64
	RMSE	270.87	91.70	96.35	124.84	71.42	61.85
Int 1/4	Mean	230.00	34.79	59.69	109.08	10.27	16.16
	Bias	-1.79	0.11	2.12	9.19	-11.13	-2.07
	RMSE	297.12	104.24	122.92	145.17	101.43	67.71
Scribner	Mean	211.39	32.28	54.98	99.29	9.57	15.27
	Bias	-2.37	-1.37	2.01	7.52	-6.65	-3.88
	RMSE	278.50	101.70	110.66	134.21	66.13	60.33
Cvob	Mean	40.79	5.85	10.26	19.91	1.85	2.91
	Bias	-0.14	-0.22	0.20	1.70	-1.29	-0.53
	RMSE	46.82	17.53	20.07	23.45	12.17	11.21
Cvib	Mean	34.15	4.95	8.65	16.57	1.54	2.44
	Bias	-0.17	-0.08	0.27	1.43	-1.40	-0.40
	RMSE	39.79	14.65	17.36	20.00	12.36	9.46

1) Volume Unit: Int 1/4 = International 1/4, Scrib = Scribner, Cvob = cubic foot volume outside bark, Cvib = cubic foot volume inside bark
2) Grade: T = total merchantable volume, 1-3 = USFS grades 1-3 volumes, 4 = tie and timber volume, 5 = cull volume

Comparing Modeling Techniques

Five different modeling approaches were compared to determine the best method for predicting both total merchantable sawtimber volume and volume by grade. The modeling approaches described in Chapter II are (1) separate weighted non-linear regression including zero observations to determine grade volume and summing the grade volume to determine total merchantable sawtimber volume, (2) separate weighted non-linear regression including zero observations to determine total merchantable sawtimber and grade_i volumes, followed by a weighted least square procedure to adjust the sum of the grade_i volumes to equal estimated total merchantable sawtimber volume, (3) separate weighted non-linear regression excluding zero observations to determine grade_i volume, followed by binary logistic regression to predict the probability of grade_i to be found in a tree. Probability and predicted volume for each grade were multiplied to find a true estimate of grade_i volume. True estimates of grade_i were then summed to determine total merchantable sawtimber volume, (4) like procedure 2 except zero observations were excluded during weighted non-linear regression modeling, and binary logistic regression was performed to find probability of grade_i. Probability and volume estimations for grade_i were multiplied, followed by a weighted lease square procedure to adjust the sum of the grade_i volumes to equal estimated total merchantable volume, and (5) the additive method proposed by Parresol (2001). The procedural system (4) out-performed the other modeling techniques examined through comparison of bias and RMSE.

Prediction equations determining total merchantable sawtimber volume were the best fits, and as techniques 1 and 3 summed grade volumes to estimate total merchantable sawtimber volume the deviation from the actual were greater than other techniques for total merchantable sawtimber volume. Technique 5, additive, had the worst results for predicting grade, with grade 1 estimating the worst and grade 5 estimating the best. As grade 1 is the most commercially important and grade 5 the least, this technique was decided to be unacceptable for estimating grade volume. Technique 2 for this example had similar results to technique 4, but complications with the technique made it unusable across broad application. The inclusion of zero observations by techniques 1 and 2 caused several complications during non-linear regression modeling, some of which were: increased non-homogeneity of variance, zeros disrupted the average trend line, and several of the various species/volume type/grade category combinations failed to converge. Do to these complications, techniques 1 and 2 were dismissed. Technique 4 was selected as the best procedure for predicting merchantable and grade volumes with the least error and best fit across broad application. Example comparative statistics for the prediction techniques 1-5 estimating total merchantable sawtimber and $grade_i$ volume of merchantable sawlog Cvib for cherrybark oak is given in Table 15.

Table 16: Comparative statistics of separate modeling techniques for merchantable sawlog cubic foot volume inside bark of cherrybark oak using dbh and total height as predictive variables.

