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The Fracture Behavior of Stitched Sandwich Composites

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The fracture behavior of stitched sandwich composites

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The purpose of this research is to evaluate the influence of through-the-thickness reinforcements on the fracture behavior of stitched sandwich composites and to develop predictive methodologies to aid in simulating their damage-tolerant capability. Sandwich composites are widely used for their high stiffness-to-weight ratio due to their unique material architecture, which is composed of two rigid, outer facesheets that are bonded to a light-weight internal core. However, sandwich composites are limited by their low interlaminar strengths and can develop core-to-facesheet separation when subjected to low out-of-plane loads. In this study, sandwich composites were manufactured with through-the-thickness reinforcements, or stitches, to act as crack-growth inhibitors and to improve interlaminar properties. Stitch processing parameters, such as the number of stitches per unit area (stitch density) and stitch diameter (linear thread density), have considerable influence on the in-plane and out-of-plane behavior of composite structures. A design of experiments (DoE) approach is used to investigate stitch processing parameters and their interaction on the fracture behavior of stitched sandwich composites.

Single cantilevered beam (SCB) tests are performed to estimate the required energy to propagate crack growth, or Mode I fracture energy, during the separation of the facesheet from
the core. Additionally, embedded optical fibers within the SCB test articles are used to determine the internal crack front variation. During testing, unique fracture morphologies are obtained and show dependency on stitch processing parameters. Furthermore, embedded optical fibers indicate that the internal crack front is approximately 10% greater than visual edge measurements, which is primarily attributed to Poisson’s effect. The DoE approach is then used to develop a statistically informed response surface model (RSM) to optimize stitch processing parameters based on a maximum predicted fracture energy. Novel analytical formulations are developed for estimating the mode I fracture energy using the J-integral approach. The DoE approach is then used to inform and validate finite element models that simulate the facesheet-to-core separation using a discrete cohesive zone modeling approach. The predicted load and crack growth response show good agreement to experimental measurements and highlights the capability of stitching to arrest delamination in stitched sandwich composites.
DEDICATION

To my brothers, Donald and Gregory; my wife, Emily; and my mother, Patricia.
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CHAPTER I
INTRODUCTION

1.1 Project Overview

Sandwich composites are widely used in the aerospace industry due to their high flexural rigidity that is associated with their unique material architecture. These structures have superior flexural rigidity due to a light-weight internal core that is bonded to thin, rigid composite facesheets (Figure 1.1). However, debonding between the facesheet and the core in sandwich composite panels can occur at relatively low out-of-plane loads. Additionally, visual inspection of the outermost surfaces provides little indication of internal delamination that may be present. In this study, sandwich composites are manufactured with through-the-thickness reinforcements, or stitches, to act as crack-growth inhibitors and to improve interfacial properties, as illustrated in Figure 1.1.

The objective of this study is to determine the influence of stitching on the fracture behavior of stitched sandwich composites. Therefore, this research develops a stitch parameter design space for future or existing aircraft programs using stitched sandwich composite parts. A design of experiments (DoE) approach is implemented to maximize the mode I fracture energy based on optimal selection of stitching parameters (stitch density and linear thread density). A flow-chart detailing the overall research program is shown in Figure 1.2. A response surface model (RSM) based on stitch design parameters is used to inform a finite element model for the prediction of crack-growth arrestment. Optimal stitch design parameters based upon maximizing
the fracture energy will be incorporated on a structural component, such as the T-38MG fighter airplane strut door.

Figure 1.1 Schematic of a stitched sandwich composite.

