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Small Signal Stability of an Unregulated Power System

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SMALL SIGNAL STABILITY OF AN UNREGULATED POWER SYSTEM

By

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Rotor angle stability is the ability of the interconnected synchronous machines of a power system to remain in synchronism. This stability problem is concerned with the behavior of one or more synchronous machine after they have been perturbed. These perturbations can be small or large depending upon the type of disturbances considered. The work presented in this thesis is focused on the power system behavior when subjected to small disturbances. The "small signal" disturbances are considered sufficiently small for the linearization of system equations to be permissible for the purpose of the analysis. The first step in the small signal stability studies is to obtain initial steady state conditions using load flow solutions. After establishing initial conditions, an unregulated mathematical model of the power system is formed. The mathematical model obtained is a set of nonlinear coupled first order differential equations. The method of small changes, called the perturbation method, is used to

linearize these nonlinear differential equations. The equations are then written in a linear state space model form. The eigenvalues and the participation factors are obtained from the state matrix and the contribution of a particular machine in a particular mode or oscillations (or eigenvalue) can be examined for the small signal stability studies.

DEDICATION

I would like to dedicate this research to my parents, brother, sister, brother-in-law and last but not least my wife who inspired and helped me to pursue higher education at Mississippi State University.

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I would like to express my sincere gratitude to all those people who provided a constant source of help and motivation throughout my work. First of all, I would like to thank my major professor and advisor Dr. Mark Halpin. I will always revere his patience, expert guidance and ability to solve intricate problems. He made my pursuit of higher education a truly enjoyable and unforgettable experience. I would also like to thank my committee members, Dr Stan Grzybowski and Dr G. Marshall Molen for their help and valuable time they spent in reviewing this work. Finally, I would like to express my deepest appreciation to my relatives and friends in India as well as at Mississippi State University for their blessings and encouragement.

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CHAPTER I

INTRODUCTION

Stability is a condition of equilibrium between opposing forces. The mechanism by which interconnected synchronous machines maintain synchronism with one another is through restoring forces, which act whenever there are forces tending to accelerate or decelerate one or more machines with respect to other machines [1]. Instability in a power system is described depending upon the system configuration and operating mode. Generally, the stability problem has been one of maintaining synchronous operation. This aspect of stability is influenced by the dynamics of machine rotor angles. But, instability may also be encountered without loss of synchronism. For example, a system can go unstable because of the collapse of load voltage. Maintaining synchronism is not an issue in this instance; instead, the concern is stability and control of voltages. In this work, the discussion is restricted to rotor angle stability.

Rotor angle stability is the ability of interconnected synchronous machines of a power system to remain in synchronism [1]. This stability problem is concerned with the behavior of a synchronous machine after it has been perturbed. Under steady state conditions, there is equilibrium between the input mechanical torque and the developed electrical torque of each machine. This equilibrium is upset during perturbation of the system. The torque unbalance is caused by a change in load, generation or any other network condition. In any case, for the system to be stable all the machines must remain operating in parallel and at the same speed. However, the statement declaring the power system to be stable is not meaningful unless the conditions under which this stability has been examined are clearly stated. This includes the operating conditions as well as the type of perturbation (which can be large or small) given to the system.

Transient stability is the ability of the power system to maintain synchronism when subjected to a *large disturbance* [1]. The resulting system response involves large excursions of generator rotor angles and is influenced by the nonlinear power-angle relationship. Stability depends on both the initial operating state of the system and the severity of the disturbance. This disturbance is usually so large that it alters the post-disturbance equilibrium conditions relative to those existing prior to the disturbance. The work presented in this thesis is focused on the power system behavior when subjected to small disturbances.

Small signal stability is the ability of the power system to maintain synchronism under *small perturbations*. Such perturbations occur continuously on the system because of the small variations in load and generation where the system continuously adjusts itself to changing conditions. Restoring forces acting on the machines help them to maintain stable conditions. The system must be able to operate satisfactorily under these conditions and successfully supply the maximum amount of load. The "small signal" disturbances are considered sufficiently small for linearization of system equations to be permissible for the purpose of analysis.

The most common form of instability between interconnected generators is loss of synchronism, monotonically, in the first few seconds following a fault due to lack of synchronizing torque and damping torque. The stability of the following types of oscillations is of concern:

- Local modes are associated with the oscillations of generating units at a particular station with respect to the rest of system. These oscillations are localized in a small part of the power system.
- Interarea modes are associated with the oscillations of many machines in one part of the system against machines in the other parts.

The first step in a stability study is to make a mathematical model of the system. The elements included in the model are those affecting the machine. The complexity of the model depends upon the type of stability study. Generally, the components of the power system that influence the electrical and mechanical torques of the machines are included in the model. Such components are the loads and their characteristics, the network during the disturbance and the parameters of synchronous machines (such as inertia of the rotating mass). Thus, the basic requirements for these studies are initial conditions of the power system prior to the start of the disturbance and the mathematical description of the main components of the system that might affect the behavior of synchronous machines.

Generally, differential equations are used to describe the various components. The system equations for small signal stability analysis are usually nonlinear. The behavior of any dynamic system, such as a power system, is described by a set of *n* first order nonlinear differential equations of the form given by (1.1).

$$\dot{\tilde{x}} = \tilde{f}\left(\tilde{x}, \tilde{u}, t\right) \tag{1.1}$$

In (1.1), f is a vector of nonlinear functions. The column vector $\tilde{x}$ is referred to as the state vector and $\tilde{u}$ is the vector of inputs to the system. These are the external signals that influence the performance of the system. The study of the dynamic behavior of the system is based on the nature of these differential equations. However, the system equations that are in the form of (1.1) are non-linear. In the context of this thesis work, a disturbance is considered to be small if the equations that describe the resulting response of the system may be linearized for the purpose of analysis. After linearization, equations described by (1.1) are represented by a state space model as defined by (1.2). Small-signal stability analysis using linear techniques provides valuable information about the inherent dynamic characteristics of the power system and assists in its design. The stability characteristics may be determined by examining the eigenvalues of the A matrix, where A is defined by (1.2).

$$\dot{\tilde{x}} = [A]\tilde{x} + [B]\tilde{u} \tag{1.2}$$

In (1.2), x is an *n* vector denoting the states of the system and A is a coefficient matrix. The system inputs are represented by the vector $\tilde{u}$ and B is a control or input matrix. The work presented in this thesis is based on the method discussed in this paragraph with some preliminary steps (discussed in next paragraph) applied to a large power system.

The first step is to do a load flow study as discussed in Chapter II to obtain the initial steady state conditions. After establishing initial conditions, an unregulated mathematical model of the power system is formed as discussed in Chapter III. The model will be an unregulated because the effects of the governor, the power system stabilizer (P.S.S) and field excitation are neglected for all machines. The mathematical model obtained is a set of non-linear coupled first order differential equations. The method of small changes, sometimes called the perturbation method, is used to linearize these non-linear differential equations. They are linearized as discussed in Chapter IV about an operating point, which is the steady state condition obtained from the load flow. The linear equations derived are assumed to be valid in the region near the quiescent condition. The equations are then written in a state-space model form as given by (1.2). The eigenvalues are obtained from the state space model as discussed in Chapter V and the free response of the system is examined.

The participation factors are then found using the eigenvalues obtained from (1.2) and are used to study the small signal stability problems of the power system. The intent of the work presented in this thesis is to focus on the participation of machines in various modes (or eigenvalues). A detailed analysis of the participation factor concept and how it helps in identifying small signal stability problems is discussed in Chapters V and VI.

CHAPTER II

POWER FLOW

To prepare the system data for a stability study, load flow studies are done. These studies establish the operating point about which the nonlinear differential equations (introduced in Chapter I) are linearized. A powerflow program finds the steady state voltage and angle at each bus for a given system. Conventional nodal or loop analysis is not suitable for power-flow studies because the input data for loads are normally given in terms of power, not impedance [2]. Also, generators are considered as power sources, not voltage or current sources. The powerflow problem is therefore formulated as a set of non-linear algebraic equations suitable for computer solution.

The information obtained from a powerflow study is essential for the continuous monitoring of the current state of the system and for analyzing the effectiveness of alternative plans for the future, such as adding new generator sites, meeting increased load demands, and locating new transmission sites and also essential for establishing *initial conditions for stability studies*. The algorithm presented here uses the Newton-Raphson method for finding solutions for the variables such as voltage magnitudes and their angles at all buses.

The Power-flow Equations

The starting point for a powerflow solution is a single-line diagram of the power system, from which the input data for computer solutions can be obtained. Input data consists of bus data, transmission line data, transformer data, generator data and load data. The following four variables are associated with bus i as shown in Figure 2.1: voltage magnitude V_i, phase angle δ_i, net real power P_i, and reactive power Q_i supplied to the bus [2].

