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The wireless network design problem

By

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The wireless network design problem

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The wireless network design problem (WNDP) considers how best to place a set of antennas so the antennas can send and receive the maximum possible amount of data subject to network-performance constraints (e.g., channel-availability constraints). To date, little research has considered how to choose the network-antenna layout that maximizes throughput under these conditions. Also, past research has mainly investigated networks with omnidirectional antennas only, not other types of antennas. A bi-level mixed-integer program is constructed to solve this problem using a cutting-plane approach. The data produced from this model demonstrate an extension of the WNDP under more realistic conditions than have been simulated previously. The questions answered by this research are as follows: (1) what are the effects on network throughput of utilizing directional or sectored antennas instead of omnidirectional antennas, and (2) what is the maximum possible throughput when imposing constraints related to differing interference types and channel availability?

Key words: Directional antennas, sectored antennas, wireless network

DEDICATION

To my mother, Melanie Howell.

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I would like to thank my mother, Melanie Howell, for supporting me throughout the process of writing this thesis. She was always available and willing to listen whenever I had concerns about how my progress was going.

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CHAPTER 1

INTRODUCTION

The wireless network design problem (WNDP) is a ubiquitous problem in the field of operations research. The WNDP considers how best to place a set of transmitters so that the transmitters can send data to a set of receivers. This analysis constitutes the first major study of the effects of varying the network's interference types, channel availability, and capacity limitations on the ability of a wireless mesh network to transmit data optimally, particularly in regard to the effects of such limitations on networks with omnidirectional antennas, networks with directional antennas, and networks with sectored antennas. This analysis's purpose is to determine the best approach in a wireless mesh network to maximize network throughput.

1.1 Background and Motivation

Wireless networks, which consist of multiple transmitting antennas that communicate data among each other, are commonplace in society. Such networks may consist of extensive, permanent physical infrastructure or of ad-hoc components that are set up only for limited times. Therefore, the optimization of network performance is a key consideration that has major implications for future technological and societal progress. However, while networks themselves are a common topic discussed in scholarly literature, most analyses have considered only omnidirectional antennas rather than also considering directional and sectored antennas; also, while elements such as interference types, channel availability, and similar limitations have been studied, the number of articles discussing such topics is quite limited, and never has any of these elements been considered in the context of the wireless network design problem under radio interference, as described in this work. Also, this analysis does not consider just single-hop networks, for which data transfer occurs only across a single connection (e.g., a modem that gives a laptop access to the Internet). Rather, this analysis will investigate the effects of the aforementioned attributes on multihop networks. The major benefit of studying multihop networks is that communications between transmitters in the network are not limited only to pairs of transmitters, but, instead, can move among multiple transmitters. Never before has a major analysis of multihop networks under a WNDP model been performed along these lines.

1.2 Literature Review

Analyses of the WNDP in particular and of wireless network interference in general are widespread in scholarly literature. Optimizing the performance of wireless networks is a perennial theme in literature, as may be seen generally in works by Shankar [2008] as well as Nicholas and Alderson [2012]. The contrasting types of networks studied have included both multihop networks and single-hop networks (per Sanchez et al. [1999] and Kasera and Ramanathan [1997]), along with client-mesh networks, infrastructure-mesh networks [Ramachandran et al., 2009], and even wireless sensor networks [Hada and Tsuchiya, 2012].

With respect to the physical configuration of networks and the ability to send communications from one antenna to another, both cyclic and acyclic directed networks have been studied [Sinha et al., 2017]. Most research in regard to physical networks, though, pertains to the design of networks - e.g., an article by Pathak and Dutta [2011] considering available literature on network design approaches, and an article by Benyamina et al. [2012], doing the same.

Particular attributes that can impose constraints on network performance have been studied - e.g., varying the interference type. For example, Iyer et al. [2009] asks the question of what the appropriate model for wireless network interference actually should be. Iyer et al.'s work discusses various models under additive and non-additive means of interference, such as additive models, capture-threshold models, protocol models, and two similar versions of interference-range models [2009]. It is illustrated that the main function of varying the interference-model type is to manipulate the conflicts that can occur among network transmitters and receivers under interference conditions. Others such as Madan et al. [2008] discuss attempted analysis using methods involving models with additive white-noise models and protocol models. Furthermore, interference in other contexts can be studied with various means of judging antennas' ability to communicate (e.g., in scenarios contemplated by Baccelli et al. [2006]). Varying interference types can have non-negligible implications; per Huang et al. [2010], varying transmission strategies among antennas and varying interference ranges can have drastic effects on communications. However, experimental analyses, such as one by Angrisani et al. [2008], have not studied networks in the same operational context and under the same criteria as are proposed in this work.

Constraints related to channels and routing are also prominent in literature. Articles discussing channels and routing include one by Carvalho and De Rezende [2010] (discussing transmission channels' characteristics during routing), one by Lee and Murray [2010] (studying network survivability when routing is limited), and another by Liu et al. [2014] (considering both power and channel-assignment parameters in network performance). Both Javed and Prakash [2013] and Burchard et al. [2017] look at different routing protocols in wireless mesh networks in their analyses of routing's effect on performance.

Methods to solve such problems also vary greatly. For example, Medal [2015] uses both standard Benders Decomposition and a branch-and-cut method with a cutting-plane approach to study wireless network performance under jamming conditions. The applications of a branch-and-cut method might also be modified and applied as a somewhat similar branch-and-price method (per Barnhart et al. [1998]) for additional analysis. For example, Li et al. [2015] uses such a branch-and-price approach along with two heuristics. Nicholas and Alderson's alternative is a sampling algorithm to solve a nonconvex, nonlinear optimization problem [2012]. Shankar [2008] solves a nonlinear convex optimization problem using a heuristic approach.

Combinations of various constraints and methods for solving problems subject to such constraints have also been studied in literature (e.g., in works such as that of Scheibe and Ragsdale [2009]), but an analysis of the effects of interference-type and channel changes along with varying antenna types seems not to exist in literature. However, even research into these sorts of network attributes, such as the work of Gokbayrak and Yildirim [2017], has not studied the effect of these parameters in tandem with sectored, directional, and omnidirectional antennas. Although some studies of network design with directional antennas exist (e.g., a study on wireless sensor network design with directional antennas by Shirazipourazad et al. [2014] or Wang et al. [2017]), literature on the WNDP for wireless mesh networks with parameters such as power is relatively limited. Some of these pieces of literature include, e.g., an article by Ramamurthi et al. [2008] studying power control and a few other parameters in mesh networks with directional antennas. Another example is a work by Li et al. [2015], which studies how to optimize attributes of a wireless mesh network with directional antennas, but not against other antenna types as described in this paper, and not in a way similar to this work's mixed-integer-program formulation. Work on sectored antennas is similarly limited (and consists mainly of articles along the lines of Noubir's work). It appears that any network with sectored antennas has mainly been studied in the general context of wireless communications rather than more specific circumstances involving the imposition of additional, non-flow-related constraints. Accordingly, more analysis needs to be conducted on the problem of optimizing network performance for a wireless network with the aforementioned characteristics.