Figure 1.2 Flow-chart of the overall research program.
1.2 T-38MG Fighter Strut Door Structural Application

The current T-38MG fighter airplane contains a titanium strut door on the interior outboard surface of each wing. The strut door schematic is shown in Figure 1.3. Boeing has recently produced a redesign of the T-38MG strut door due to the excessive cost associated with fabricating the doors with titanium. This redesign incorporates through-the-thickness stitching to reinforce the out-of-plane properties of the strut door. However, the proposed composite design does not satisfy all of the Federal Aviation Administration (FAA) certification requirements for damage tolerant bonded composite structures. The composite strut door is composed of stitched carbon/epoxy facesheets with an internally bonded close-cell Rohacell™ foam core. This research proposes to improve the current design by introducing stitching through the foam core. These improvements are the following:

1. Using a stitched composite structure will increase cost savings.
2. Stitching will reinforce the bonded foam core and arrest delamination to satisfy FAA requirements.
3. Stitching will allow pathways for resin infusion from a single side. The current composite fabrication process infuses from both sides of the structural component.

To develop an efficient redesign, this research proposes to use a DoE approach to investigate stitching parameters to improve the out-of-plane response of sandwich composites. Optimal design parameters will be used to fabricate a new strut door for the T-38MG fighter with improved through-thickness reinforcement to arrest delamination in the bonded regions with foam core.
1.3 Research Scope

The dissertation herein presents the experimental and computational methodologies at a coupon-level for future use in predicting the structural-scale crack-growth arrestment. The research strategy to achieve these goals is illustrated in Figure 1.4. This study consists of three primary steps: 1) literature review, 2) experimental phase, and 3) response surface and computational modeling phase. A literature review is performed to understand the state of the art regarding the influence of stitching on the out-of-plane behavior of polymer matrix composites. In particular, this review provides insight into the mode I and mode II fracture properties, interlaminar properties, and impact behavior of stitched composites. Experimental methods of stitched composites are investigated to inform an experimental test plan that uses DoE, which minimizes the required number of experiments to fully characterize the out-of-plane behavior of stitched sandwich composites within a selected design space. Based on the DoE approach, a
RSM is developed to determine optimum stitch parameters. These parameters and experimental test results are used to inform a finite element model for predicting crack-growth arrestment.

1.4 The Dissertation

This section outlines the dissertation and includes a summary of each of the remaining chapters. Each of the remaining chapters are formatted such that they represent articles that have been or are to be submitted to leading journals within the field of composite materials.

Chapter 2 is a literature review that describes the current state of the art on the out-of-plane behavior of stitched composite materials. This paper summarizes results from over one hundred papers on the influence of stitch parameters on fracture energy, interlaminar strength, and impact characteristics of stitched composite laminates, sandwich composites, and
high-temperature composites. Additionally, charts and tables are provided that summarize the typical materials and stitch processing parameters used by researchers within the open literature.

Chapter 3 describes the design of experiments approach that is used to characterize the influence of through-the-thickness reinforcements on the mode I fracture energy. A response surface model was developed to predict the fracture energy of stitched sandwich composites based on the selected input factors (stitch density, linear thread density, and facesheet thickness). Unique fracture surface morphologies are exhibited during mode I testing that are dependent on stitch processing parameters. Lastly, an optimum selection of stitch processing parameters can be obtained to maximize the mode I fracture energy.

The analytical formulations to determine the mode I fracture energy of stitched sandwich composites is described in Chapter 4. Current test standards to estimate the fracture energy of composite materials assume small-scale yielding near the crack front and do not consider large plastic zone sizes due to large-scale bridging. Therefore, this study explores the use of the J-integral approach to better approximate the fracture energy in a stitched sandwich composite specimen that develops large-scale bridging.

Chapter 5 describes the computational approach to simulate the facesheet-to-core separation during single cantilevered beam tests. A discrete cohesive zone modeling approach is used to simulate the separation of the facesheet to the core. A trilinear traction-separation law was used to represent the failure process of the through-the-thickness reinforcement within a 2D and 3D finite element model. Good agreement was obtained between the experimental measurements and predicted finite element analysis results. Experimental measurements were obtained from SCB tests that are discussed in Chapter 3.
Chapter 6 describes the methodology to estimate internal crack growth within mode I fracture specimens using embedded optical fibers near the crack interface. In this chapter, a numerical approach to estimate the internal fracture toughness using a Lagrangian cross-correlation method is demonstrated. The strain measurements from embedded optical fibers are correlated to the internal location of the delamination front. Double cantilevered beam test articles were fabricated from non-crimped carbon fabric using a vacuum-assisted resin transfer molding process. Estimated internal crack lengths, fracture toughness, and flexural moduli are in excellent agreement with surface measurements. The variation in the strain energy release rate along the delamination front is obtained using multiple optical fiber passes.