		Grade					
Procedure		T[1]	1[1]	2[1]	3[1]	4[1]	5[1]
1	Mean	62.37	27.22	12.89	16.61	2.41	3.22
	Bias	-2.44	-0.04	-0.34	-1.42	-0.64	0.01
	RMSE	15.77	18.63	13.65	14.13	4.88	5.24
2	Mean	59.73	27.13	12.42	15.98	1.92	2.28
	Bias	0.20	0.05	0.14	-0.80	-0.15	0.95
	RMSE	9.69	17.74	13.33	14.23	4.83	6.67
3	Mean	50.41	27.35	14.07	5.20	0.29	3.49
	Bias	9.52	-0.17	-1.52	9.98	1.48	-0.26
	RMSE	16.42	17.26	13.97	18.53	4.98	5.34
4	Mean	59.73	27.59	11.99	14.75	2.10	3.32
	Bias	0.20	-0.40	0.57	0.44	-0.32	-0.08
	RMSE	9.69	17.14	13.54	14.13	5.00	6.04
5	Mean	57.78	16.02	15.07	17.61	4.01	5.07
	Bias	2.24	11.20	-2.50	-2.41	-2.23	0.70
	RMSE	11.18	27.47	15.11	15.37	5.11	5.51

1) Grade: T = total merchantable volume, 1-3 = USFS grades 1-3 volumes, 4 = tie and timber volume, 5 = cull volume

CHAPTER IV

DISCUSSION

Total Merchantable Sawtimber Volume

Total merchantable sawtimber volume models for each species group provide excellent volume predictions as shown by plotting the fitted model estimates of cubic foot volume inside bark for each species group over observed volume (Figure 2). This result was expected because input variables for the models are the same as those used to calculate observed volume. The addition of these volume prediction equations for bottomland hardwood species groups will add much needed information for landowners making management decisions. In addition, they will also aid volume estimates for specific products in an area. In recent years, there have been numerous feasibility studies on available woody material for conversion to energy products (Schultz et. al. 2008), (Grebner et. al. 2008), (Grebner et. al. 2009). While these models only estimate merchantable tree bole volume and not branches, bark, and foliage, they can help determine the potential for markets that may be of higher value than the energy market when incorporated with the volume by grade models.

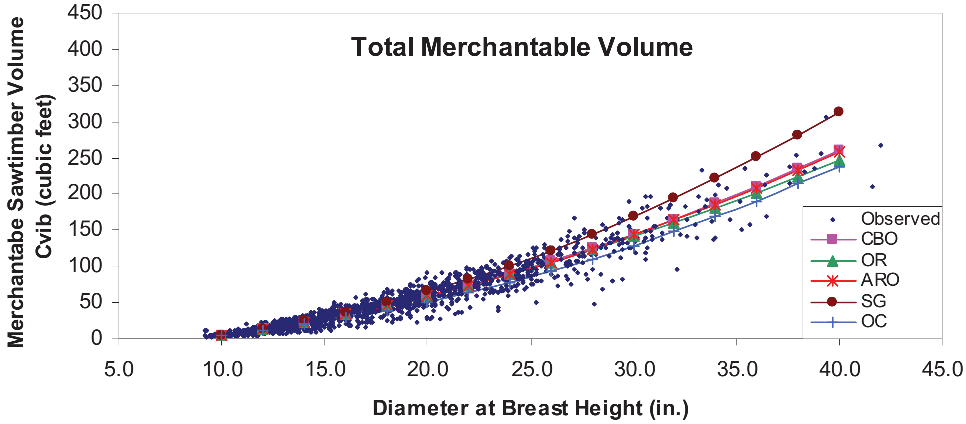

Figure 2: Observed and predicted individual tree merchantable sawlog cubic foot inside bark volume by species group based on diameter at breast height and total tree height.

Volume by Grade Category

The estimation of volume by grade allows the manager to consider more product categories when estimating the value of a stand. The more information that is available on a stand the better possibility the decisions will be. The grade distribution models are not intended to determine individual tree grade volumes, but rather a summed total volume per grade for an entire stand. Because grade within individual trees varies greatly, it is more accurate to make predictions on a stand or pre acre level.

Grade 1

Grade 1 percentage of merchantable sawlog cubic foot volume inside bark plotted against dbh for each species group is shown in Figure 3. General trends show, grade 1 category requires long term development within the stand to establish a majority of the stand volume. There is a low percentage of grade 1 volume during early stand development when trees have a dbh of 15 inches or less. Grade 1 requires minimum scaling diameters of 13 inches for butt logs and 16 inches for upper logs (Rast et al. 1973). When a tree grows larger that 15 inches dbh, grade 1 percentage of volume increases continuously as dbh increases. Many of the early logs that were once not able to be categorized as grade 1 become large enough in size to meet the criteria, thus increasing the percentage of volume for grade 1 as dbh increases. Minor defects once limiting a log from the grade 1 category often become overgrown with clear wood as a tree increases in dbh, and the percentage of volume for grade 1 increases. Species group models display an upward trend of grade 1 percentage of volume as dbh increases following the trend of observed data.