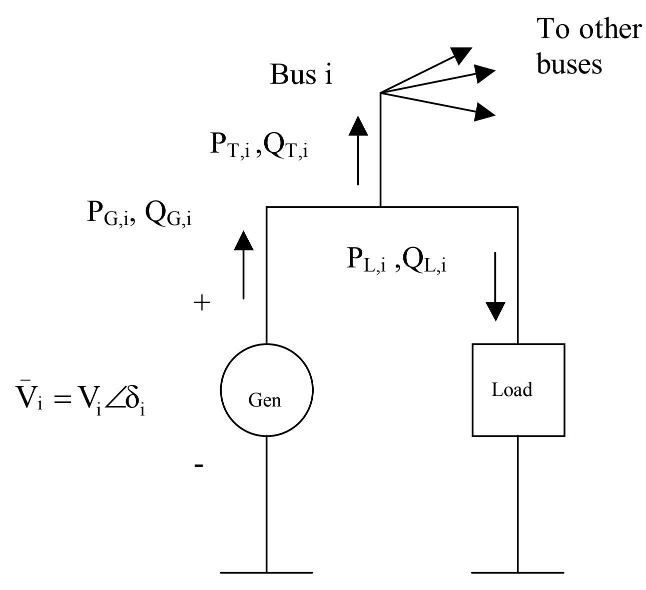

Figure 2.1: Bus i

At each bus, two of these variables are specified as input data, and the other two are unknowns to be computed by the powerflow program. Based on Kirchhoff's current law, real and reactive power flow equations at bus i can be written as shown in (2.1).

$$P_{G,i} = P_{L,i} + P_{T,i}$$

$$Q_{G,i} = Q_{L,i} + Q_{T,i} \tag{2.1}$$

In (2.1), subscript "G" represents generator, "L" represents load and "T" represents transmission network. P_T and Q_T are the real and the reactive power respectively, flowing from bus i into the transmission network. These powers (both real and reactive) are expressed in terms of the voltage variables and the network impedances as shown in (2.2).

$$P_{T,i} = \sum_{j=1}^{N} V_i V_j Y_{ij} \cos(\delta_i - \delta_j - \gamma_{ij}) \quad i=1, 2, 3\ldots\ldots N$$

$$Q_{T,i} = \sum_{j=1}^{N} V_i V_j Y_{ij} \sin(\delta_i - \delta_j - \gamma_{ij}) \quad i=1, 2, 3\ldots\ldots.N \tag{2.2}$$

In (2.2), N represents the number of buses in the system, and Y_{ij} represents the admittance connected between bus i and bus j and γ_{ij} represents the angle on that admittance. The net real and reactive power-flow at each bus i as shown in (2.3) is obtained by substituting (2.2) in (2.1).

$$P_{G,i} - P_{L,i} - \sum_{j=1}^{N} V_i V_j Y_{ij} \cos(\delta_i - \delta_j - \gamma_{ij}) = P_i(\tilde{V}, \tilde{\delta}) = P_i = 0$$

$$Q_{G,i} - Q_{L,i} - \sum_{j=1}^{N} V_i V_j Y_{ij} \sin(\delta_i - \delta_j - \gamma_{ij}) = Q_i(\tilde{V}, \tilde{\delta}) = Q_i = 0 \tag{2.3}$$

These are nonlinear algebraic equations. For an N bus system, there will be 2N non-linear equations with real coefficients. These equations are solved using the Newton-Raphson method (abbreviated as the NR method). For a large power system with many unknown variables (voltages and angles), the NR method has the form shown in (2.4).

$$\begin{bmatrix}\delta_1\\ \cdots\\ \delta_N\\ V_1\\ \cdots\\ V_N\end{bmatrix}_{new} = \begin{bmatrix}\delta_1\\ \cdots\\ \delta_N\\ V_1\\ \cdots\\ V_N\end{bmatrix}_{old} + \begin{bmatrix}\frac{\partial P_1}{\partial \delta_1} & \cdots & \frac{\partial P_1}{\partial \delta_N} & \frac{\partial P_1}{\partial V_1} & \cdots & \frac{\partial P_1}{\partial V_N}\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ \frac{\partial P_N}{\partial \delta_1} & \cdots & \frac{\partial P_N}{\partial \delta_N} & \frac{\partial P_N}{\partial V_1} & \cdots & \frac{\partial P_N}{\partial V_N}\\ \frac{\partial Q_1}{\partial \delta_1} & \cdots & \frac{\partial Q_1}{\partial \delta_N} & \frac{\partial Q_1}{\partial V_1} & \cdots & \frac{\partial Q_1}{\partial V_N}\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ \frac{\partial Q_N}{\partial \delta_1} & \cdots & \frac{\partial Q_N}{\partial \delta_N} & \frac{\partial Q_N}{\partial V_1} & \cdots & \frac{\partial Q_N}{\partial V_N}\end{bmatrix}^{-1}_{\substack{\tilde{V}=\tilde{V}_{old}\\ \tilde{\delta}=\tilde{\delta}_{old}}} \begin{bmatrix}-P_1\left(\tilde{V}_{old},\tilde{\delta}_{old}\right)\\ \cdots\\ -P_N\left(\tilde{V}_{old},\tilde{\delta}_{old}\right)\\ -Q_1\\ \cdots\\ -Q_N\left(\tilde{V}_{old},\tilde{\delta}_{old}\right)\end{bmatrix} \quad (2.4)$$

In (2.4), a vector quantity is represented by the symbol " ~" on the top. The values of real and reactive power in (2.4) are found using current values of V and δ. Equation (2.4) is solved iteratively to get final values of voltages and angles. But, before using (2.4), two modifications are made for slack and PV buses in the Jacobian matrix.

Every system has a "slack bus." The purpose of this bus is to serve as a reference for both voltage magnitudes and their angles. This bus must be a bus where a generator is connected. In per-unit, the phasor voltage at the slack bus is usually specified as $1.0\angle 0^o$ but may have different magnitudes depending on the nature of the study being conducted. In any case, both the voltage magnitude and angle are known at the slack bus. As two of the unknowns in (2.4) are now known, rows and columns corresponding to the voltage and the angle at the slack bus can be eliminated from (2.4). Mathematically, these rows and columns can be simply neglected.

Any generator bus (known as a PV bus) other than the slack bus is characterized by a known voltage magnitude and real power generation. The voltage angle at a PV bus is not known. Therefore for a PV bus, only the row and the column corresponding to the voltage magnitude are eliminated from (2.4). Mathematically, this row and column can be simply neglected.

Stopping Criteria for the NR Solution

After the modifications for slack and PV buses are made, (2.4) is solved iteratively to get the final voltage magnitudes and the angles at all buses. The iterative process to determine successively better approximations to the solution for all "unknown" voltage magnitudes and angles uses a mechanism to determine when the approximate solution is close enough. This mechanism is called a "stopping criteria" and is based on the mismatch vector, which is the column vector of real and reactive powers in (2.4). After each iteration, the absolute value of every entry in the mismatch vector is compared with some tolerance specified for convergence. The tolerance of 0.001 is typically used.

Assumptions for the Powerflow Program

The powerflow program built to establish initial conditions for the small signal stability studies is based on the specific set of assumptions as follows:

1. The program considers capacitor banks to be present (or absent) right from the start of the iterations.
2. The effects of phase shifting transformers and load tap changers, if any, are not taken into account.

The values obtained from the power flow program are used as initial conditions in dynamic stability studies of multimachine systems. For testing the program, the standard IEEE 118 bus [3] and 9 bus systems [4] are used. After establishing all initial conditions such as bus voltages and bus angles using the powerflow program, the reduced mathematical model for the system can be formed which includes generator nodes only.

CHAPTER III

REDUCED NETWORK MODEL

The classical model of a synchronous machine is used to study the stability of a power system for a period of time of one second or less because during this period the system dynamic response is dependent largely on stored kinetic energy in rotating masses. The classical model is the simplest model used in studies of power system dynamics and requires a minimum amount of data. Therefore, studies can be conducted in a relatively short time and at minimum cost. Studies based on simplified models can provide useful information and are often used as preliminary studies to identify problem areas that require further study with more detailed modeling.

Classical Representation of a Synchronous Machine in Stability Studies

The generator model shown in Figure 3.1 [4] is a common "voltage behind a reactance" model of a synchronous machine. The internal voltage magnitude E is controlled by machine speed and excitation. Because of inertia, speed changes are minimal and also exciter response is limited during one second. During this one second period, therefore, the internal voltage magnitude remains constant. The reactance X_d' is the direct axis transient reactance.

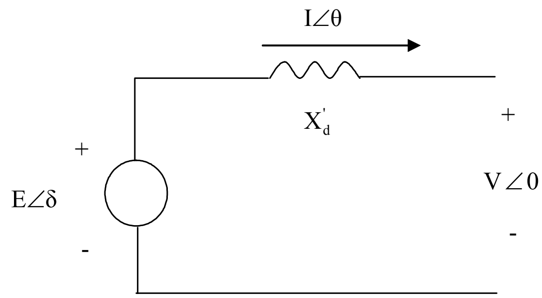

Figure 3.1: Representation of a Synchronous Machine by the Classical Model

This value is used when the machine has just experienced a disturbance. Usually, this value is applicable from 3 cycles to 1 second. The reduced mathematical model build is based on this machine model and the specific set of assumptions discussed in the next section.

Assumptions used for building Mathematical Model of a Power System [4]

In developing equations or a mathematical model of a multimachine power system, the following assumptions were made:

1. Mechanical power input P_m is constant;
2. Machine internal voltage E shown in Figure 3.1 (voltage behind the transient reactance) is constant;
3. The mechanical rotor angle of a machine coincides with the angle of the voltage behind the transient reactance;
4. X_d', is the direct axis transient reactance; and
5. Loads are represented by passive impedance.

Assumption 1 would be unrealistic if the time period of disturbance is considered greater than one second. Assumption 5 is made for convenience in stability studies. Loads have their own dynamic behavior, which is usually not precisely known and varies from constant impedance to constant power. The model formed using these assumptions is useful for stability analysis but is limited to the study of disturbances for only the "first swing" or for periods on the order of one second [4]. Based on these assumptions, a complete mathematical model is constructed as described in the next section.