Chapter 7 describes the use of determining the internal crack front within stitched sandwich composites using the Lagrangian cross-correlation method. Optical fibers were embedded within stitched sandwich composites prior to resin infusion and near the through-thickness reinforcements. During single cantilevered beam tests, the internal strain distributions were monitored, and subsequent internal crack growth was determined. Comparisons are made to existing finite element results obtained from Chapter 5.

Chapter 8 summarizes the contributions of this research to the current state-of-the-art. Furthermore, recommendations for future work on the topic are discussed. Supplemental information is provided in the appendices.
2.1 Introduction

Lightweight materials are essential in the aerospace industry to reduce fuel consumption and emissions, and to increase aircraft range and payload. In particular, fiber-reinforced composite materials are extensively used due to their tailorability and their high in-plane specific strengths and stiffnesses. However, their applicability can be limited by their low interlaminar shear and interlaminar tensile strengths; this is due to a property mismatch between the composite laminae [1] and the matrix being the primary load carrier for out-of-plane loads. For example, Nie et al. [2] reported the in-plane tensile and interlaminar shear strengths for a carbon/silicon carbide composite are 234.3 MPa and 29.1 MPa, respectively. Increasing the property mismatch, as a result of increasing the angle between subsequent plies, was shown to lower interlaminar shear strength and delamination resistance under out-of-plane loading [3]. These low interlaminar properties in layered composites can result in delaminations, which can be due to impacts or other excessive interlaminar loads during part assembly or service operations. These composite parts typically undergo costly nondestructive evaluations and, depending on the delamination size and location, may need to be replaced.

Novel approaches have been developed to reinforce composite materials in the through-the-thickness direction to prevent delamination. These methods include z-pinning [4-7],
Z-pinning is typically performed by embedding small diameter carbon rods in a polymeric foam preform. An ultrasonic horn is used to drive the pins through the thickness of an uncured prepreg laminate prior to cure [11]. The needling process uses downward-barbed needles to re-orient in-plane short fiber mats in the through-thickness direction of dry composite preforms [8]. Weaving, stitching, and tufting are traditionally performed using an industrial sewing machine or loom to introduce through-thickness reinforcement before resin infusion. Tufting is a form of stitching that uses non-interconnected stitches to reinforce polymer composites in the through-thickness direction. Overall, stitching is a simple 3D reinforcement method that provides similar interlaminar improvements to 3D weaving [5, 9] and can be used for any conventional dry composite preform that is commercially available. This capability allows greater design flexibility in the composite layup, making stitching of great interest in improving the performance of composite aerospace structures.

The stitching process involves sewing aramid, carbon, polyester, glass threads, or yarns, into a non-crimped fabric (NCF) or woven dry preform using an industrial or robotic sewing machine. A thread indicates a twisted assemblage of tows, where a tow is an untwisted collection of fiber filaments. Threads can be twisted together to provide enhanced tensile thread strength. NCFs are fabrics that consist of multiple layers of parallel tows that are held together using non-structural (low filament count) threads, typically using a polyester material. Stitching can be performed using prepregs prior to cure, but damage such as in-plane fiber rupture and stitch needle breakage can occur during the stitching process [9]. The formation of the stitch during the sewing process is referred to as the stitching style. The most common styles of stitches are the modified lock stitch, lock stitch, and chain stitch, as shown in Figures 2.1(a), 2.1(b), and 2.1(c),
respectively. The lock stitch is an interlock of stitching thread at the mid-plane of the composite laminate and has been shown to create stress-concentrations and reduce in-plane properties [1, 12]. The stress concentrations have been mitigated by using a modified lock stitch where the thread interlock is on the surface of the preform. Both the lock and modified lock stitches require a needle thread and a bobbin thread, which are on opposite sides of the laminate. The bobbin thread is used to interconnect adjacent stitching on both sides of the laminate to enhance the out-of-plane behavior as compared to a non-interconnected stitching style such as tufting. For the chain stitch, a single thread is used to stitch fabric from one side of the composite laminate. Stitches are periodically spaced and characterized by the stitch pitch (P) and spacing (S), as shown in Figure 2.1(d). The stitch pitch (P) is defined as the distance between two adjacent stitches along the same stitch seam, and the stitch spacing (S) is the distance between two adjacent seams of stitching.