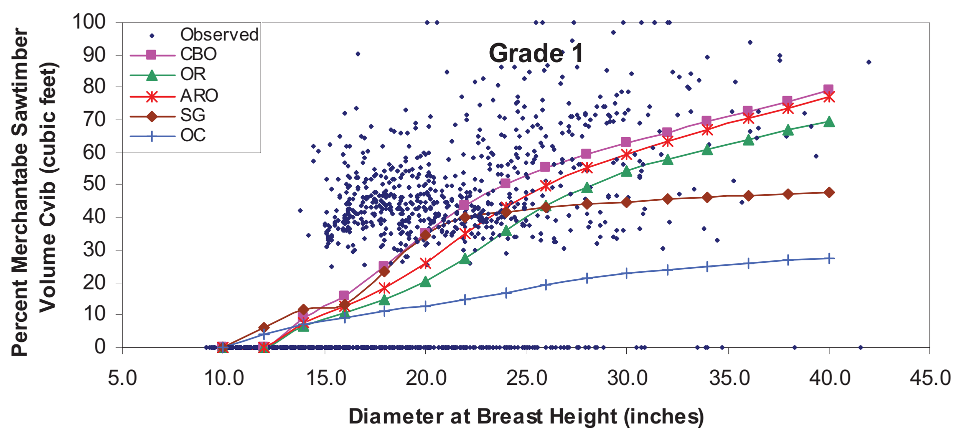

Figure 3: Observed and predicted individual tree grade 1 percentage merchantable sawlog cubic foot inside bark volume by species group based on diameter at breast height and total tree height.

Grade 2

Grade 2 holds a relatively constant percentage of volume across dbh classes greater than 12 inches. This trend can be explained by the general improvement of grade as dbh increases and grade 2 being in the middle of this advancement. As dbh increases, many grade 3 logs will improve in quality and develop into grade 2 logs. Simultaneously, grade 2 logs will improve in quality and develop into grade 1 logs. This constant recruitment and advancement of logs in the grade 2 category allows the category maintain a relatively constant percentage of volume. A slight decrease in the grade 2 trend can be seen as dbh increases. As dbh increases greater than 22 inches, the advancement of trees into grade 1 begins to out weigh the recruitment of trees from grade 3 due to fewer trees available for advancement as the stand ages. Grade 2 percentage of merchantable sawlog cubic foot volume inside bark plotted against dbh for each species group is shown in Figure 4.

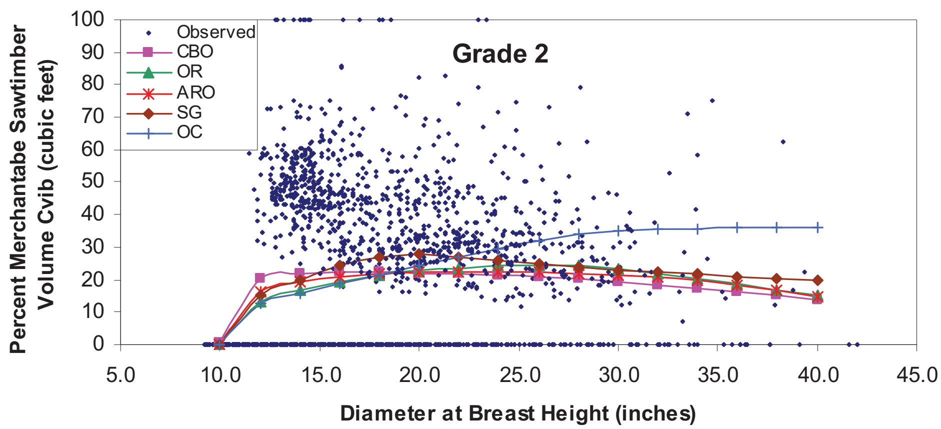

Figure 4: Observed and predicted individual tree grade 2 percentage merchantable sawlog cubic foot inside bark volume by species group based on diameter at breast height and total tree height.

Grade 3

As both grades 1 and 2 have size limitations, it makes biological sense that the majority of volume at early stand development (a dbh of 14 inches or less) is in the grade 3 category. Percentage of volume in grade 3 rapidly decreases as dbh increases and grades 1 and 2 are able to incorporate the stand's volume. The declining trend of grade 3 continues with the increase of dbh. While grade 3 does not completely subside to the other grades, added clear wood covers many old defects on logs as dbh increases. Unlike grade 2, grade 3 does not have lower grades that tend to improve and advance in quality. This lack of lower advancement can be seen in Figure 5 with the continuous downward trend of percentage of volume as dbh increases.