Mathematical Model of Multimachine system

An electrical network as shown in Figure 3.2 has n nodes with active sources and r nodes where there are no generators.

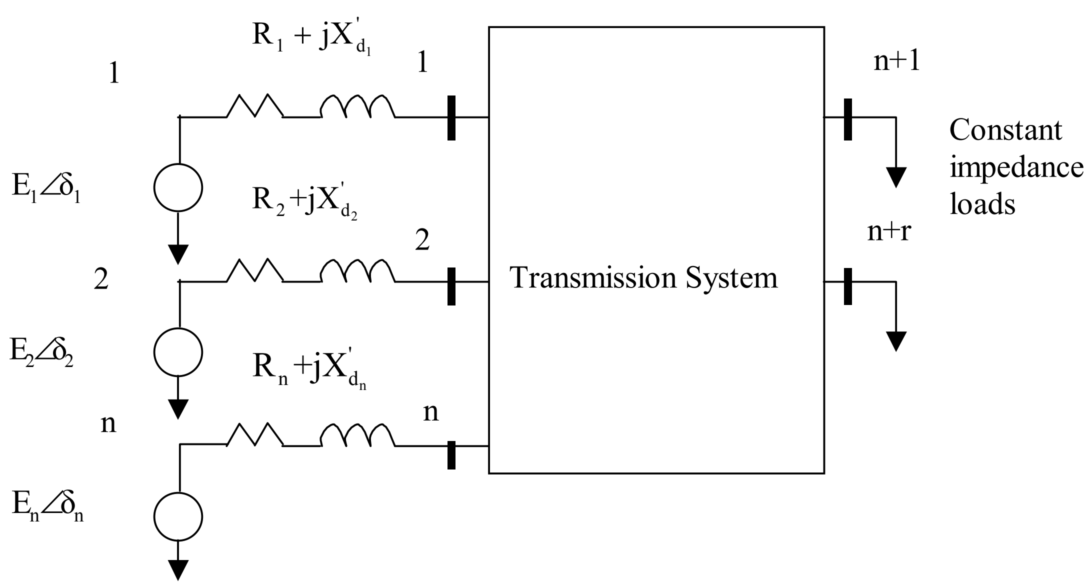

Figure 3.2: Representation of a Multimachine System (Classical Model)

The admittance matrix of the transmission network looking into the network from the generator internal nodes (1, 2…n) is given by (3.1). The current and the voltage vector ($\bar{E}$) are of dimension ((n + r) * 1) and the admittance matrix is of dimension ((n + r) * (n + r)).

$$\tilde{\bar{I}} = [\bar{Y}]\tilde{\bar{E}} \qquad (3.1)$$

The admittance matrix $\bar{Y}$ shown in (3.1) is constructed using the following steps:

1. The diagonal entries $\bar{Y}_{ii}$ of the admittance matrix are the sum of all the admittances connected to node i.

2. The off-diagonal entries $\bar{Y}_{ij}$ of the admittance matrix are the sum of the negatives of all admittances between node i and node j.

3. Additional nodes are provided for the internal generator voltages (1, 2, 3… n) and the appropriate values for $X_d^{'}$ are connected between these nodes and generator terminal nodes.

The admittance matrix built using these steps is a complete mathematical model except that the loads, which are generally given in P+jQ form, are yet to be included. These load powers are converted into equivalent passive impedances as discussed in the next section and are then included in the admittance matrix.

Representing Loads by Equivalent Admittances

For a bus having a voltage $\bar{V}_L$ to which a load $P_L + jQ_L$ is connected, the load is represented by the static admittances G_L and B_L as shown in Figure 3.3 [4]. The needed data for the buses are obtained from the power-flow study (discussed in Chapter II).

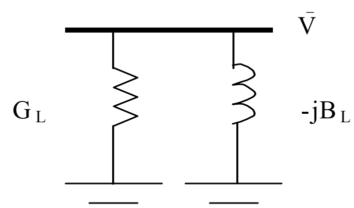

Figure 3.3: A Load Represented by Passive Impedance

The equivalent shunt admittance at this bus is given by (3.2).

$$\overline{Y_L} = \frac{P_L}{V_L^2} - j\frac{Q_L}{V_L^2} \tag{3.2}$$

$$\begin{bmatrix} \bar{I}_1 \\ \vdots \\ \bar{I}_n \\ \bar{I}_{n+1} \\ \vdots \\ \bar{I}_{n+r} \end{bmatrix} = \begin{bmatrix} \bar{Y}_{1,1} & \cdots & \bar{Y}_{1,n} & \bar{Y}_{1,n+1} & \cdots & \bar{Y}_{1,n+r} \\ \vdots & \ddots & \cdots & \cdots & \cdots & \vdots \\ \bar{Y}_{n,1} & \cdots & \bar{Y}_{n,n} & \bar{Y}_{n,n+1} & \cdots & \bar{Y}_{n,n+r} \\ \bar{Y}_{n+1,1} & \cdots & \bar{Y}_{n+1,n} & \bar{Y}_{n+1,n+1} & \cdots & \bar{Y}_{n+1,n+r} \\ \vdots & \cdots & \cdots & \cdots & \ddots & \vdots \\ \bar{Y}_{n+r,1} & \cdots & \bar{Y}_{n+r,n} & \bar{Y}_{n+r,n+1} & \cdots & \bar{Y}_{n+r,n+r} \end{bmatrix} \begin{bmatrix} \bar{V}_1 \\ \vdots \\ \bar{V}_n \\ \bar{V}_{n+1} \\ \vdots \\ \bar{V}_{n+r} \end{bmatrix} \tag{3.3}$$

The equivalent shunt admittances for each load of the system are added to their corresponding diagonal entries of the admittance matrix specified in (3.1). The detailed admittance matrix model after including the loads is given in (3.3).

Reducing the System Model

Careful evaluation of (3.3) reveals that there are only n nodes with machines connected and the remaining r nodes have no active sources connected to them. All the nodes except for the internal generator nodes are eliminated and the admittance matrix reduced from the dimension ((n + r)*(n + r)) to the dimension (n * n). This reduction (Kron's reduction) is done by matrix operations considering the fact that all the nodes have zero injection currents except for the generator internal nodes. This property is used to obtain the network reduction. After re-ordering the nodes such that those with machines connected to them are given numbers 1-n and all the other nodes (without machines) are numbered n+1-r, equation (3.3) can be written as (3.4).

$$\begin{bmatrix} \tilde{\bar{I}}_n \\ \hline \tilde{\bar{I}}_r \end{bmatrix} = \left[\begin{array}{c|c} [\bar{Y}_{nn}] & [\bar{Y}_{nr}] \\ \hline [\bar{Y}_{rn}] & [\bar{Y}_{rr}] \end{array}\right] \begin{bmatrix} \tilde{\bar{V}}_n \\ \hline \tilde{\bar{V}}_r \end{bmatrix} \tag{3.4}$$

In (3.4), the subscript n is used to denote generator nodes and the subscript r is used to denote the remaining nodes. Because there are no generators connected to the remaining r nodes, the current injection from these nodes into the network is zero and so $\bar{I}_r$ (which is an (r x1) vector) is zero. Equations (3.5) and (3.6) can be obtained by expanding (3.4).

$$\tilde{\bar{I}}_n = [Y_{nn}]\tilde{\bar{V}}_n + [Y_{nr}]\tilde{\bar{V}}_r \tag{3.5}$$

$$0 = [Y_{rn}]\tilde{\bar{V}}_n + [Y_{rr}]\tilde{\bar{V}}_r \tag{3.6}$$

Substituting $\tilde{\bar{V}}_r$ of (3.6) in (3.5) and solving for $\tilde{\bar{I}}_n$ gives (3.7).

$$\tilde{\bar{I}}_n = ([\bar{Y}_{nn}] - [\bar{Y}_{nr}][\bar{Y}_{rr}]^{-1}[\bar{Y}_{rn}])\tilde{\bar{V}}_n \quad (3.7)$$

The matrix $([\bar{Y}_{nn}] - [\bar{Y}_{nr}][\bar{Y}_{rr}]^{-1}[\bar{Y}_{rn}])$ is the desired reduced matrix $[\bar{Y}]$. It has the dimensions (n x n) where n is the number of generators. The Kron reduction discussed in this section is only possible if loads are treated as constant impedance; otherwise the identity of each load bus must be retained.

The reduced $[\bar{Y}]$ matrix defined by (3.7) is a complete mathematical model of the power system that includes machines, transmission elements and loads. The next step after obtaining the reduced admittance matrix is to formulate equations of motion for each of the generators.