1.3 Contributions

This work studies the WNDP, attempting to maximize a wireless, multihop mesh network's throughput. The WNDP includes an analysis of a network with sectored, directional, or omnidirectional antennas, and it is subject to limitations imposed by changing the interference type and channel availability in the network. The goal of this article is to determine what changes occur to maximum throughput based on imposing constraints related to interference type and channel availability in a network using sectored antennas, a network using directional antennas, and a network using omnidirectional antennas. This article's contributions include an analysis of such scenarios based on a mixed-integer programming formulation with a Benders Decomposition approach. The available number of antennas to be placed in any wireless-network array is also varied so that the mixed-integer-program formulation is forced to determine the placement of antennas that maximizes throughput. The analysis includes empirical data that helps a network designer to determine what conditions are most suitable for increasing the throughput of a wireless network.

Below, Section 2 describes the WNDP in depth. Section 3 includes a model formulation, while Section 4 contains the methodology used to determine an optimal solution for the model. Section 5 discusses the results of experiments required to determine what conditions lead to optimal network performance. Finally, Section 6 contains information about future work that might prove beneficial.

CHAPTER 2

PROBLEM DESCRIPTION

The WNDP in this context consists of a mixed-integer-programming problem. The objective of the problem is to maximize the flow of data within the network by optimally placing a set of transmitters.

2.1 Basic Parameters

The network operator's objective is to maximize the throughput between a source node or nodes *s* and a sink node or nodes *t* that have been previously selected. Let $\mathbf{x}$ consist of a vector that shows where the network's transmitters will be located, and let the set X denote the set of all feasible transmitter location vectors. Let $\mathbf{y}$ denote a flow in the network and let y_a represent the network throughput produced for a given flow. In accordance with these parameters, the problem can be determined to be simply the maximization of the flow through the network from source s to sink t.

2.2 Three-Layer Network Representation

The network studied in the WNDP possesses three layers: a physical layer, a connectivity layer, and an interference layer. The physical layer consists of the set of antennas that comprise the network; the location of each transmitter is fixed pursuant to a given "map" of transmitter locations. Let each transmitter be represented as a node i in the graph, with i being present in N, the set of all nodes. The variable d_{ij} refers to the geometric distance between nodes i and j in set N. Each node has two intrinsic abilities: to communicate with another node within range c_i, and to interfere with other nodes' communications, within range h_i. The distances to which these ranges extend are a function of the type of antenna placed at each node. The maximum communication rate between any two nodes i and j is U_{ij}.

The connectivity layer consists of the transmissions of data packets among pairs of nodes. A connectivity graph can be constructed to model the connectivity layer, such that $G = (N, A)$; A constitutes the set of arcs in the network and is indexed by k. Connectivity between any two nodes i and j is illustrated by the construction of an arc between the two nodes if the latter is within the communication range of the former. (This is the case if $d_{ij} \leq c_i$ for this particular pairing of nodes at the angle between the nodes.) A helpful means to illustrate these connections is a connectivity graph, such as the following in Figure 2.1.

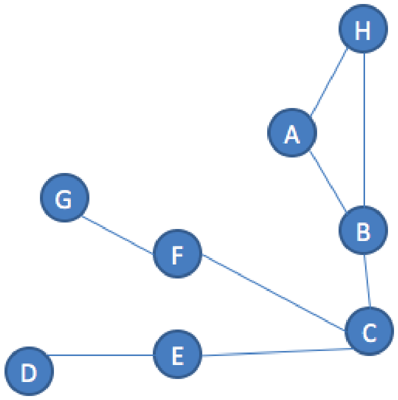

Figure 2.1

Connectivity graph example

The benefit of this sort of graph is that is easily illustrates to the observer which antennas can connect to one another. For example, in Figure 2.1, antennas at D and E can communicate because nodes D and E are connected by an edge (arc) in the graph. When performing analysis on the network, a connectivity graph concisely determines which antennas can connect to each other. In addition, the interference layer of the network consists of the interference that occurs when data-packet transmissions intrude upon each other's path. Under a protocol model of interference, a network cannot have multiple data packets present at the same location in space at one time. Accordingly, a transmitter can only receive one communication at any given point in time, and a transmitter cannot send and receive communications simultaneously. The interference layer can be illustrated using a conflict graph. In fact, an example of a conflict graph is shown in Figure 2.2. The benefit of a conflict graph is that it illustrates which combinations of nodes cannot communicate simultaneously. Based on the connectivity graph in Figure 2.1, nodes D and E can connect, and so can nodes C and E. However, the conflict graph shows that nodes D and E and nodes C and E cannot connect with each other simultaneously because the two combinations would share an endpoint (at node E).

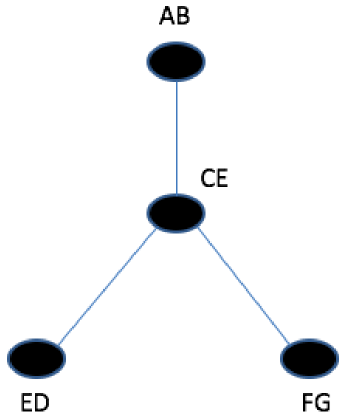

Figure 2.2

Conflict graph example

Nonetheless, it could similarly be the case that the transmitting antennas' wireless signals interfere with each other. If nodes A and B and nodes C and E were attempting to communicate at the same time, the conflict graph shows that that combination would likewise fail to communicate without interference. In any case, the conflict graph determines which sets of nodes cannot be active at the same time without causing interference, which leads to failed communication among antennas.

2.3 Antenna Radiation Patterns

The three types of antennas studied in this work are omnidirectional antennas, directional antennas, and sectored antennas. The signals broadcasted by these antennas are usually referred to as radiation patterns.

An omnidirectional antenna has a radiation pattern extending a uniform distance (gain) in all directions. An example of an omnidirectional antenna's radiation pattern may be seen in Figure 2.3 below.

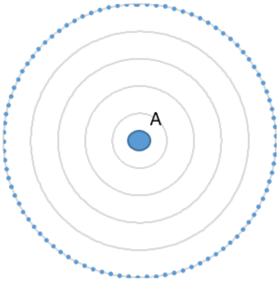

Figure 2.3

Omnidirectional antenna's radiation pattern

In the case of an antenna at A in the figure above, the radiation pattern extends 1 unit of distance out from A. If another antenna is situated within 1 unit of distance from location A, that antenna can receive transmissions from A.