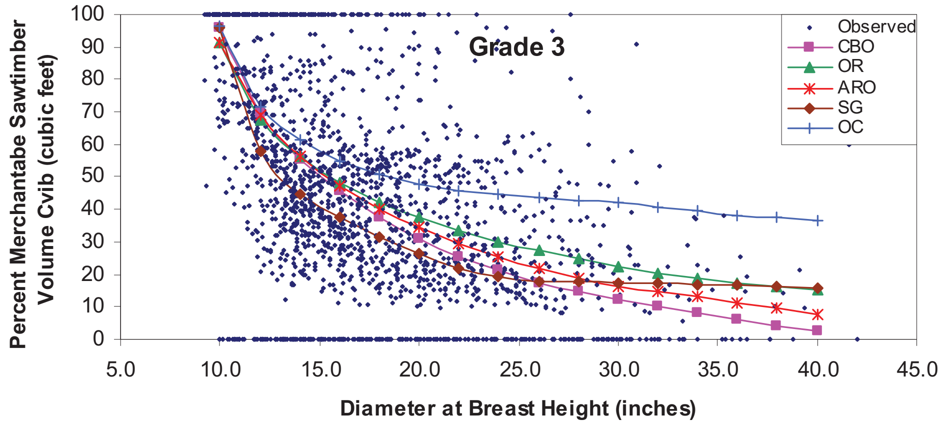

Figure 5: Observed and predicted individual tree grade 3 percentage merchantable sawlog cubic foot inside bark volume by species group based on diameter at breast height and total tree height.

Grade 4

The percentage of volume for grade 4 (tie and timber) exhibits a downward trend flattening out to zero around 26 inches dbh for all species groups except sweetgum, which can be seen in Figure 6. Sweetgum remains relatively constant across all dbh classes between 10 and 20 percent volume. Grade 4 was never a major component of the observed volume found in the bottomland hardwood sites being studied. Study site quality was generally very good and thus merchantable log sections were typically of grade 3 or better quality. With such low volume of grade 4 on these sites, it could be assumed that a harvest would not yield enough to separately merchandize tie/timber. Thus, during an actual harvest this volume may be marketed as poor grade 3, left as grade 5, or sold as pulpwood.

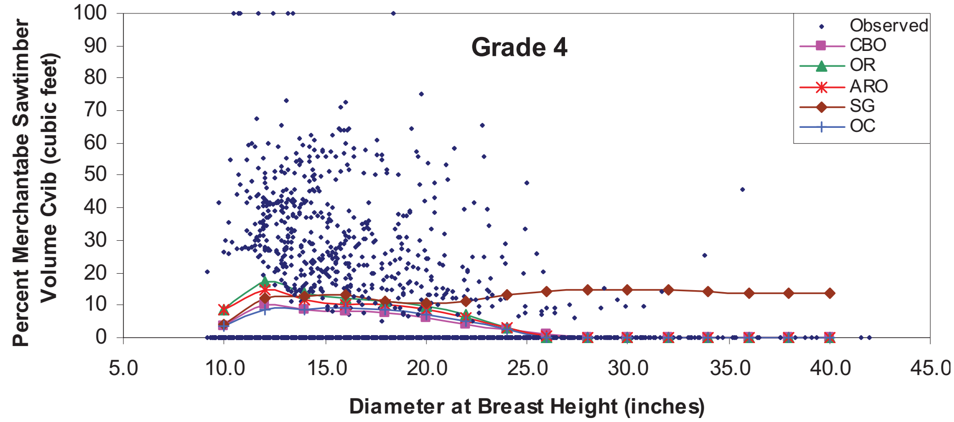

Figure 6: Observed and predicted individual tree grade 4 percentage merchantable sawlog cubic foot inside bark volume by species group based on diameter at breast height and total tree height.

Grade 5

Grade 5 exhibits a slight downward trend in volume as dbh increases. Since known dead trees were not selected to be graded, only small sections of a tree tended to be grade 5. Many of the observed grade 5 small sections were only 2 to 5 feet in length. These sections were often fluted butts, bulges, forks, large branch meeting the bole, and rot. These defects are found during all stages of development helping to explain the little change in trend across dbh classes. Figure 7 shows the trends of grade 5 percent merchantable sawlog cubic foot volume inside bark versus dbh.