Equations of Motion for the Multimachine System

The electrical power injected into the network at node i, which is the electrical power output of machine i, is given by (3.8).

$$P_{e,i} = \mathrm{Real}\left(\bar{E}\bar{I}^*\right) \quad (3.8)$$

Equation (3.9) is obtained by substituting (3.1) in (3.8).

$$P_{e,i} = \mathrm{Real}\left\{\bar{E}_i \sum_{j=1}^{n} \bar{Y}_{ij}^{*} \bar{E}_j^{*}\right\} \quad (3.9)$$

In (3.9), n is the number of generators. Expanding and taking the real part of (3.9) gives (3.10), the electrical power output of the generator i.

$$P_{e,i} = \sum_{j=1}^{n} E_i E_j Y_{ij} \cos\left(\delta_i - \delta_j - \gamma_{ij}\right) \qquad i=1,2,3........n \quad (3.10)$$

The equation of motion (corresponding to the classical swing equation) is given in (3.11).

$$\frac{d\omega_i}{dt} = \frac{\pi f}{H_i}\left\{P_{m,i} - P_{e,i}\right\}$$
$$\frac{d\delta_i}{dt} = \omega_i \tag{3.11}$$

In (3.11),

H_i is the inertia constant in seconds;

f is the frequency in hertz and is constant (not affected by changes in ω);

$P_{m,\,i}$ is the mechanical power input for machine i;

$P_{e,\,i}$ is the electrical power output of machine i; and

ω_i is the relative angular velocity of the rotor of machine i.

Combining (3.10) and (3.11) gives (3.12). Equation (3.12) is a complete set of equations of motion for machine i.

$$\frac{d\omega_i}{dt} = \frac{\pi f}{H}\left\{P_{m,i} - \sum_{j=1}^{n} E_i E_j Y_{ij} \cos\left(\delta_i - \delta_j - \gamma_{ij}\right)\right\} \tag{3.12.1}$$

$$\frac{d\delta_i}{dt} = \omega_i \tag{3.12.2}$$

Among (3.12), (3.12.2) is a nonlinear first order differential equation and (3.12.2) is a linear first order differential equation. Equation (3.12) can be expressed in the form of (3.13).

$$\dot{\tilde{x}} = \tilde{f}\left(\tilde{x}, \tilde{u}, t\right) \tag{3.13}$$

In (3.13) $\tilde{f}$ is a set of nonlinear functions of the elements of the state vector $\tilde{x}$ which is a vector of dimension (2n x 1) and is given by (3.14).

$$x^t = [\omega_1, \delta_1, \omega_2, \delta_2, \ldots\ldots\ldots, \omega_n, \delta_n] \quad (3.14)$$

Calculating Machine Internal Voltages (E) and Angles (δ)

The internal voltage E of a generator and the machine angle δ may be computed from the pre-transient terminal voltage V and the terminal voltage angle α. To make calculations easier, the terminal voltage is temporarily used as reference as shown in Figure 3.4 [4].

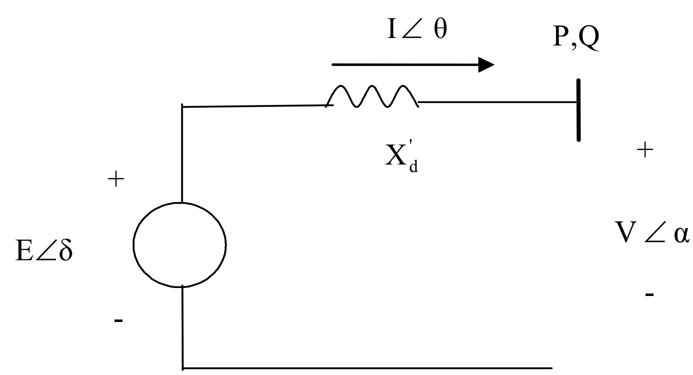

Figure 3.4: Generator Representation for Calculating δ

The complex power and current flowing in the circuit shown in Figure 3.4 is given by (3.15) and (3.16) respectively (again terminal voltage is used here as a reference).

$$P + jQ = \bar{V}\,\bar{I} \quad (3.15)$$

$$I_1 + jI_2 = (P - jQ)/V \tag{3.16}$$

Applying Kirchhoff's voltage law in Figure 3.4 gives (3.17) and substituting (3.16) in (3.17) gives (3.18).

$$E\angle\delta' = \overline{V} + jX'_d\bar{I} \tag{3.17}$$

$$E\angle\delta' = \left(V + QX'_d/V\right) + j\left(PX'_d/V\right) \tag{3.18}$$

The initial generator angle δ_o is then obtained by returning the reference back to system slack machine as shown in (3.19).

$$\delta_o = \delta' + \alpha \tag{3.19}$$

After obtaining the values of reduced admittance matrix given by (3.3) and the machine internal voltages and angles using (3.18) and (3.19), (3.12) must be linearized. Once (3.12) is linearized, a state space model can be constructed and eigen techniques can be employed to evaluate stability conditions.

CHAPTER IV

LINEARIZATION

The set of equations derived using the classical model and given in (3.12) are n coupled nonlinear first order differential equations. They are linearized about some operating point before building the state space model. The method of analysis used to linearize the differential equations describing the system behavior is to assume small changes in δ and ω (which are the state variables here) and then solving for that small perturbation. Equations for these variables are found using first order approximation method or method of small changes, sometimes called the perturbation method about some quiescent operating point with higher order terms neglected. This means that a first-order approximation is made for the system equations. The new linear equations thus derived are assumed to be valid in a region near the quiescent condition.

Once the non-linear coupled first order differential equations given in (3.12.1) are linearized, a state space model can be constructed which is in the form given in (4.1).

$$\dot{\tilde{x}} = [A]\tilde{x} + [B]\tilde{u} \tag{4.1}$$

The dynamic response of a linear system defined by (4.1) is determined by its characteristic equation given in (4.2).

$$\det\{(s[I]-[A])\} = 0 \tag{4.2}$$

The free response of the system is determined by the roots of (4.2). From a point of view of small signal stability, the free response (which means there is no input or the disturbances causing the changes disappear) gives all information regarding the dynamic performance of the system. Before applying the linearization technique to a large multimachine system, it is first demonstrated on a single machine infinite bus system in the following section.

Single Machine Infinite Bus System

The equations of motion for a single machine infinite bus (SMIB) situation as shown in Figure 3.1 are given in (4.3).

$$\frac{d\omega}{dt} = \frac{\pi f}{H}\left\{P_m - \frac{EV}{X}\sin\delta\right\} \tag{4.3.1}$$

$$\frac{d\delta}{dt} = \omega \tag{4.3.2}$$

In (4.3),

E is the machine internal voltage;

V is the voltage for the infinite bus;

X is the total reactance between E and V;

P_m is the mechanical power input to the machine;

δ is the rotor angle;

f is the frequency in hertz and is constant regardless of any changes in ω; and

H is the inertia constant of the machine expressed in seconds.

Equation (4.3.1) has a trigonometric nonlinearity, which is removed by making first order approximations or by applying a Taylor series expansion. Suppose there is a small disturbance (perturbation) in the system and the change in machine angle is $\bar{\delta}$. The new machine angle after the disturbance is given by $\delta = \delta_o + \bar{\delta}$. Using similar notation for ω, the new equations of motion described in (4.3) are given by (4.4).

$$\frac{d\delta_o}{dt} + \frac{d\bar{\delta}}{dt} = \omega_o + \bar{\omega} \tag{4.4.1}$$

$$\frac{d\omega_o}{dt} + \frac{d\bar{\omega}}{dt} = \frac{\pi f}{H}\left(Pm - \frac{VE}{X}\sin(\delta_o + \bar{\delta})\right) \tag{4.4.2}$$

In (4.4), the change in quantities is represented with the symbol "bar" on top and the subscript "o" represents the initial values of those quantities. In (4.4), it is assumed that before the disturbance the initial quantities on the right hand side (R.H.S) are exactly equal to the initial quantities on the left hand side (L.H.S.). Using the trigonometric identity $\sin\left(\delta_o + \bar{\delta}\right) = \sin\delta_o \cos\bar{\delta} + \cos\delta_o \sin\bar{\delta}$ and assuming $\bar{\delta}$ is so small such that $\cos\bar{\delta} \to 1$ and $\sin\bar{\delta} \to \bar{\delta}$, equation (4.4.2) can be rewritten as given in (4.5)

$$\frac{d\omega_o}{dt} + \frac{d\bar{\omega}}{dt} = \frac{\pi f}{H}\left(P_m - \frac{VE}{X}\left(\sin\delta_o + \cos\delta_o \bar{\delta}\right)\right) \tag{4.5}$$

Using the assumption that before the disturbance the initial quantities on the R.H.S are exactly equal to initial quantities on the L.H.S., (4.5) can be written as (4.6).

$$\frac{d\bar{\omega}}{dt} = -\frac{\pi f}{H}\left(\frac{VE}{X}\cos\delta_o \bar{\delta}\right) \tag{4.6}$$

Equation (4.6) is a linear equation because the state variable (ω) can be expressed explicitly and linearly in terms of the other state variable (δ). To verify that this method is a first order approximation, a Taylor series expansion was performed on (4.3.1). The details of this expansion are shown in the following section.