However, the angle between a transmitter and a receiver can matter when considering networks composed of antennas with directional radiation patterns. Simply put, directional radiation patterns do not extend a uniform distance in all directions. Rather, their radiation patterns are varied, and the exact distance of the gain is dependent on the angle between the transmitter and a given receiver. An example of a directional radiation pattern may be seen in Figure 2.4 below. Directional radiation patterns, generally speaking, usually have main lobes that extend the farthest distance out from the transmitter at the center as well as other lobes that might extend shorter distances in particular directions. In Figure 2.4, the main lobe extends straight up from the broadcasting antenna at A, while other lobes extend in various other directions.

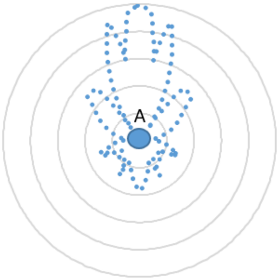

Figure 2.4

Directional antenna's radiation pattern

Directional antennas with radiation patterns such as these have both potential benefits and potential detriments compared to omnidirectional antennas. On one hand, by sending a varied radiation-pattern signal out from the transmitter that does not extend to the same distance in all directions, it is possible that a directional antenna could save some energy by sending out less signal. On the other hand, it is certainly possible that a radiation pattern that is directional might not cover the same areas that an omnidirectional radiation pattern would reach, because its signal would not go as far in certain directions. Thus, a tradeoff exists between using directional and using omnidirectional antennas.

The final type of antenna used for experiments would be the sectored antenna. A sectored antenna is an antenna whose radiation pattern is a sector of a circle. In other words, the radiation pattern is a portion of an omnidirectional antenna's radiation pattern, but is actually a directional antenna's radiation pattern with only one main "sector" of gain. In Figure 2.5, the sector in questions points toward the first quadrant of the Cartesian coordinate system.

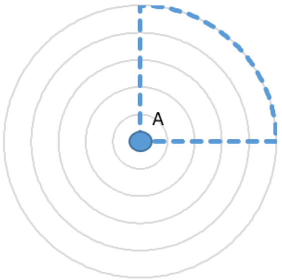

Figure 2.5

Sectored antenna's radiation pattern

The benefit of using sectored antennas is that they represent something of a hybrid approach, with many benefits both of omnidirectional antennas and directional antennas. As Noubir reports, sectored antennas can potentially improve network connectivity, which would appear to be a result of their design.

2.4 Computing Network Throughput Under Interference Conditions

The interference layer is modeled using a conflict graph, as shown by Medal [2015] and discussed earlier. To summarize, a conflict graph, denoted G', has a node for each arc that lies in the connectivity graph, and then constructs an arc between such nodes if the arcs' simultaneous transmissions would interfere with each other. Medal [2015] indicates that multiple sets of arcs from the conflict graph will not interfere with one another if the arcs in each set are parts of an independent set in G'. Maximal throughput exists only in the case in which maximal independent sets are utilized. Let I_k denote the set of maximal independent sets that contain the arc k.

The routing of data in the network for this problem can be defined in terms of two vector quantities. The first is $\mathbf{w}$, where $\mathbf{w} = (w_\sigma)_{\sigma\epsilon I}$. This is the usage vector, which identifies the proportion of time that the arcs in set I_σ are all active. Next, $\mathbf{y}$ is a vector such that $\mathbf{y} = (y_k)_{k\epsilon A}$, whereby $\mathbf{y}$ is a vector that designates the average flow y on arc k. The value of this flow y on arc k is limited by the value of the average capacity on arc k, which is simply going to be the product of the arc's given capacity, U_k, and the proportion of time that the arc is actually active as part of a set of antennas, $\sum_{\sigma\epsilon I_k} w_\sigma$.

However, when looking at how data travels through a given wireless network under interference conditions, the connectivity and conflict graphs illustrate just how communication and interference can occur, not how those attributes are brought about from a network's physical infrastructure. Figure 2.6 illustrates how communications occur in a network of this type and how connectivity and interference can take place. In Figure 2.6, an antenna at location A in the center of a circle is broadcasting a radiation pattern that is illustrated by the lower-right area emanating from the location at A. Any other antenna contained within this area is able to receive data from A under the protocol model of interference. In the figure, it is apparent that communications *may* be successful between A and B, since B is within A's radiation pattern, but communications would be unsuccessful between A and C, since C is not located within A's radiation pattern.

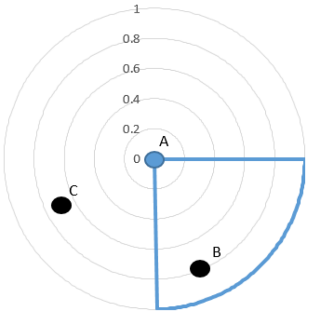

Figure 2.6

Network connectivity example

2.5 Interference Models

In this paper, a protocol model of interference and an interference-range-based model are utilized; these models were constructed per Iyer et al. [2009]. Under the 802.11 protocol, which is commonplace in wireless networks, no node can send or receive data at the same time as another node unless the two nodes are located outside of one another's interference range. For the example related to Figure 2.7 below, A's communications to B would not be considered successful unless B is able to send an acknowledgment signal to A, with A being located within B's radiation pattern. Additionally, because C's radiation pattern (shown with the dashed line) and A's radiation pattern (shown with the solid line) both cover B, then B cannot receive from A if C is also broadcasting at the same time. Specifically, Iyer et al. [2009] states that, for the protocol model, a link can exist between a transmitting node and a receiving node if connectivity can be established and if interference can be avoided.

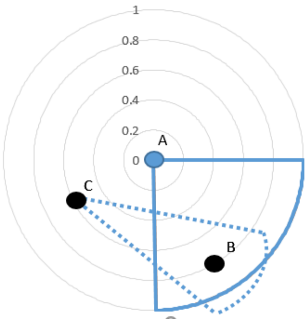

Figure 2.7

Network interference example

As an example, consider a transmitter at A attempting to communicate with a receiver at B. Meanwhile, transmitter C and receiver D are also located nearby and are the only other antennas in the area. The protocol model says that communications are successful if the distance between C and B is a multiplier $(1 + \Delta)$ greater than the distance between A and B. Also, the distance between A and B must be no more than A's predefined communication range. To summarize, no other antenna must be broadcasting around B, and A must be close enough for its radiation pattern to cover B, if communications from A to B are to be successful.