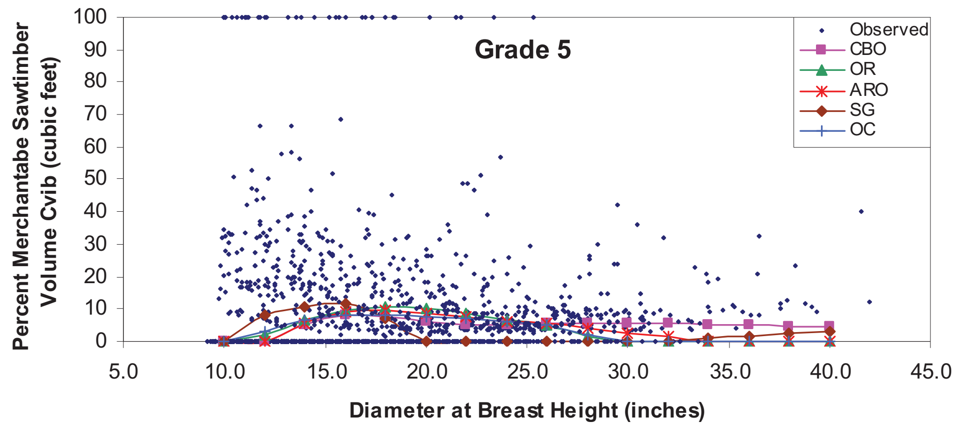

Figure 7: Observed and predicted individual tree grade 5 percent merchantable sawlog cubic foot inside bark volume by species group based on diameter at breast height and total tree height.

Species Groups

Differences in grade trends were found among species groups. Growth requirements and stand development patterns play a role in how grade development evolves during the life of a stand and how species groups respond. Grade development in each species group is discussed in the following sub-sections.

Cherrybark Oak, Other Red Oak, and All Red Oak

All three red oak species groups (cherrybark oak, other red oak, and all red oak) behave in a similar way in terms of how percentage of grade is distributed over time. During early development of a tree, size limitations for grade 1 and 2 explain the majority of volume encapsulated in the grade 3 category. As a tree grows larger, both in diameter and height, the previous restrictions to higher quality grades are removed and the percentage of grade 3 drops quickly. The drop in grade 3 coincides with the increase in grades 1 and 2. While grade 2 holds a relatively constant percentage of volume across a wide range of dbh classes, grade 1 continues to increase in percentage as a tree grows in diameter. Trends for grades 1 and 3 react inversely to each other across the development of a tree. These trends were observed across all stands in this study. Percentage trends of grades 1 through 5 merchantable sawlog cubic foot volume inside bark over dbh for cherrybark oak, other red oak, and all red oak can be seen in Figures 8, 9, and 10 respectively.

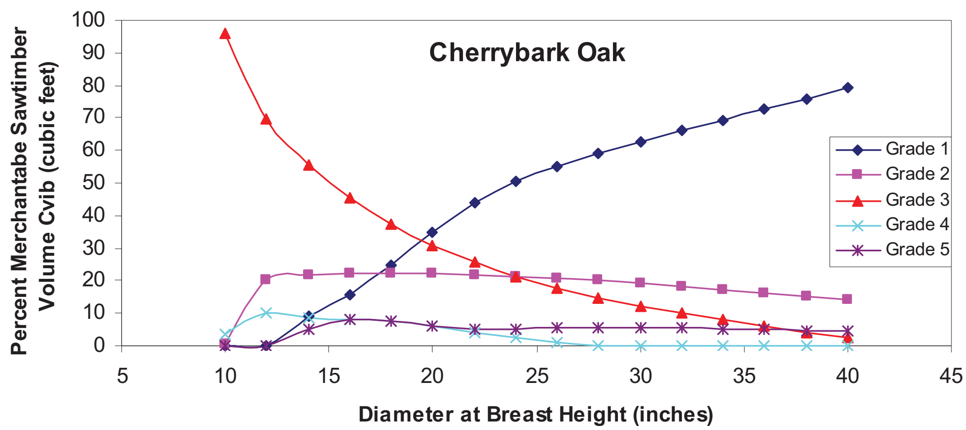

Figure 8: Predicted individual tree merchantable sawlog cubic foot volume inside bark percentage by grade categories for cherrybark oak based on diameter at breast height and total tree height.