Linearization using Taylor Series

The Taylor series expansion of any function is given by (4.7).

$$f(\delta) = a_o + a_1(\delta - \delta_o) + a_2(\delta - \delta_o)^2 + \dots\dots\dots + a_n(\delta - \delta_o)^n \qquad (4.7)$$

In (4.7), δ_o is the value about which the linearization is to be performed and the coefficients a_o, $a_1 \dots a_n$ are defined by (4.8).

$$\begin{aligned} a_o &= f(\delta_o) \\ a_1 &= \left\{\frac{df}{d\delta}\right\}\Bigg|_{\delta=\delta_o} \\ a_2 &= \left\{\frac{d^2f}{d\delta^2}\right\}\Bigg|_{\delta=\delta_o} \\ &\vdots \\ a_n &= \left\{\frac{d^nf}{d\delta^n}\right\}\Bigg|_{\delta=\delta_o} \end{aligned} \qquad (4.8)$$

By forcing the coefficients a_2-a_n to zero, first order approximation is used for linearization .The other coefficients a_0 and a_1 described in (4.8) are calculated for (4.3.1) and are given in (4.9).

$$a_o = f(\delta_o) = \frac{\pi f}{H}\left\{P_m - \frac{EV}{X}\sin\delta_o\right\}$$
$$a_1 = \left\{\frac{df}{d\delta}\right\}\Bigg|_{\delta=\delta_o} \Rightarrow -\frac{\pi f}{H}\left\{\frac{EV}{X}\right\}\cos\delta_o \tag{4.9}$$

Equation (4.10) can be obtained by substituting (4.9) in (4.7).

$$f(\delta) = \underbrace{\frac{\pi f}{H}\left\{P_m - \frac{EV}{X}\sin\delta_o\right\}}_{K_1} - \underbrace{\frac{\pi f}{H}\left\{\frac{EV}{X}\right\}\cos\delta_o}_{K_2}(\delta - \delta_o) \tag{4.10}$$

Comparing (4.1) and (4.10) reveals that K_1 is nothing but the [B] term that is the input term and can be discarded because stability of machine is controlled by K_2 which is the [A] term in (4.1). Also, K_1 in (4.10) is zero because during steady state a machine's mechanical input and developed electrical power are equal. In (4.10), $(\delta-\delta_o)$ is nothing but change in the machine angle, which is $\bar{\delta}$ (by convention used for change in machine angle). Equation (4.10) can be written as (4.11) by discarding the input terms and by replacing $(\delta-\delta_o)$ with $\bar{\delta}$.

$$f(\delta) = \frac{d\omega}{dt} = -\frac{\pi f}{H}\left\{\frac{EV}{X}\right\}\cos\delta_o\,\bar{\delta} \tag{4.11}$$

Thus both (4.6) and (4.11) are exactly the same and neither have a trigonometric nonlinearity. The method used for deducing both of the results is a first order approximation.

Linearizing the Multimachine Equations of Motion

The equations of motion (3.8) developed in Chapter III are given by (4.12).

$$\frac{d\omega_i}{dt} = \frac{\pi f}{H}\left\{P_{m,i} - \sum_{j=1}^{n} E_i E_j Y_{ij}\cos\left(\delta_i - \delta_j - \gamma_{ij}\right)\right\}$$
$$\frac{d\delta_i}{dt} = \omega_i \tag{4.12}$$

Assuming perturbation values are very small, (4.12) can be solved for these small values of disturbances. Suppose there is a small disturbance in the system and the value of machine angle changes from δ_o to $\bar{\delta}$. The new machine angle is given by $\delta = \delta_o + \bar{\delta}$ and the new equations of motion are given by (4.13).

$$\frac{d\delta_{i,o}}{dt} + \frac{d\bar{\delta}_i}{dt} = \omega_{i,o} + \bar{\omega}_i \tag{4.13.1}$$

$$\frac{d\omega_{i,o}}{dt} + \frac{d\bar{\omega}_i}{dt} = \frac{\pi f}{H}Pm_i - \frac{\pi f}{H}\sum_{j=1}^{n} E_i E_j Y_{ij}\cos\left(\delta_{i,o} - \delta_{j,o} - \gamma_{ij} + \bar{\delta}_i - \bar{\delta}_j\right) \tag{4.13.2}$$

Before the disturbance the initial values on the R.H.S are exactly equal to quantities on the L.H.S; therefore (4.13.1) can be written as (4.14.1). Expanding (4.13.2) gives (4.14.2).

$$\frac{d\bar{\delta}_i}{dt} = \bar{\omega}_i \tag{4.14.1}$$

$$\frac{d\omega_{i,o}}{dt} + \frac{d\bar{\omega}_i}{dt} = \frac{\pi f}{H}P_{m,i} - \frac{\pi f}{H}\left(\sum_{j=1}^{n} E_i E_j Y_{ij}\cos\left(\delta_{i,o} - \delta_{j,o} - \gamma_{ij}\right)\cos\left(\bar{\delta}_i - \bar{\delta}_j\right) - \sin\left(\delta_{i,o} - \delta_{j,o} - \gamma_{ij}\right)\sin\left(\bar{\delta}_i - \bar{\delta}_j\right)\right) \tag{4.14.2}$$

Assuming the disturbance is small such that the trigonometric identities given in (4.15) can be used, (4.16) can be obtained by substituting (4.15) in (4.14.2).

$$\begin{aligned} &\left(\bar{\delta}_i - \bar{\delta}_j\right) \to 0 \\ &\sin\left(\bar{\delta}_i - \bar{\delta}_j\right) \to \bar{\delta}_i - \bar{\delta}_j \\ &\cos\left(\bar{\delta}_i - \bar{\delta}_j\right) \to 1 \end{aligned} \tag{4.15}$$

$$\frac{d\bar{\omega}_i}{dt} = \frac{\pi f}{H_i}\left(\sum_{j=1}^{n} E_i E_j Y_{ij} \sin\left(\delta_{i,o} - \delta_{j,o} - \gamma_{ij}\right)\left(\bar{\delta}_i - \bar{\delta}_j\right)\right) \tag{4.16}$$

The complete linearized set of equations of motion is shown in (4.17).

$$\frac{d\bar{\omega}_i}{dt} = \frac{\pi f}{H_i}\left(\sum_{j=1}^{n} E_i E_j Y_{ij} \sin\left(\delta_{i,o} - \delta_{j,o} - \gamma_{ij}\right)\left(\bar{\delta}_i - \bar{\delta}_j\right)\right) \tag{4.17.1}$$

$$\frac{d\bar{\delta}_i}{dt} = \bar{\omega}_i \tag{4.17.2}$$

The new linear equations (4.17) derived are assumed to be valid in a region near the quiescent operating point, which is the pre-disturbance steady state operating condition.

The Unregulated Multimachine System

The set of equation (4.17) gives a complete set of equations of motion of a machine and can be applied to all the machines connected in a power system. This model is an unregulated model because effects of the governors, the power system stabilizers (PSS) and field excitation systems are not taken in account. Equation (4.17) applies to any number of nodes where voltages are known and it is a linearized set of equations. However, this set of equations is not a set of equations of 2n independent equations.

Because for the same values of i and j, $\left(\bar{\delta}_i - \bar{\delta}_j\right)$ in (4.17.1) is zero. Hence, (4.17) reduces to a set of 2n-1 independent equations [4].

Equation (4.17.1) can be written as (4.18) where P_{ij} is defined by (4.19).

$$\frac{d\bar{\omega}_i}{dt} = \frac{\pi f}{H_i}\left(\sum_{j=1}^{n} P_{ij}\left(\bar{\delta}_i - \bar{\delta}_j\right)\right) \qquad i = 1, 2, 3 \ldots n \tag{4.18}$$

$$P_{ij} = E_i E_j Y_{ij} \sin\left(\delta_{i,o} - \delta_{j,o} - \gamma_{ij}\right) \tag{4.19}$$

The complete set of linearized equations of motion for machine n is given by (4.20).

$$\frac{d\bar{\omega}_n}{dt} = \frac{\pi f}{H_n}\left(\sum_{j=1}^{n} P_{nj}\left(\bar{\delta}_n - \bar{\delta}_j\right)\right) \tag{4.20.1}$$

$$\frac{d\bar{\delta}_n}{dt} = \bar{\omega}_n \tag{4.20.2}$$

Subtracting the n^{th} machine equations (4.20) from the i^{th} machine equations (4.18) gives (4.21)

$$\frac{d\bar{\omega}_i}{dt} - \frac{d\bar{\omega}_n}{dt} = \frac{\pi f}{H_i}\left(\sum_{j=1}^{n} P_{ij}\left(\bar{\delta}_i - \bar{\delta}_j\right)\right) - \frac{\pi f}{H_n}\left(\sum_{j=1}^{n} P_{nj}\left(\bar{\delta}_n - \bar{\delta}_j\right)\right) \tag{4.21.1}$$

$$\frac{d\bar{\delta}_i}{dt} - \frac{d\bar{\delta}_n}{dt} = \bar{\omega}_i - \bar{\omega}_n \tag{4.21.2}$$

The set of equations (4.21) can be rewritten as shown in (4.22).

$$\frac{d\bar{\omega}_{in}}{dt} = \frac{\pi f}{H_i}\left(\sum_{j=1}^{n} P_{ij}\left(\bar{\delta}_i - \bar{\delta}_j\right)\right) - \frac{\pi f}{H_n}\left(\sum_{j=1}^{n} P_{nj}\left(\bar{\delta}_n - \bar{\delta}_j\right)\right) \tag{4.22.1}$$

$$\frac{d\bar{\delta}_{in}}{dt} = \bar{\omega}_{in} \tag{4.22.2}$$

In (4.22), $\bar{\omega}_{in}$ and $\bar{\delta}_{in}$ are given by $(\bar{\omega}_i - \bar{\omega}_n)$ and $(\bar{\delta}_i - \bar{\delta}_n)$ respectively. Because $\bar{\delta}_i$ and $\bar{\delta}_j$ are changes in angles with respect to the n[th] machine, $(\bar{\delta}_i - \bar{\delta}_j)$ can be defined by (4.23). Using (4.23), (4.22.1) can be modified as shown in (4.24).