Meanwhile, Iyer et al. [2009] suggests that an interference-range model can be considered as a model for wireless channel interference. Taking the same example with antennas A, B, C, and D, the interference-range model says that communications are successful if the distance from C to B is greater than a defined interference range and if the distance from A to B is less than a defined communication range. Basically, if no other antenna is broadcasting close enough to D for it to interfere with D's reception, and if A is close enough to B to be able to transmit, then A can communicate data to B under the interference-range model. The interference-range model can, in some cases, even be seen as a simplified version of the protocol model, if one assumes that the interference range is greater than the communication range of the respective antennas.

CHAPTER 3

MATHEMATICAL MODEL

3.1 Formulation

For this problem, let $\mathbf{x} = (x_i)_{i \epsilon N}$ be a vector consisting of binary variables, with $x_i = 1$ if a transmitter is placed at node i or 0 otherwise, for each node i in the set of all nodes, N. Two nodes can constitute an arc, (i, j), in the set of all arcs, A. The forward star (FS) of arc k, equivalent to (i, j), is defined as the flow on arc k toward the sink from the direction of the source, and the node's reverse star (RS) is defined as the flow on arc k toward the source from the direction of the sink.

The mixed-integer program formulation is the following:

$$max \; y_a \tag{3.1a}$$

subject to:

$$\sum_{k \in \mathrm{FS(i)}} y_k - \sum_{k \in \mathrm{RS(i)}} y_k \; = \begin{cases} y_a & i = s \\ 0 & i \; \in \mathbb{N} \setminus \{s, \, t\} \; \forall i \, \epsilon \, \mathbb{N} \quad [\alpha_i] \\ -y_a & i = t \end{cases} \tag{3.1b}$$

$$0 \ \leq \ y_k \leq \ \sum_{\sigma \epsilon I_k} \Bigg(w_\sigma \Bigg) U_k \ \forall k \ \epsilon \mathrm{A} \quad [\beta_k] \tag{3.1c}$$

$$w_\sigma \geq 0 \ \forall \sigma \epsilon I \tag{3.1d}$$

$$\sum_{\sigma \epsilon I} \Bigg(w_\sigma \leq 1 \qquad [\gamma] \tag{3.1e}$$

$$0 \leq y_{ij} \leq U_k x_i \ \forall k = (i, j) \epsilon A \qquad [\delta_{ij}] \tag{3.1f}$$

$$0 \leq y_{ij} \leq U_k x_j \ \forall k = (i, j) \epsilon A \qquad [\varepsilon_{ij}] \tag{3.1g}$$

$$\sum_{i \epsilon I} \Bigg(r_i x_i \leq R \tag{3.1h}$$

The objective function of the problem will consist of the maximization of throughput in the wireless network. Basically, this will consist of equation 3.1a. We will compute the optimal throughput for a wireless network in which the placement of transmitters is fluid.

One constraint that must be enforced is in terms of the flow throughout the network, as shown with equation (3.1b). Overall, the total flow entering the network must equal the total flow leaving the network, and the net flow at any intermediate node must have a value of zero (since only the source and sink have non-zero supply and demand values, respectively).

A constraint must be implemented so that the average flow on any arc is non-negative but also no greater than the proportion of time the arc is actively transmitting (*w*) times the arc's capacity, U_{ij}; this would basically require equation (3.1c). Equation (3.1d) and equation (3.1e) ensure that the usage vector, $\mathbf{w} = (w_\sigma)_{\sigma \epsilon I}$, is a proportion that adds up to one.

Also, one of the constraints to be utilized is a constraint on the flow of data on each arc. Equation (3.1f) and equation (3.1g) require that no flow can move along the arc between two nodes unless a transmitter is present at each node. Finally, equation (3.1h) ensures that the total number of transmitters is subject to a budget R which is measured against by imposing a cost of r_i for placing an antenna. The dual variables required to solve this problem are denoted by the variables shown in brackets above.

3.2 Extension to Multiple Channels

The utilization of multiple arcs among pairs of nodes in the connectivity graph is sufficient for the purpose of allowing multiple channels for communications. These arcs should not cause any interference so long as such arcs do not share an endpoint in common. In other words, different transmissions passing through the area surrounding a given network will not cause interference with one another's transmissions unless the transmissions coincide at the same node (which serves as an endpoint of an arc (i, j) on which the data is traveling).

CHAPTER 4

SOLUTION METHODOLOGY

The approach that will be utilized to solve the problem of optimizing throughput involves Benders Decomposition, which allows the user to find solutions to extremely large linear programming problems. Essentially, this methodology compares the true solution for a version of the original problem to an approximated solution that is found via the generation of supporting hyperplanes in a reformulated version of the function from equation (3.1a).

In order to use Benders Decomposition, the original problem, 3.1, is formulated as a two-level problem in which both levels are maximization. Next, the dual of the inner maximization problem is taken. The resulting formulation is as follows:

$$max_{\mathbf{x}\epsilon X}\ g(\mathbf{x}) \tag{4.1a}$$

where

$$g(\mathbf{x}) = min_{\beta\geq 0,\,\gamma\geq 0}\ \gamma + \hat{x}_i U_{ij}\delta_{ij} + \hat{x}_i U_{ij}\varepsilon_{ij} \tag{4.1b}$$

$$\text{s.t.} \qquad \alpha_i - \alpha_j + \beta_k + (1 - \hat{x}_i)(1 - \hat{x}_j) \geq 0 \, \forall (i,j) = k \epsilon A \tag{4.1c}$$

$$\alpha_t - \alpha_s \geq 1 \tag{4.1d}$$

$$\gamma - \sum_{k \epsilon I_\sigma} \Bigg(U_k \beta_k \geq 0 \;\; \forall \sigma \epsilon I \tag{4.1e}$$

$$\beta_k \geq 0 \, \forall k \epsilon A \tag{4.1f}$$

$$\delta_{ij} \geq 0 \, \forall (i,j) \epsilon A \tag{4.1g}$$

$$\varepsilon_{ij} \geq 0 \, \forall (i,j) \epsilon A \tag{4.1h}$$

The Benders master problem is as follows:

$$max_{\theta \geq \mathbf{0}, \mathbf{x} \epsilon \mathbf{X}} \, \theta \tag{4.2a}$$

$$\text{s.t.} \qquad \theta \leq \gamma^\omega + x_i U_{ij} \delta_{ij}^\omega + x_j U \varepsilon_{ij}^\omega \; \forall \omega = 1, \, ..., \, \mathit{\Omega} \tag{4.2b}$$

For this master problem, Ω is the number of extreme point solutions of (4.1). Constraints (4.2b) are supporting hyperplanes of $g(x)$, also referred to as Benders optimality cuts. Because of the number of extreme point solutions that can exist, the cuts are computed via the Benders subproblem, determined via the dual of the original problem above (3.1).