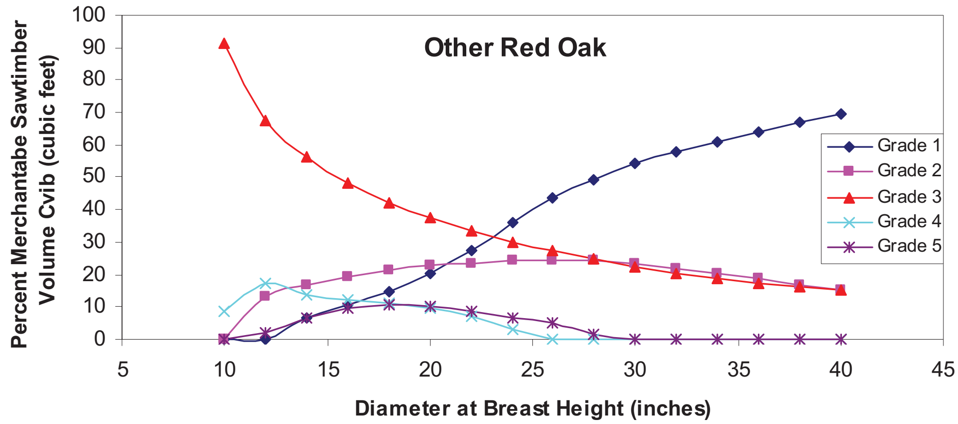

Figure 9: Predicted individual tree merchantable sawlog cubic foot volume inside bark percentage by grade categories for other red oak species based on diameter at breast height and total tree height.

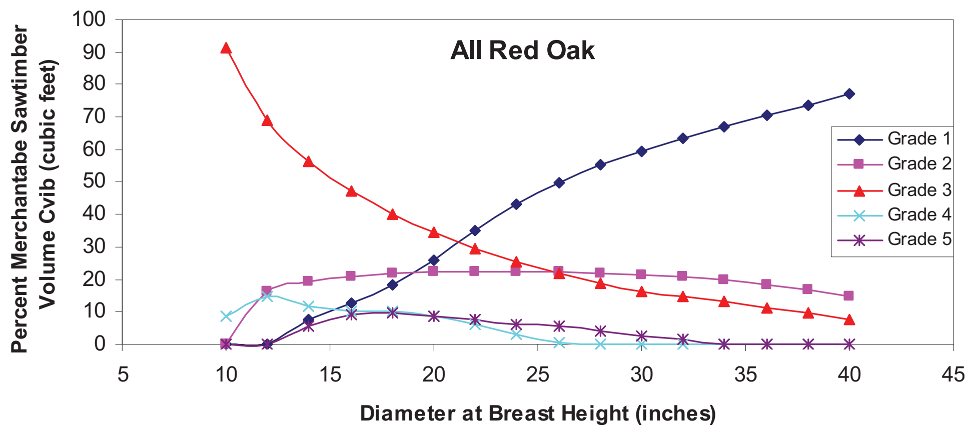

Figure 10: Predicted individual tree merchantable sawlog cubic foot volume inside bark percentage by grade categories for all red oak species based on diameter at breast height and total tree height.

Sweetgum

Sweetgum responded differently to the development of grade compared to the red oak species groups (Figure 11). Grade 3 progresses in a similar way to the red oaks with a quick drop early and a slower decline once trees reach 20 inches in dbh. Grade 2 holds a relatively constant percentage, but tends to be slightly greater than the red oak groups.

The slight difference in grade 2 is partially due to the lower level of grade 1 development. The grade 1 sweetgum displays an incline similar to the red oak species groups in the 13 to 18 inch dbh range, but then levels off across larger dbh classes. Sweetgum is shade intolerant and reacts differently to competition than the red oak species. The stress from shade tolerant competition continues to increase through time, but once a stand contains sweetgum trees with dbh of 18 inches, competition from shade tolerant species is fully evident. The increase in competition creates stress in the sweetgum which invites many defect producing agents, such as epicormic branching, swell, bulges, animal and insect wounds, rot and tree death. The sweetgum species group is the only group that has an increase in grade 4 percentage as dbh increases. Many of the byproducts from increased competition help explain this trend. While grade 1 percentage begins to level, grade 4 begins to increase slightly as dbh increases. Sweetgum that are not large enough to compete with shade tolerant species in early stand development are reflected in the increasing trend of grade 5 with the 10 to 16 inch dbh range. As these trees die and are no longer in the stand, grade 5 percentage decreases. Grade 5 re-emerges and begins to increase slightly at 32 inch dbh. Increased competition can explain some of the higher dbh grade 5 volumes together with a naturally shorter sweetgum life expectancy than oaks.