Figure 2.1 Stitching styles showing (a) modified lock [11], (b) lock [11], (c) chain [11] and (d) modified lock stitch schematic showing stitch parameters [13].

Stitch processing parameters have considerable influence on the mechanical performance in both the in-plane and out-of-plane directions. These processing parameters include the thread
material, linear thread density (mass per unit length of the thread), thread finish, stitch density or the number of stitches per unit area \((1/(P\cdot S))\), stitching distribution or pattern, stitch style, and stitch pretension [1]. Various configurations of these parameters have produced contradictory results with respect to their in-plane mechanical behavior, but generally stitching composites results in an approximately 10% reduction in the in-plane mechanical properties [9, 10]. Therefore, careful attention to stitch parameters is needed for an effective design of 3D reinforced composite structures.

The Pultruded Rod Stitched Efficient Unitized Structure (PRSEUS) concept [14], developed by Boeing, NASA, and the United States Air Force, uses a novel selective stitching approach as a means of joining major structural elements (skin, stringer, and frames) prior to resin infusion and subsequent cure. The reduction of in-plane properties is minimized by stitching structural elements where the delamination due to interlaminar peel stresses at the joined regions are the most critical [15]. Delamination due to interlaminar peel stresses are the primary form of failure in overlapping joints, and stitching has shown to increase the lap joint strength by approximately 60% to 175% [15-23]. Quasi-static structural testing of notched PRSEUS panels has also shown that the selective stitching approach allows unitized structural members to effectively arrest delaminations from barely visible impact damage and translaminar crack growth [14]. As a result of this research, a significant amount of exploration of the out-of-plane behavior of stitched polymer composites was performed in 2007 [15, 18, 19, 24-36] and 2008 [2, 20, 37-48]. Few stitched composite studies are found between 2000-2004, as shown in Figure 2.2. Prior to 2000, NASA enacted the Affordable Composite Technology (ACT) program to develop a database of composite technology for implementation in production aircraft. A
A considerable amount of research on stitched composite structures was performed during this time and is summarized in reference [49].

Figure 2.2   Number of stitched composites peer-reviewed journal papers and government technical documents from 1987-2019.

The in-plane mechanical properties of stitched polymer matrix composites (PMCs) with respect to stitch processing parameters have been well characterized and reviewed [1, 9, 10]. However, the fracture behavior and associated interlaminar properties have not received as much attention and are not well understood for a variety of composite material systems and stitch parameters. For example, ceramic matrix composites (CMCs) have relatively low in-plane mechanical properties due to their manufacturing processes [50], but their interlaminar and flexural strengths have been shown to increase by approximately 20% by adding through-thickness stitching compared to an unstitched structure [2]. In this study, a review of the fracture behavior (mode I, mode II, and mixed-mode) of stitched composites, and the necessary experiments needed for their characterization are presented. Additionally, a summary of the
latest advancements in stitched composites with a focus on the out-of-plane behavior (interlaminar strength and impact) and for select material systems (PMCs, CMCs, carbon/carbon composites and sandwich composites) is provided.