Because of the need for a large number of w variables in the dual of 4.1, however, a column generation master problem must be implemented as well. The column generation master problem serves as a throughput optimization problem, to calculate throughput and provide dual variables as part of the solution methodology. A column generation master problem of the following form is constructed:

$$g(x) = max_{\mathbf{y} \geq \mathbf{0}} \, y_a \tag{4.3a}$$

s.t. (3.1b)

$$0 \leq y_k \leq (\sum_{\sigma \epsilon \bar{I}} \Bigg(w_\sigma) \, U_k, \, \forall k \epsilon A \tag{4.3b}$$

$$\sum_{\sigma \epsilon \bar{I}} \Bigg(w_\sigma \leq 1 \tag{4.3c}$$

$$w_\sigma \geq 0, \, \forall \sigma \epsilon \bar{I} \tag{4.3d}$$

To determine which data to choose, though, the Benders subproblem must have an additional component; it is therefore composed of two parts: the aforementioned column generation master problem and a separation problem that prices the "best" independent set into the basis. With this formulation, it should particularly be noted that $\bar{I}$, the index of the σ variable, refers to the restricted set of maximum weight independent sets that are introduced as new columns using the pricing-problem formulation found below. This formulation can also be referred to as a column generation subproblem or as a separation problem. New columns related this problem are generated by using the maximum weight independent sets related to this problem, which correspond to the optimality cuts that are desired to help to find an optimal solution to the original problem. By using this column-generation procedure, one is not forced to find all potential independent sets relating to transmitter activity, but, rather, can approximate potential independent sets via $\bar{I}$ until such time that an optimal independent set is found and documented. Per this method, the objective-function value is found by imposing the reduced cost on including a given arc in the independent set whose column values are being investigated to see if they should be added. When an independent set is chosen, it is provided to the column generation master problem so that throughput can be determined; then, dual variable values are "fed" back into the pricing-problem formulation to determine which independent set is to be added next (if possible). For this pricing problem, $N(G')$ is the set including the nodes of the conflict graph, and $A(G')$ is the set comprising the arcs of the conflict graph. Additionally, the value of β_k is already fixed.

$$z(\beta) = max \sum_{k \epsilon N(G')} \Bigg(\hat{\beta}_k U_k v_k \tag{4.4a}$$

$$\text{s.t.} \quad v_k + v_{k'} \leq 1 \, \forall (k, k') \epsilon A(G') \tag{4.4b}$$

$$v_k \epsilon \{0, 1\} \, \forall k \epsilon N(G') \tag{4.4c}$$

In this pricing problem, the variable v_k is 1 if arc k is included in a potential independent set and 0 otherwise. Equation (4.4b) ensures that two adjacent arcs in the conflict graph are not chosen, which would violate the definition of an independent set. Once the independent-set solution is found, they are examined to be added to the constraints of the column generation master problem. Any set added to the problem, as designated by the pricing problem, is added in the form of a column via the column generation master problem. Therefore, the benefit of this method involves avoiding the need to include all of the potential combinations of arcs in independent sets, by only adding what is absolutely needed to benefit the output (throughput) maximally. Instead, the independent sets that are determined to be independent via the pricing problem are added as needed to the column generation master problem.

Now that the entire Benders procedure has been discussed in depth, the reader can view the interactions among its component parts in the following figure, Figure 4.1, which shows the manner in which the subparts of the Benders problem are executed.

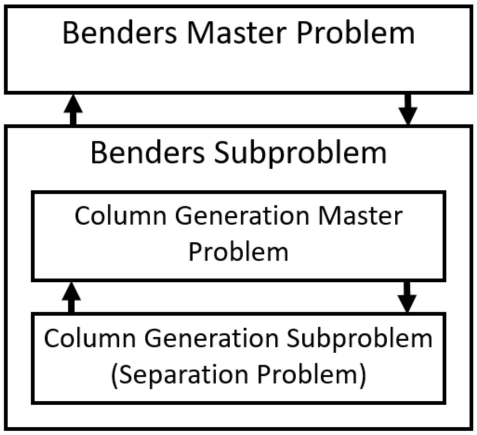

Figure 4.1

Benders procedure

CHAPTER 5

COMPUTATIONAL RESULTS

5.1 Questions Posed

The optimization model discussed in the previous section was implemented in order to provide insight into several scenarios answering the questions posed in the following table.

Table 5.1

Questions posed and information needed to run experiments

Question Posed	**Data Utilized**
What is the effect on throughput of increasing the number of channels available?	1, 2, and 3 channels
What is the effect on throughput of using different types of antennas?	Omnidirectional, directional, and sectored antennas
What is the effect on throughput of using different models of interference?	Protocol-based model and interference-range-based model

Each experiment was implemented using an $n\ x\ n$ network grid, for which the baseline is a 5 x 5 network grid with a number of nodes equivalent to the product of its rows and its columns. The nodes were arranged inside of a unit square. The distance on the side of each square was 51. The origin and destination points (i.e., the source and sink nodes) were placed at the midpoints of or on the corners of the unit square arrangement. Accordingly, there were four potential locations at corner points and four potential locations at midpoints on the sides at which origin and destination nodes may be situated. For each potential pair of nodes for the origin and destination, respectively, an origin node-destination node pair was designated in each direction, allowing for 8 origin node-destination node pairs in total. A default distance range for communications was chosen to be 17.7828, which was found by setting the interference range equal to the product of the communication range and the multiplier $(1 + \Delta)$ in the interference-range model, as described by Iyer et al. [2009]; doing so allowed for some comparability in the protocol model and the interference-range model by viewing the latter as a simplication of the former. The arc capacity was chosen as 1 for all arcs k in the set A.

The baseline parameter values are defined in Table 5.2 below, and, unless otherwise noted, are the values used during the running of all experiments.

Table 5.2

Baseline values used to run experiments

Parameter Name	**Baseline Parameter Value**
Dataset	5 x 5 Grid
Communication Pairs	16
Communication Range	17.7828
Interference-Model Type	Interference Range
Number of Transmitters in Budget	25
Number of Channels	1

5.2 Channel Constraints

One option to attempt to improve a wireless network's performance is to ensure that the network's antennas are able to operate on multiple channels to send data and receive data. The benefit is that multiple channels do not interfere with each other. Also, making more channels available does not require additional infrastructure (e.g., antennas) to be placed within the confines of a network. Furthermore, since the channels do not interfere with each other, one might avoid the interference that might come about from placing more antennas within the perimeter of an existing network array.