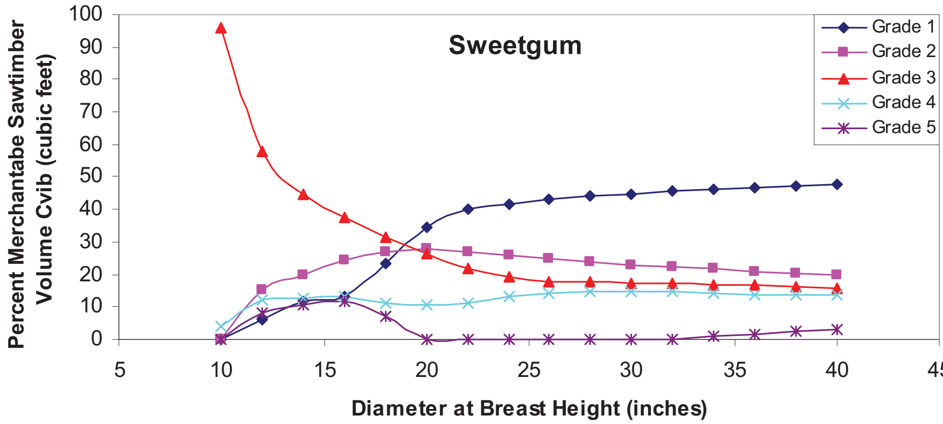

Figure 11: Predicted individual tree merchantable sawlog cubic foot volume inside bark percentage by grade categories for sweetgum species based on diameter at breast height and total tree height.

Other Commercial Species

Percentage merchantable sawlog cubic foot volume inside bark trends of grades 1 through 5 over dbh for the other commercial species group can be seen in Figure 12. White oak, sycamore, elm, sugarberry, green ash, yellow-poplar, and hickory are examples of species found in the other commercial species group. Many of these species have growth patterns which differ from one another, and this is because the only requirement to be categorized in the other commercial species group was to not be red oak or sweetgum. Hickories observed in the study often were found to have several branches on the lower portion of the bole. The lower braches cause defects which are undesirable for sawtimber. Sugarberry and sycamore can often be observed with sweep in the merchantable bole causing grade limitations. As the species in this group differ, have less uniform growth patterns, and comprise some species that are less desirable for sawtimber, grade 1 never composes a large percentage of the overall merchantable sawtimber volume and remains behind grades 2 and 3 across all dbh classes. Grade 3 has the characteristic early drop in percentage, but declines less quickly than in the other species groups. Grade 3 remains an important category across all ranges of dbh. Due to many of the less desirable species and growth patterns, grade 2 is often the best attainable grade. Given a relatively low percentage of grade 1 volume and the improvement of grade with dbh, grade 2 volume continues to increase with dbh as large as 33 inches. Grade 2 volume for the other species group begins to level off around 28 inches dbh. Grades 4 and 5 exhibit level trends around 10 percent volume until these species reach a dbh of approximately 25 inches. Trees that survive to be 25+ inches in dbh often have developed wood properties greater than grades 4 and 5, and the predictions decline to towards zero percent.

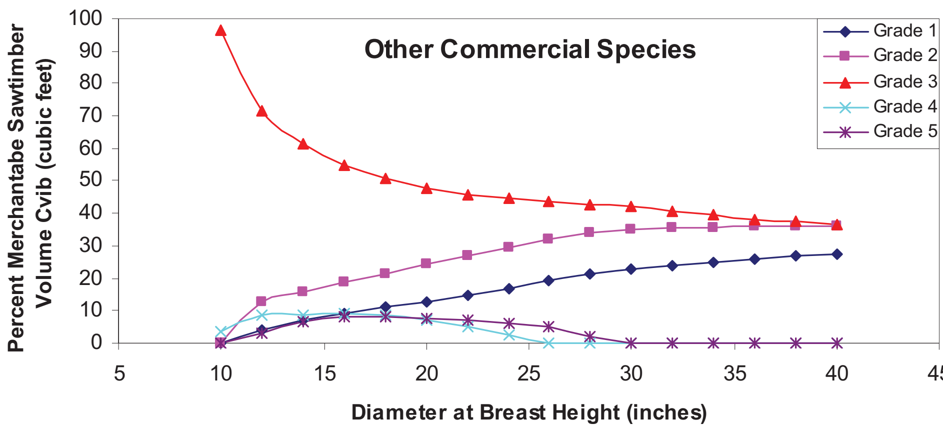

Figure 12: Predicted individual tree merchantable sawlog cubic foot volume inside bark percentage by grade categories for other commercial species based on diameter at breast height and total tree height.