2.2 Stitching Parameters for Fracture Characterization

Studies focused on different stitch parameters to evaluate the fracture and interlaminar properties of stitched polymer matrix composites are summarized in Table 2.1. The percentage of papers in each category was estimated based on the total number of unique papers. In many of these studies, linear thread density (40.6%) and stitch density (56.3%) have been investigated because these two stitch parameters primarily influence the out-of-plane behavior of layered composites. As such, the influence of stitch density and linear thread density on the out-of-plane behavior are the primary stitch parameters discussed in this review. Studies that used various thread and stitch densities are shown in Table 2.2. This listing includes papers that did not investigate stitch parameters, but evaluated a single set of stitch parameters on the out-of-plane response. Additionally, the ranges represent a standard deviation of stitch density and linear thread density from all referenced papers that evaluated the effectiveness of stitches on the out-of-plane behavior of PMCs. Moderate levels of stitch density (0.0025 stitches/mm$^2$ to 0.05 stitches/mm$^2$) have been investigated in a majority (85.4%) of studies. Additionally, most of these studies (90%) investigated linear thread densities that range between 19 denier to 1800 denier. Very few articles report the influence of other stitching parameters such as pretension, stitching style, and thread twist. Pretension has shown to significantly influence out-of-plane properties through finite element modeling [51] and analytical approaches [52]. Experimentally quantifying the amount of pretension is extremely difficult and validation of these modeling approaches requires more attention. Several studies have also investigated
different stitching thread materials, such as Kevlar™ [19, 53-55], glass [19], carbon fibers [19, 53-55], Vectran™ [55], polyamide [56], and phenoxy [56]. Lastly, researchers have also used analytical [51] and finite element modeling [57] approaches to investigate the influence of stitch thread stiffness on the delamination resistance of polymer matrix composites.

Table 2.1  
Studies that evaluated influence of stitching parameters on the out-of-plane composite behavior.

<table>
<thead>
<tr>
<th>Stitch Parameter</th>
<th>References</th>
<th>Percent of Papers (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Thread Density</td>
<td>[19, 53, 54, 58-67]</td>
<td>40.6</td>
</tr>
<tr>
<td>Stitch Density</td>
<td>[2, 52-54, 58, 61-64, 67-75]</td>
<td>56.3</td>
</tr>
<tr>
<td>Stitch Pattern</td>
<td>[29, 30, 46, 52, 64, 76]</td>
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<tr>
<td>Pretension</td>
<td>[51, 52]</td>
<td>6.3</td>
</tr>
<tr>
<td>Stitch Style</td>
<td>[19, 77]</td>
<td>6.3</td>
</tr>
<tr>
<td>Twist</td>
<td>[19]</td>
<td>3.1</td>
</tr>
<tr>
<td>Ply Orientation</td>
<td>[19]</td>
<td>3.1</td>
</tr>
</tbody>
</table>

Table 2.2  
Studies on stitch and linear thread densities.

<table>
<thead>
<tr>
<th>Stitch Parameter</th>
<th>Range</th>
<th>References</th>
<th>Percent of Papers (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stitch Density (Stitches/mm²)</td>
<td>0.0025-0.050</td>
<td>[2, 19, 25, 28, 29, 41, 42, 46, 52-56, 58-64, 66-75, 78-82]</td>
<td>85.4</td>
</tr>
<tr>
<td></td>
<td>0.051-0.0950</td>
<td>[37, 53, 54, 58, 64, 77, 78, 83]</td>
<td>19.5</td>
</tr>
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<td></td>
<td>0.0951-0.140</td>
<td>[53-55, 57, 58, 61, 62, 64, 67, 69, 74, 75, 82, 84, 85]</td>
<td>36.6</td>
</tr>
<tr>
<td></td>
<td>&gt;0.140</td>
<td>[2, 78]</td>
<td>4.9</td>
</tr>
<tr>
<td>Linear Thread Density (Denier)</td>
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<td>56.7</td>
</tr>
<tr>
<td></td>
<td>601-1200</td>
<td>[53-55, 59-61, 64, 65, 77]</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>1201-1800</td>
<td>[28, 29, 52, 59, 61, 64, 70, 80, 84]</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>&gt;1800</td>
<td>[61, 66, 71, 72]</td>
<td>13.3</td>
</tr>
</tbody>
</table>
Mode I fracture energy, or the energy required to promote an opening mode of crack growth, can be increased by incorporating through-the-thickness stitching [19, 42, 51, 57, 59]. Under mode I conditions, stitches bridge the opposing crack faces and induce internal traction stresses to resist crack growth. Researchers have noted that stitching does not influence the initial fracture energy when the delamination front is approximately 10 mm from the initial stitch row [68]. The fracture energy during propagation and near a stitch seam results in a significantly greater mode I fracture energy than at crack initiation. The relative influence of stitch density on the normalized mode I fracture energy for select linear thread densities and stitching thread materials from published data is shown in Figure 2.3. The maximum fracture energy of a stitched composite laminate is normalized by the fracture energy of its unstitched counterpart. Low stitch densities correspond to relatively large distances between adjacent stitches, whereas high stitch densities correspond to small distances between adjacent stitches. Overall, increasing the stitch density linearly increases the mode I fracture energy for each material due to higher traction loads from stitch bridging during delamination propagation. During failure of the stitch under mode I delamination, multiple stitch rows have been observed to fail during double-cantilevered beam (DCB) testing [59]. Additionally, other studies [28, 29] have found that the relative pattern or distribution, while maintaining a constant stitch density, influences the mode I fracture energy. Stitching has also been shown to provide the same delamination resistance for any in-plane fiber orientation [42].
Figure 2.3  Influence of stitch density on the normalized mode I fracture energy for select stitching thread materials.