$$(\bar{\delta}_i - \bar{\delta}_j) = (\bar{\delta}_i - \bar{\delta}_n) - (\bar{\delta}_j - \bar{\delta}_n) \tag{4.23}$$

$$\frac{d\bar{\omega}_{in}}{dt} = \left(\sum_{j=1}^{n-1} \Lambda_{ij} \left(\bar{\delta}_j - \bar{\delta}_n \right) \right) \tag{4.24}$$

In (4.24), the summation is done to (n-1) only because the equation of the n[th] machine is already subtracted from the equation of the i[th] machine. The coefficients Λ_{ij} depend on the machine inertias and the power coefficients given by (4.19) and are defined by (4.25).

$$\text{For } i = j, \quad \Lambda_{ij} = \frac{\pi f}{H_n} P_{ni} + \sum_{j=1, i \neq j}^{n} \frac{\pi f}{H_i} \left(P_{ij} \right) \quad i=1, 2, 3\ldots n-1. \tag{4.25.1}$$

$$i \neq j, \quad \Lambda_{ij} = \frac{\pi f}{H_n} P_{ni} - \frac{\pi f}{H_i} P_{ij} \quad i=1, 2, 3...n-1. \tag{4.25.2}$$

Equation (4.24) represents (n-1) first order differential equations and the complete set of equations of motion for the unregulated multimachine model is given by (4.26)

$$\frac{d\bar{\delta}_{in}}{dt} = \bar{\omega}_{in} \tag{4.26.1}$$

$$\frac{d\bar{\omega}_{in}}{dt} = \left(\sum_{j=1}^{n-1} \Lambda_{ij} \left(\bar{\delta}_j - \bar{\delta}_n \right) \right) \tag{4.26.2}$$

The system equations (4.26.1) and (4.26.2) can be written in state space model form as shown in (4.27). In (4.27), $\bar{\omega}_{in}$ and $\bar{\delta}_{in}$ are changes in angle and speed of i^{th} machine with respect to the n^{th} machine.

$$\underbrace{\begin{bmatrix} \frac{d\delta_{1n}}{dt} \\ \frac{d\delta_{2n}}{dt} \\ \cdots\cdots \\ \frac{d\delta_{(n-1)n}}{dt} \\ --- \\ \frac{d\omega_{1n}}{dt} \\ \frac{d\omega_{2n}}{dt} \\ \cdots\cdots \\ \frac{d\omega_{(n-1)n}}{dt} \end{bmatrix}}_{\frac{dx}{dt}} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & | & 1 & 0 & 0 \\ 0 & 0 & 0 & | & 0 & 1 & 0 \\ 0 & 0 & 0 & | & 0 & 0 & 1 \\ - & - & - & | & - & - & - \\ \Lambda_{11} & \Lambda_{12} & \Lambda_{1,(n-1)} & | & 0 & 0 & 0 \\ \Lambda_{21} & \Lambda_{22} & \Lambda_{2,(n-1)} & | & 0 & 0 & 0 \\ \Lambda_{(n-1),1} & \Lambda_{(n-1),2} & \Lambda_{(n-1),(n-1)} & | & 0 & 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \delta_{1n} \\ \delta_{2n} \\ \cdots\cdots \\ \delta_{(n-1)n} \\ --- \\ \omega_{1n} \\ \omega_{2n} \\ \cdots\cdots \\ \omega_{(n-1)n} \end{bmatrix}}_{X} \qquad (4.27)$$

This has practical significance because angles and speeds must be physically expressed and measured with respect to defined a point of reference, which is taken as the n^{th} machine in this work. Equation (4.27) is in the form of the linear state space model $\dot{\tilde{x}} = [A]\tilde{x}$. To obtain the free response of the system, eigenvalues of [A] in (4.27) are examined as discussed in the next chapter.

CHAPTER V

EIGENANALYSIS

The dynamic equations governing the performance of the power system are nonlinear. They are linearized about an operating point (i.e. steady state condition) for small signal stability studies. The linearized model may be put in the form of a set of linear, first order differential equations (as discussed in Chapter IV) with constant coefficients having the form as shown in (5.1).

$$\dot{\tilde{x}} = [A]\tilde{x} \tag{5.1}$$

In (5.1),

x is a state vector of dimension 2(n-1);

A is the state matrix of size 2(n-1) x 2(n-1); and

n is the number of generators.

The state equation given by (5.1) is evaluated by taking the Laplace transform [1]. The new equation derived in s domain is given by (5.2).

$$s\tilde{X}(s) - \tilde{X}(0) = [A]\tilde{X}(s) \tag{5.2}$$

Rearranging (5.2) and solving for X(s) gives (5.3).

$$\tilde{X}(s) = (s[I] - [A])^{-1}\tilde{X}(0) \tag{5.3}$$

Equation (5.3) can be rewritten as (5.4) which is the Laplace transform of $\tilde{x}$.

$$\tilde{X}(s) = \frac{\text{adj}(s[I]-[A])}{\det(s[I]-[A])}\tilde{X}(0) \tag{5.4}$$

The poles of the transfer function given by (5.4) are the roots of the equation given by (5.5).

$$\det(s[I]-[A]) = 0 \tag{5.5}$$

The values of s that satisfy (5.5) are known as eigenvalues (modes) of the matrix A. The eigenvalues may be real or complex. If A is real, complex eigenvalues always occur in conjugate pairs.

Eigenvalues and Stability

The time dependent characteristic of a mode corresponding to an eigenvalue λ is given by $e^{\lambda t}$ [1]. Therefore, the stability of the system is determined by the eigenvalues as follows:

1. If all the eigenvalues have negative real parts all modes decay with time and the system is said to be globally stable.
2. If any one eigenvalue has a positive real part, the corresponding mode grows exponentially with time and eventually dominates the system behavior. Such a system is said to be unstable.
3. If any one of the eigenvalues has a real part that is zero, the system will have an undamped oscillatory response.

Eigenvectors

Eigen analysis also includes the computation of eigenvectors, which are often not as well understood as eigenvalues but are very important quantities for analyzing the system. For any eigenvalue λ_i, the column vector φ_i that satisfies (5.6) is called the right eigenvector for λ_i [2].

$$[A]\tilde{\varphi}_i = \lambda_i \tilde{\varphi}_i \qquad i=1, 2, 3\ldots\ldots\ldots n \tag{5.6}$$

The eigenvector φ_i is defined by (5.7).

$$\tilde{\varphi}_i = \begin{bmatrix} \varphi_{1i} \\ \varphi_{2i} \\ \vdots \\ \varphi_{ni} \end{bmatrix} \tag{5.7}$$

The right eigenvector show the distribution of the modes of response (eigenvalues) through the state variables. The right eigenvector also gives the mode shape, i.e., the relative activity of state variables when a particular mode is excited or how the mode shows up in the time behavior of each state variable. Similarly, the row vector ψ_i that satisfies (5.8) is called the left eigenvector of [A] associated with the eigenvalue λ_i.

$$\tilde{\psi}_i[A] = \lambda_i \tilde{\psi}_i \tag{5.8}$$

The left eigenvectors, together with the initial conditions of the system state vector x, determine the magnitudes of the modes [5,6].

Participation Factors

The problem in using eigenvectors for identifying the relationship between the states and the modes is that they are dependent on units and scaling associated with state variables. As a solution to this scaling problem, the participation factor P is defined in (5.9) and (5.10) and is useful in identifying those states which have the most influence on any mode [1]. They are non-dimensional and their use removes all of the problems encountered when using right eigenvectors for deciding the importance of any state.

$$P = \begin{bmatrix} p_1 & p_2 & \cdots & p_n \end{bmatrix} \tag{5.9}$$

$$\tilde{p}_i = \begin{bmatrix} p_{1i} \\ p_{2i} \\ \vdots \\ p_{ni} \end{bmatrix} = \begin{bmatrix} \varphi_{1i}\psi_{i1} \\ \varphi_{2i}\psi_{i2} \\ \vdots \\ \varphi_{ni}\psi_{in} \end{bmatrix} \tag{5.10}$$

The element $p_{ki} = \varphi_{ki}\psi_{ik}$ is called a participation factor. It is a measure of the relative participation of the k^{th} state variable in the i^{th} mode, and vice versa [1]. In general, participation factors are useful in identifying those states which have the most influence on any mode. The higher the value of participation factor of a state for a corresponding mode, the more active is the state in that mode as compared to other states. The participation factor is a very important concept which helps in identifying small-signal stability problems for the large power systems. These problems may be either local or global in nature [2].

Local problems involve a small part of the power system. They are associated with rotor angle oscillations of a single generator or a single plant against the rest of the power system. These types of oscillations are called local mode oscillations. Local problems may also be associated with the oscillations between the rotors of a few generators close to each other. Such oscillations are called interplant oscillations. Global problems involve oscillations of a group of generators in one area swinging against a group of generators in another area. Such oscillations are called interarea oscillations.

The participation factor helps in identifying these problems. The absolute values of participation factors reveal which machines are involved in a particular mode. It also reveals which machine or machines could go out of step for any known mode or modes that might cause problem in the power system. The participation factors are also sometimes used to identify the areas in the power system where any mode or (oscillation) has most of its effect. A detailed analysis of the participation factor concept for two power systems (IEEE 118 bus system [3] and a 9 bus system [4]) and how it helps in identifying small signal stability problems is discussed in Chapter VI.

CHAPTER VI

RESULTS

This Chapter is intended to provide system data for the 9-bus system and IEEE 118 bus system that are used to validate the proposed approaches. Moreover, results of a detailed powerflow studies and eigenanalysis for small signal stability studies using participation factors are also provided. The 9 bus test system [4] is used for verification and then the method developed here in this thesis is applied to the larger IEEE 118 bus system.