Future Technology Transfer and Research

The log grade volume prediction models constructed in this project will become an integrated part of a Mississippi State University-USFS web-based growth and yield decision support system (DSS). Previously completed stand level growth and yield component models by Iles (2008) and current work on modeling diameter distributions will be integrated with the log grade models, a financial analysis calculator, and user interface. The overall objective of the DSS is to have a complete bottomland hardwood growth and yield system online that is easy to use and readily available to the public.

The diameter distribution models will use the stand level prediction of numbers of trees per acre and distribute them into diameter classes. As the stand level models currently can predict total height by species group (needed as an input for the grade models), the grade prediction models still require dbh as an input. The diameter distribution models will fill the missing predicted dbh variable needed to complete the grade predictions. Once all three projects are complete, the system will have the ability to predict stand growth forward with outputs of site index, trees per acre by species group, diameter and total height of trees, and total merchantable sawtimber and grade volumes.

Response predictions to management options are also needed to expand the use of this growth and yield system as a management tool. Response to thinning treatments is the next logical step. This research should incorporate several thinning densities to determine optimal density levels at a given age to promote growth. The addition of management options into the DSS will promote better understanding and management implications for the landowners of this valuable resource in the southern United States.

CHAPTER V

CONCLUSIONS

The bottomland hardwood log grade volume distribution model system quantifies the amount of total merchantable sawtimber and grade volume based on either dbh and total height or dbh and merchantable height. The total merchantable sawtimber volume models can function as stand-alone models, but the grade distribution models work as a system and require the use of the total merchantable sawtimber volume models to function properly. The system is not intended to be used to obtain grade volume estimates for individual trees, but rather to sum all predicted volumes to find the total tract or per acre volumes by grade category. Model input variables were selected based on statistical significance, ease of measurement, and compatibility with previously developed stand level bottomland hardwood growth and yield models.

Five different modeling approaches were examined to determine the best method to predict both total merchantable sawtimber volume and volume by grade. The modeling approaches were: (1) separate weighted non-linear regression which included zero observations to determine $grade_i$ volume and summing the $grade_i$ volumes to determine total merchantable sawtimber volume, (2) separate weighted non-linear regression which included zero observations to determine total merchantable sawtimber and $grade_i$ volumes, followed by a weighted least square procedure to adjust the sum of the $grade_i$ volumes to equal estimated total merchantable sawtimber volume, (3) separate weighted non-linear regression which excluded zero observations to determine $grade_i$ volume, followed by binary logistic regression to predict the probability of a $grade_i$ to be found in a tree. Probability and predicted volume for each grade were multiplied to find a true estimate of $grade_i$ volume. True estimates of $grade_i$ were then summed to determine total merchantable sawtimber volume, (4) like procedure 2 except zero observations were excluded during weighted non-linear regression modeling, and binary logistic regression was performed to find probability of $grade_i$. Probability and volume estimations for $grade_i$ were multiplied, followed by a weighted lease square procedure to adjust the sum of the $grade_i$ volumes to equal estimated total merchantable volume, and (5) the additive method proposed by Parresol (2001). Of the modeling techniques examined, the procedural system (4) estimated total merchantable sawtimber and grade volumes best.

Bias and RMSE fit statistics were examined to determine how well the individual grade models fit the observed data. Procedural system (4) was determined to have good to excellent fit statistics across all species groups, volume types, and input variables with bias ranging from -2.97 to 1.23 for total merchantable sawtimber, -42.79 to 2.07 for grade 1, -4.11 to 4.10 for grade 2, -3.75 to 47.84 for grade 3, -12.01 to 1.62 for grade 4, and -2.07 to 8.05 for grade 5 volumes.

Growth and yield models are a valuable tool used by land managers to estimate current and future values, make sound silvicultural decisions, and develop management schedules. The addition of the log grade volume distribution model will improve the way bottomland hardwood forests are managed. The ability to quantify the standing timber will lead to better understanding of how these dynamic forest complexes develop and their potential value.

Stand level bottomland hardwood growth and yield models have been constructed to predict site index, trees per acre, arithmetic mean diameter, quadratic mean diameter, and total tree height for various species groups. Diameter distribution models are currently being developed to link the stand level prediction models with the log grade volume distribution models. The final hardwood growth and yield model will be the first publicly available system in the southeastern U.S.

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