The baseline parameter used for communications in such a network includes just one channel. However, test datasets with one, two, and three channels were separately studied, all else equal, to determine if the use of an increasing number of channels would benefit the network's operator by improving network performance (i.e., throughput).

The values determined during experiments from differing channel values may be found in the following table, Table 5.3.

Table 5.3

Channel experiment results

Interference Type	**Number of Channels**	**Total Throughput**	**Percent Change**
Interference Range	1	0.75	-
	2	1.50	100%
	3	1.81	21%
Protocol	1	1.13	-
	2	1.78	58%
	3	2	12%

The experimental results appear to indicate that the performance (i.e., the throughput) of a wireless network under multiple interference types is improved as the number of channels is increased. Nonetheless, the increasing throughput values increase at a decreasing rate as the number of channels is boosted. For instance, while increasing the number of channels from one to two in the baseline model incurs an increase of 100% of throughput, a further increase leads just to 21% additional throughput under experimental conditions.

Also, there is a disparity between the percent change of the throughput for the two interference types studied by this work (interference range versus protocol). Although the total throughput estimate is higher under the protocol model of interference for any number of channels, per Table 5.3, it is conversely true that the percent change of the throughput estimate when increasing the number of available channels is greater for the interference-range model than for the protocol model. One reason for this occurrence might be that, under the protocol model, communications are successful if the distance between two antennas is a multiplier $(1 + \Delta)$ larger than the distance between another set of antennas, while, under the interference-range model, communications are successful based on a fixed communication range, not a multiplier. Relying on a multiplier rather than fixed values might allow for greater flexibility. In any case, the percent change in estimated throughput for the interference-range model for each increase in the number of available channels is almost double that for the protocol model.

5.3 Varying Antenna Types

Each antenna located at a particular node was assigned as a particular type of antenna with a distinct radiation pattern. During each type of experiment, only one type of antenna was used at all potential node locations. All experiments were performed under the baseline parameters unless otherwise stated. However, one should note that, in Table 5.4, the varied parameters are the budget of transmitters for the 5 x 5 array (composed of 25 possible locations for antennas) and the varying antenna types (comprised of an omnidirectional radiation pattern, a directional radiation pattern, and three different sectored radiation patterns). In the case of the sectored radiation patterns, the table indicates to which of the four quadrants of the unit circle the sectors extend, in terms of degrees rather than radians.

Table 5.4

Antenna-type experiment results with 1 channel for the interference-range model

				Antenna Type		
		Omni-directional	**Directional**	**Sectored (0-90°)**	**Sectored (0-180°)**	**Sectored (90-180°)**
	5	0.65	0.54	0.54	0.65	0.25
Budget	15	0.75	0.7	0.7	0.75	0.25
	25	0.75	0.7	0.7	0.75	0.25

Based on the available data, it is apparent that the maximum amount of throughput from these differing antenna types is produced under the omnidirectional-antenna model. Slightly less throughput is produced when using directional antennas, and sectored antennas can have a lot of variation based on the directions in which their signals (radiation patterns) are pointed.

A particularly interesting result of the experiments is that the variation of sectored antennas' broadcasting regions (i.e., the directions in which their radiation patterns are sent out) can lead the sectored antennas' experiments' throughput values to resemble those from omnidirectional and directional antennas in some cases. For instance, when the sectored antennas cover half of the circle, from 0 degrees to 180 degrees, the total throughput is the same as in the omnidirectional case. In addition, when the sector covers only the area from 0 to 90 degrees, the total throughput is equal to the throughput produced in the directional case. Conversely, sectored antennas that are pointed in other directions (e.g., 90 to 180 degrees) can have much different throughput compared to any other existing case studied during experiments.

These attributes can also extend to experiments with varying antenna types under non-baseline conditions. For instance, the data in Table 5.5 shows that the characteristic of having the highest predicted throughput with omnidirectional antennas continues to be true, even when a parameter such as the number of available channels is modified.

Table 5.5

Antenna-type experiment results with 1 channel for the protocol model

		Antenna Type				
		Omni-directional	**Directional**	**Sectored (0-90º)**	**Sectored (0-180º)**	**Sectored (90-180º)**
Budget	5	0.89	0.72	0.72	0.894	0.33
	15	1.09	0.99	0.99	1.09	0.33
	25	1.125	1.05	1.05	1.125	0.33

Additionally, the antenna-type experiments' results hold true even under differing standards of channel availability. Table 5.6 illustrates that omnidirectional connectivity is superior to connectivity with directional antennas, per the results of experiments with the interference-range model. Connectivity with omnidirectional antennas is never exceeded by that of sectored antennas, either. Nonetheless, directional antennas' estimated throughput is not too far behind that provided by omnidirectional antennas, although it is still inferior.

Table 5.6

Antenna-type experiment results with 2 channels for the interference-range model

				Antenna Type		
		Omni-directional	**Directional**	**Sectored (0-90°)**	**Sectored (0-180°)**	**Sectored (90-180°)**
Budget	5	1.43	1.37	1.37	1.43	0.5
	15	1.5	1.4	1.4	1.5	0.5
	25	1.5	1.4	1.4	1.5	0.5

Table 5.7 confirms all of these assertions continue to hold true in the case of a differing number of channels and a differing interference type compared to the experimental baseline.

Table 5.7

Antenna-type experiment results with 2 channels for the protocol model

		Antenna Type				
		Omni-directional	**Directional**	**Sectored (0-90º)**	**Sectored (0-180º)**	**Sectored (90-180º)**
	5	1.75	1.66	1.66	1.74	0.5
Budget	15	1.77	1.72	1.72	1.77	0.5
	25	1.78	1.72	1.72	1.78	0.5

Obviously, as previously discussed, connectivity with omnidirectional antennas is never exceeded by connectivity with any other type of antenna. However, the utilization of some sectored antennas with particular radiation patterns can result in basically the same amount of throughput as the amount produced by using omnidirectional antennas. Everything depends on the orientation and size of the sectored antennas' radiation patterns.

In any case, to summarize, the results presented indicate that, in all cases studied, the omnidirectional-antenna scenarios provide the greatest amount of throughput, with directional antennas providing a close second in terms of predicted throughput. However, sectored antennas with various radiation patterns can provide estimated throughput equal to or less than the maximum estimated throughput provided by omnidirectional antennas.

5.4 Varying Interference Types

Further experiments were conducted on models consisting of the protocol-based model or the interference-range-based model as described by Iyer et al. [2009]. The total throughput produced under these models, with the effects measured across multiple budgets and channels, was studied in depth. Only omnidirectional antennas were used, as a rough proxy for all types of antennas, since omnidirectional antennas were determined to provide the greatest throughput of all of the antenna types studied during experiments. A graphic demonstrating the throughput produced by each type of model is visible in Figure 5.1.