Test System 1: Complete Data for System 1

A small signal stability study is presented here on a small nine bus power system. A single line diagram for the system is shown in Figure 6.1[4]. The bus data for this system is given in Table 6.1. This data is the input data for the powerflow analysis. Each bus is assigned a type (as discussed in Chapter II) which corresponds to a slack bus, a PV bus or a load bus. A slack bus is denoted by number '3,' a PV bus is denoted by '2' and a load bus is denoted by '0'(as shown in Table 6.1). The branch and the generator data for this system are given in Tables 6.2 and 6.3 respectively. All the quantities in Tables 6.1 and 6.2 are in p.u. on a 100 MVA base and the system frequency is 60 Hz.

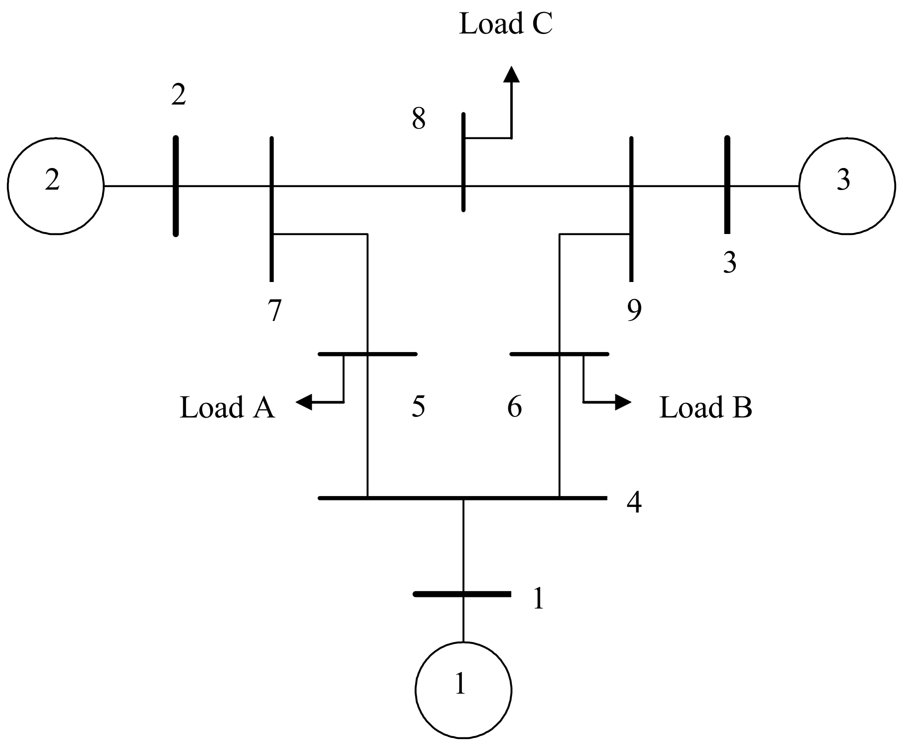

Figure 6.1: 9 Bus System

Table 6.1

BUS DATA FOR 9 BUS SYSTEM

Bus No.	Bus Type	V (p.u)	Angle (Degrees)	P_L (p.u.)	Q_L (p.u.)
1	3	1.040	0	0.000	0.000
2	2	1.025	0	0.000	0.000
3	2	1.025	0	0.000	0.000
4	0	1.000	0	1.250	0.000
5	0	1.000	0	0.900	0.500
6	0	1.000	0	0.000	0.300
7	0	1.000	0	0.000	0.000
8	0	1.000	0	1.000	0.350
9	0	1.000	0	0.000	0.000

Table 6.2

BRANCH DATA FOR 9 BUS SYSTEM

From bus	To bus	R(p.u)	X(p.u)	B(p.u)
1	4	0.0000	0.0576	0.000
2	7	0.0000	0.0625	0.000
3	9	0.0000	0.0586	0.000
4	5	0.0100	0.0850	0.176
4	6	0.0170	0.0920	0.158
5	7	0.0320	0.1610	0.306
6	9	0.0390	0.1700	0.358
7	8	0.0085	0.0720	0.149
8	9	0.0119	0.1008	0.209

Table 6.3

GENERATOR DATA FOR 9 BUS SYSTEM

Generator	1	2	3
Rated MVA	247.5	192.0	128.0
kV	16.5	18.0	13.8
Inertia Constant (H) in seconds	23.64	6.40	3.01
X'_d (p.u.)	0.0608	0.1198	0.1813

Load Flow Results

To obtain initial conditions for the stability analysis, the load-flow solution was obtained. Voltages and angles at each bus and the real and reactive power generation from the converged solution are tabulated in Table 6.4 and are compared with the values presented in [4] for the verification of the power flow program.

Table 6.4

AFTER LOAD FLOW SOLUTION

Bus No.	Voltages (p.u.) (Calculated by powerflow program)	Voltages (p.u.) (Given in [4])	Angles (degrees) (Calculated by powerflow program)	Angles (degrees) (Given in [4])	P_G (p.u.)	Q_G (p.u.)
1	1.040	1.040	0.0	0.0	0.716	0.270
2	1.025	1.025	9.3	9.3	1.630	0.067
3	1.025	1.025	4.7	4.7	0.850	-1.109
4	1.026	1.026	-2.2	-2.2	0.000	0.000
5	0.996	0.996	-4.0	-4.0	0.000	0.000
6	1.013	1.013	-3.7	-3.7	0.000	0.000
7	1.026	1.026	3.7	3.7	0.000	0.000
8	1.016	1.016	0.7	0.7	0.000	0.000
9	1.032	1.032	2.0	2.0	0.000	0.000

These voltages and angles are used to find the internal voltage magnitudes and the initial rotor angles for the machines. Also, these voltages are used to represent loads by passive impedance models as discussed in Chapter III.

Network Modeling & Reduction

The Y matrix is obtained for the system and the nodes other than the generator nodes are eliminated using the Kron reduction technique as discussed in Chapter III. The reduced admittance matrix obtained is of size 3 x 3 because there are three generators in the system shown in Figure 6.1. The reduced Y matrix is given in (6.1).

$$\bar{Y}_{reduced} = \begin{bmatrix} 0.845 - j2.988 & 0.287 + j1.513 & 0.210 + j1.226 \\ 0.287 + j1.513 & 0.420 - j2.724 & 0.213 + j1.088 \\ 0.210 + j1.226 & 0.213 + j1.088 & 0.277 - j2.368 \end{bmatrix} \tag{6.1}$$

State Space Formulation

The state space equation $\dot{\tilde{x}} = [A]\tilde{x}$ discussed in Chapter IV is formulated in two different ways. Before calculating the A matrix, power coefficients defined by (4.19) are calculated as discussed in Chapter IV. These power coefficients can be calculated in two different ways:

1. Each element of (6.1) can be represented in the form G + jB. The power coefficients are found using the exact values of the admittances as given in (6.1). Using these power coefficients the A matrix of (6.2) is obtained. Thus (6.2) is the exact A matrix for the given system.

$$\begin{bmatrix} \dot{\delta}_{13} \\ \dot{\delta}_{23} \\ \dot{\omega}_{13} \\ \dot{\omega}_{23} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -102.49 & -60.386 & 0 & 0 \\ -33.953 & -151.51 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_{13} \\ \delta_{23} \\ \omega_{13} \\ \omega_{23} \end{bmatrix} \tag{6.2}$$

2. As suggested in [4], the power coefficients can also be found using the imaginary part B of the admittances (the real parts G are neglected as they are small numbers as compared to imaginary parts). These power coefficients are used in formulating the A matrix defined in (6.3) for the given system. This approximate approach is used only to verify that the program and approach presented here in this thesis produces the same results as compared to the results given in [4] for the system shown in Figure 6.1.

$$\begin{bmatrix} \dot{\delta}_{13} \\ \dot{\delta}_{23} \\ \dot{\omega}_{13} \\ \dot{\omega}_{23} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -104.10 & -59.52 & 0 & 0 \\ -33.84 & -153.46 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_{13} \\ \delta_{23} \\ \omega_{13} \\ \omega_{23} \end{bmatrix} \tag{6.3}$$

Eigenanalysis

Eigenvalues for (6.3) are calculated using MATLAB and are compared with the values given in [1]. They are tabulated in Table 6.5.

Table 6.5

EIGENVALUES FOR 9 BUS SYSTEM

	Eigenvalues (λ_1, λ_2)	Eigenvalues (λ_3, λ_4)
Calculated using (6.3)	±j13.41	±j8.80
Given in [4]	±j13.41	±j8.80
Calculated using (6.2)	±j13.36	±j8.69

The eigenvalues given in [4] and calculated here using (6.3) are exactly same. The eigenvalues using (6.2) are also shown in Table 6.5. The difference in eigenvalues calculated using (6.2) and (6.3) is very small, but eigenvalues calculated using (6.2) are more accurate (no approximation is done).

Participation Factor Analysis

The participation factors for the eigenvalues given in Table 6.5 are given in (6.4).

$$P=\begin{bmatrix} 0.131 & 0.131 & 0.369 & 0.369 \\ 0.369 & 0.369 & 0.131 & 0.131 \\ 0.131 & 0.131 & 0.369 & 0.369 \\ \underbrace{0.369}_{\lambda_1} & \underbrace{0.369}_{\lambda_2} & \underbrace{0.131}_{\lambda_3} & \underbrace{0.131}_{\lambda_4} \end{bmatrix} \begin{matrix} \delta_{13} \\ \delta_{23} \\ \omega_{13} \\ \underbrace{\omega_{23}}_{\text{states}} \end{matrix} \tag{6.4}$$

Careful evaluation of the participation matrix given in 6.4 reveals that machine 2 has higher participation in the oscillatory mode corresponding to eigenvalues λ_1 and λ_2 and machine 1 has higher participation in the oscillatory mode corresponding to λ_3 andλ_4.

The system shown in Figure 6.1 is a small system and all the machines are closely coupled to each other. This can be concluded from (6.4) because for each mode all the machines tend to accelerate together. If the mode (λ_1, λ_2) were to be controlled for some reason, then it would be better to do it using machine 2 than machine 1. Also if for some reason mode (λ_1, λ_2) showed instability, then the participation factors for this mode shows that machine 2 is more likely to go unstable.

Test System 2: IEEE 118 Bus System

The approach presented and validated using the 9 bus system [4] in this thesis was applied to a larger system (IEEE 118 bus system [3]). The bus data and the branch data for the IEEE 118 bus system were obtained from [3]. The single line diagram for IEEE 118 bus is shown in Figure 6.2. The significance of the markings shown in Figure (6.2) will be discussed later in this section.

To establish initial conditions for the stability analysis, the load flow solution was obtained. The voltages and angles are used to find the internal voltage magnitudes (E) and initial rotor angles for the machines. Also, these voltages are used to represent loads by passive impedances as discussed in Chapter III. The admittance matrix is updated using these load impedances and the nodes other than generator nodes are eliminated using Kron reduction. The reduced admittance matrix is of size 54 x 54 because there are 54 generators in the system shown in Figure 6.2.

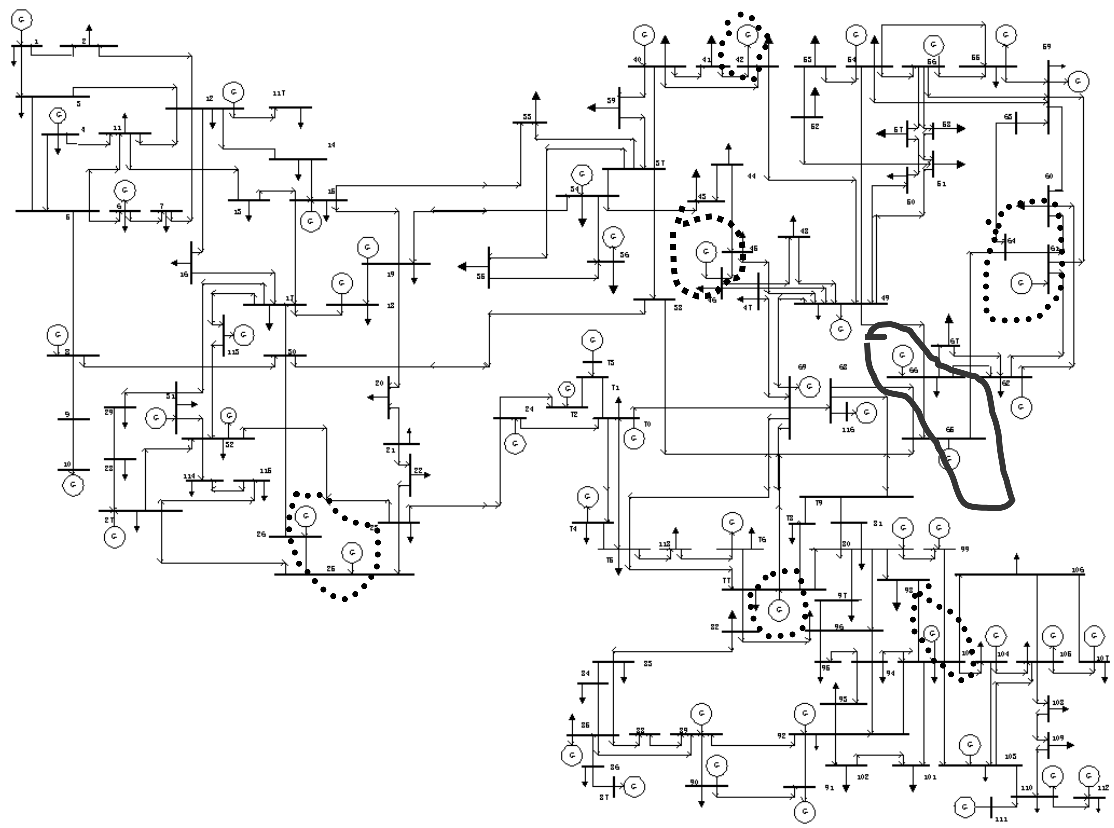

Figure 6.2: Single Line Diagram for the IEEE 118 Bus System

Once the reduced admittance matrix was formulated, the linearized equations of motion were written in the form of the linear state space model as described in the previous section for the 9 bus system shown in Figure 6.1. Eigenvalues for the linear state space model are obtained using MATLAB. These eigenvalues are tabulated in Table 6.6.

These eigenvalues are used to calculate participation factors for each of the modes. The participation factors of mode 1 (0 ± j98.0392) are examined here in detail. The active machines or the dominant machines for this particular mode are those, which have high participation factors as compared to the other machines and are given in Table 6.7. It can be concluded from Table 6.7 that machines at bus numbers 65 and 66 are the most active machines for this mode.

Table 6.6

EIGENVALUES FOR IEEE 118 BUS SYSTEM

Mode	Eigenvalues	Mode	Eigenvalues	Mode	Eigenvalues
1	0±j98.0392	19	0 ±j41.7531	37	0 ±j13.3938
2	0±j94.1947	20	0 ±j41.0973	38	0 ±j27.0260
3	0± j76.8659	21	0 ±j40.4203	39	0 ±j26.6094
4	0 ±j70.1881	22	0 ±j37.5655	40	0±j14.9891
5	0 ±j66.3154	23	0 ±j37.3889	41	0 ±j16.1164
6	0 ±j64.5323	24	0 ±j35.0766	42	0 ±j17.5880
7	0 ±j64.1454	25	0 ±j33.9808	43	0±j18.2203
8	0 ±j61.7852	26	0 ±j32.7656	44	0 ±j24.5334
9	0 ±j61.1468	27	0 ±j31.5543	45	0 ±j19.3640
10	0 ±j60.0641	28	0 ±j31.6074	46	0 ±j19.7477
11	0 ±j55.2594	29	±5.6051	47	0±j23.7117
12	0 ±j52.2651	30	0 ±j 5.0279	48	0 ±j20.6060
13	0 ±j49.8762	31	0±j7.8985	49	0 ±j22.9185
14	0 ±j47.9607	32	0 ±j 7.9191	50	0 ±j21.9633
15	0 ±j48.6638	33	0 ±j 8.7052	51	0±j23.3556
16	0 ±j45.3494	34	0. ±j27.4635	52	0 ±j21.0800
17	0 ±j43.2173	35	0 ±j12.4317	53	0 ±j21.8868
18	0 ±j42.4910	36	0 ±j12.8326		

If this mode were to be controlled for some reason, then it would be better to do it with the machines at buses 65 and 66. These machines (at buses 65 and 66) are shown in Figure 6.2 by encircling them with a solid line curve. The machines at buses 26,42,46,49,61,77 and 100 that are less active for this mode are shown in Figure 6.2 by encircling them with a dashed line curve.

Table 6.7

PARTICIPATION FACTORS

S. No.	Machines at (Bus No.)	Participation factor
1	65	0.2958300
2	66	0.1742900
3	26	0.0001136
4	42	0.0001340
5	77	0.0070816
6	61	0.0041741
7	46	0.0002080
8	49	0.0218090
9	100	0.0001899

It can also be observed from Figure 6.2 that the farther a machine is from the most active machine or (machines), the less significant will be its contribution in that mode. If for some reason mode 1 shows instability, then the participation factors of the machines for mode1 show that the machines at buses 65 and 66 are more likely to go out of step as compared to the other machines.

CHAPTER VII

CONCLUSION

In interconnected power systems, the most common form of instability for the interconnected generators is loss of synchronism. Its study requires the solution of the nonlinear differential equations. To study small signal stability, a method was developed here which can be applied to a large n machine power system.

The first step was to build an unregulated mathematical model of the power system. The model built was an unregulated model because the effects of the governor, the power system stabilizer (P.S.S) and the field exciter were neglected for each machine. The mathematical model obtained was a set of nonlinear coupled first order differential equations. The method of small changes, sometimes called the perturbation method, was used to linearize the nonlinear differential equations. They were linearized about an operating point which was the steady state condition obtained from a load flow solution. The linear equations derived were assumed to be valid in the region near the quiescent condition. The equations were then written in a state-space model form and the eigenvalues were obtained from the state space model. Using these eigenvalues and the corresponding eigenvectors and the participation factors, the free response of any system could be examined. The results for the 9 bus system were compared and verified with the results given in [4]. The participation factors were then calculated using these eigenvalues and were used to study small signal stability problems such as local mode oscillations and interarea mode oscillations. The same techniques were also applied to a larger IEEE 118 bus system. The absolute values of participation factors reveal which machines are involved in a mode or which machines could go out of step. This participation identification avoids changing a machine via its control that does not participate in any particular mode.

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