Figure 5.1 demonstrates the throughput found experimentally for each model of interference. At first glance, it is easy to observe that the throughput is estimated to be greater under the protocol model of interference than under the interference-range model of interference; this fact is true for any given number of channels made available during any experiment. Total throughput is 2 data units at most when studied under the protocol model, while it is limited only to about 1.8 when predicted under the interference-range model.

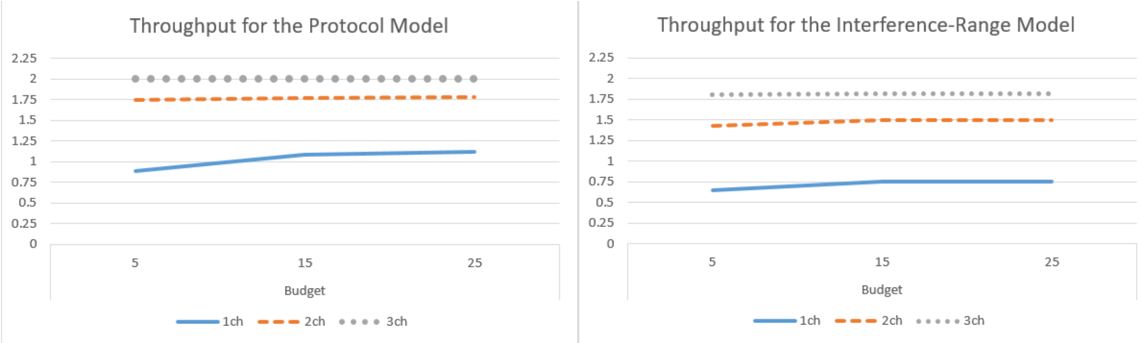

Figure 5.1

Throughput for cases involving different types of interference

Furthermore, the same is true regardless of the number of available channels imposed during the experiments. For instance, the throughput calculated with 2 channels under the protocol model far exceeds the throughput estimated with 2 channels under the interference-range model, and, in fact, is almost the same as the predicted throughput with 3 channels under the interference-range model. In any case, it is quite evident that the protocol-model experiments allow for superior total data transmission compared to the interference-range-model experiments.

It is additionally clear that the throughput value for a given budget of antennas (5, 15, or 25 antennas) is larger in the protocol case than in the interference-range case for the same budget. No matter the number of channels for any given experiment, the estimated throughput value remained greater for the protocol model for a particular antenna budget. Also, with respect to the three budget options, the graphs in Figure 5.1 also demonstrate that the increase in projected throughput from a budget of 5 antennas to a budget of 15 antennas is non-zero, and, in the case of the one-channel scenarios, pretty obvious, judging from the positive slope of the line. However, the increase in estimated throughput provided by increasing the number of antennas in the budget from 15 to 25 turned out to be negligible or even zero, as evidenced by the lines with almost zero or zero slope from the 15-antenna scenarios to the 25-antenna scenarios for a given number of channels. However, the premise that predicted throughput is greater under the protocol-based model rather than the interference-range-based model continues to hold true for a particular antenna budget, all else equal.

5.5 Runtime

One should also consider the runtime of each experiment when varying the number of available channels, the antenna type, and the interference-model type. The experiments were all conducted on a 64-bit Dell Latitude E6540 laptop computer with 16 GB of RAM. All experiments were performed using the Gurobi mathematical programming solver with Python interface. These experiments to investigate total runtime were conducted with a time limit of a maximum of 7,200 seconds, or 2 hours, of runtime; experiments still running at the end of that time were automatically terminated. For experiments terminating after the 2-hour period, the gap between the lower bound and upper bound generated under the solution methodology was reported as a percentage; this gap is equal to the difference between the upper bound and the lower bound divided by the lower bound. The respective runtime values (in seconds) and various gap percentages for non-terminated experimental runs may be examined in Table5.8; for this table, for a given number of channels (Ch) and a given antenna budget (Budget), the data is given for various interference-model types (protocol and interference range) and antenna types (omnidirectional [Omni], directional [Dir], and sectored [Sect]).

Table 5.8

Runtime for experiments under varying conditions

			Antenna Type				
Interference Type	Ch	Budget	Omni	Dir	Sect (0-90º)	Sect (0-180º)	Sect (90-180º)
Interference Range	1	5	1,836.13	123.09	124.04	1,356.89	4.91
		15	2,993.98	156.53	156.77	1,801.01	4.83
		25	27.77	18.89	13.08	26.62	4.83
	2	5	23%	2,780.18	2,812.52	20%	5.13
		15	1,100.55	844.85	855.72	805.39	5.69
		25	303.83	113.05	109.78	207.86	5.77
	3	5	46%	24%	15%	42%	5.92
		15	6,425.49	1,279.58	1,278.02	5,255.53	6.11
		25	897.07	328.05	332.35	650.71	5.95
Protocol	1	5	6,269.83	198.41	198.42	3,836.65	3.72
		15	129%	46%	22%	112%	3.7
		25	28.8	11.44	17.39	28.56	4.25
	2	5	77%	34%	20%	70%	4.02
		15	39%	52%	42%	31%	4
		25	270.05	145.22	165.07	289.08	4.56
	3	5	146.18	6,682.33	6,825.24	104.87	4.11
		15	763.03	763.03	1,071.71	103.81	4.06
		25	148.74	259.96	261.04	104.59	4.86
Average runtime			1,631.65	978.90	1,015.80	1,120.89	4.80
Average gap			63%	39%	25%	55%	-

The data in Table 5.8 provide several notable pieces of information. First, whether under baseline or other parameters, the type of antenna whose experiments required the most time to run is the omnidirectional antenna. As the table reports, the average runtime for an omnidirectional-antenna experiment to reach optimality is more than 1,600 seconds. Furthermore, in the case in which optimality is not reached (and a gap is present between the upper bound and the lower bound found via the solution methodology), omnidirectional antennas have the greatest average gap of all types of antennas studied.

As previously discussed, there is a correspondence between the throughput provided by certain types of antennas. For example, sectored antennas with a range from 0º to 180º provided equivalent throughput. However, the runtime data suggest that, in a majority of cases, a sectored-antenna configuration covering the area from 0º to 180º can provide the same amount of throughput as omnidirectional antennas with less runtime. The average runtime of omnidirectional antennas (1,631.65 seconds) is greatly diminished to 1,120.89 seconds for such sectored antennas without the throughput value being diminished. Further, the average gap for these sectored antennas, which is 55%, is less than the average gap for omnidirectional antennas (63%).

A somewhat similar situation exists for sectored antennas with a range from 0º to 90º and directional antennas. Although the latter have a lower average runtime than the former (978.90 seconds versus 1,015.80 seconds), the additional runtime of less than 60 seconds for these sectored antennas allows for a reduction in the gap between the upper bound and the lower bound from 39% to 25%. In other words, for antennas providing the same amount of throughput, even if the average time to reach optimality is slightly higher for sectored antennas, there is still a much smaller gap when using sectored antennas, in the cases in which optimality is not attained after 7,200 seconds.

A few more trends may be seen in the data from Table 5.8; one of these is that optimality is less likely to be reached when the antenna budget is limited (e.g., 15). In fact, when the budget allows 15 antennas, one-third of all of the experimental scenarios leave a gap at the end of the 2-hour period, including two-thirds of the scenarios under protocol interference. However, for a budget of 5 antennas, a gap is actually more likely under the interference-range model. In general, optimality is not reached much less often (after 7,200 seconds) under the protocol-based interference model, with a higher average gap under the protocol-based interference model as well for experiments not solved to optimality.

Finally, one should note that optimality appears to be reached slightly more quickly with the protocol-based model of interference rather than the interference-range-based model, with an average runtime for completed experiments of 871 seconds for the former versus 899 seconds for the latter. However, on the other hand, the average gap found for experiments not solved to optimality after two hours is significantly larger for the protocol-based model, at 56% on average versus 28% for the interference-range-based model.

5.6 Conclusions

Because wireless network performance is so significant to modern-day technology, improving the performance of wireless networks of all kinds will continue to be important as time goes on. This article attempts to provide some insights by studying the wireless network design problem and looking at how the manipulation of certain parameters, such as the number of channels, affects a wireless network's estimated throughput. These tasks were accomplished through the introduction of a unique mixed-integer program and solving the program via Benders Decomposition. Next, experiments were conducted by varying a single parameter at a time, to determine the effects of changing such parameters.

After performing experiments and gathering computational results, a few key conclusions can be made about acquiring the maximum throughput value possible. First, modifying the number of channels is probably the easiest way to improve predicted throughput. Without having to provide additional physical infrastructure, such as more antennas, increasing the number of channels available can result in as much as a doubled maximum throughput value, per the data in Table 5.3. Even with diminishing marginal returns when increasing from 2 to 3 channels versus increasing from 1 to 2 channels, the increase in estimated throughput still exceeded 10% in all scenarios. Thus, there would be a benefit to increase the number of available channels in a network, at least until the throughput is "saturated" (i.e., has reached a maximum value regardless of more channel availability).

Also, antennas are not required at every location of the possible 25 locations in order to maximize predicted throughput, whether under the baseline scenario or under other parameters. Imposing a budget of fewer than the maximum of 25 antennas (e.g., 15) can result in the same amount of connectivity, such as in the case of a 2-channel interference-range model. Even in the protocol scenario, with a lower budget than 25, the maximum amount of throughput under experimental conditions only decreases by a few percentage points compared to the baseline. An important implication of this fact is that, if antennas have a sufficient communication range in certain directions, not all locations in a node array are necessarily required to have antennas in order to maximize predicted throughput. However, there is a lower threshold at some budget level, as evidenced by the fact that a 5-

antenna budget leads to significantly less estimated throughput than a 15-antenna budget, generally speaking. Of course, no connectivity could occur with a budget of less than two antennas, since connectivity requires at least a transmitter and a receiver. Thus, some number of antennas defines the threshold below which throughput begins to drop, while some number of antennas defines the threshold above which no additional throughput can be achieved.

Furthermore, sectored antennas can provide the same amount of connectivity in some cases if antennas are oriented in a calculated fashion. As previously indicated, some sectored antennas with radiation patterns oriented in particular directions accomplished the same amount of estimated network throughput as omnidirectional antennas or as directional antennas did. Obviously, the total predicted throughput is dependent on the exact size and breadth of the radiation pattern of any sectored antenna, but it seems quite noteworthy that one kind of antennas with radiation patterns only a fraction of the size of other kinds of antennas' radiation patterns could transmit the same amount of data overall under experimental conditions.

An additional issue presented is that a protocol-based interference model generally provided a larger amount of throughput during experimentation than an interference-range-based model did. This result held true for all antenna budget levels, all numbers of available channels, and almost all antenna types. The protocol-based models also had a slightly lower average runtime for all experiments that solved to optimality within the 7,200-second threshold. In any case, the experimental results predict that the protocol-based model would provide superior throughput.

CHAPTER 6

FUTURE WORK

Although this work has gained some useful insights, some future work remains.

For one thing, one ought to consider the number of available channels during future analysis. During the experiments that were conducted for this work, the number of available channels eventually reached a level of diminishing marginal returns, but the "saturation point" (in terms of the lowest number of channels providing maximum estimated throughput) needs to be investigated further. In fact, by looking into the saturation point, one might be able to minimize energy losses from having too many channels available that have no effect or only a negligible effect on total throughput.

One should also investigate similar scenarios for node configurations with varying densities. Rather than simply studying the 5 x 5 baseline array utilized in this work, future scholars can look at scenarios with greater number of antennas within a given area (e.g., 7 x 7 arrays). Obviously, as more antennas are placed within a given area, for a given communication range and a given interference range for each antenna, a tradeoff occurs between increased connectivity (based on shorter distances between antennas) and increased interference (also based on shorter distances between antennas). With respect to interference, future researchers might consider looking at additional models of interference identified by Iyer et al. [2009], such as additive interference, instead of only the protocol and interference-range models discussed in this work.

Exploring the effects on throughput of not making all antennas have the same radiation pattern during a given experiment might prove valuable in the future. Doing so could be accomplished by randomly selecting a radiation pattern for each antenna placed in a location, or perhaps having a node's radiation pattern be a design decision. It might also be interesting to examine scenarios in which antennas' positions are not fixed, but, rather, are changeable; such a case could be accomplished by making the position of antennas subject to change as a variable, perhaps with several discrete options for an antennas' orientation.

Also, potential improvements to the Benders Decomposition solution approach might prove useful. For this paper, some experiments did not conclude during the defined 7,200-second period, and many of those unfinished experiments reported large, double-digit gaps between the lower and upper bounds defined by the Benders methodology. The initial experimental approach of this work considered some improvements, such as the imposition of trust-region stabilization (see Medal [2015] for an example). However, the runtime improvements from such an approach were negligible to non-existent. Perhaps an approach involving such methods as Pareto optimality cuts could help to improve the Benders procedure. Alternatively, a non-Benders perspective, such as a branch-and-price algorithm to serve as the solution methodology, might be beneficial.

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