The normalized fracture energy can be further improved by increasing the linear thread density of the stitching thread. The influence of linear thread density on the normalized steady-state mode I fracture energy for select stitch densities and stitching thread materials is shown in Figure 2.4. Overall, increasing the linear thread density proportionally increases the normalized fracture energy. Additionally, the increase in fracture energy appears to be highly dependent on the stitching thread material. For an E-glass thread material (0.04 stitches/mm²), increasing the linear thread density from 600 denier to 1200 denier increases the estimated fracture energy by approximately 174%. A 400 denier carbon thread shows a more significant increase (~900%) in the normalized fracture energy for stitched densities greater than 0.04/mm². These studies may indicate that there is a stitch parameter interaction between linear thread density, stitch density, and stitch material on the mode I fracture energy.
Figure 2.4 Influence of linear thread density on the normalized mode I fracture energy for select stitching thread materials.

Increasing the linear thread density can result in a reduction in the in-plane mechanical properties due to increased fiber waviness and the formation of resin pockets near the stitch [9]. For example, Heb et al. [31] reported a 10% to 14% reduction in the in-plane properties for carbon/epoxy laminates using 612 denier and 1224 Denier E-glass stitching. Also, untwisted carbon fiber threads within woven carbon fabric have shown to increase mode I interlaminar fracture energy without impacting in-plane properties [19]. A uniform distribution of untwisted filaments within the displaced region of in-plane fibers is developed, thereby decreasing the resin-rich pockets near the stitching regions. Heb et al. [19] reported that the steady-state mode I fracture energy is primarily controlled by the thread diameter or linear thread density. However, this finding is somewhat contrary to other reported data, as shown in Figures 2.3 and 2.4. The mode I fracture energy also appears to be highly dependent on thread stiffness and stitch tensile strength [51, 57]. For example, mode I fracture energy is increased by a factor of 2 and 15 when using 612 denier E-glass thread and 756 denier carbon thread, respectively. These results agree
with several computational studies that have focused on the influence of thread stiffness, failure load, and crack length [51, 57]. In particular, Glaessgen et al. [57] used a virtual crack closure technique to investigate the influence of thread stiffness on the mode I strain energy release rate for double cantilever beam (DCB) specimens. Increasing the thread stiffness caused an increase in the stitch failure force [57].

2.3 Mode II Fracture Energy

Under mode II fracture conditions, delaminations develop due to internal shear stresses between the composite laminae and result in a sliding action between two opposing crack faces. During a mode II delamination, the stitches resist the crack front through a “plowing” action, in which the stitches deform the surrounding matrix near the delaminated interface. The additional energy expended to resist crack growth is primarily due to the deformation of the surrounding matrix and not the failure of the through-thickness stitching [86]. This plowing action is a result of a snubbing phenomenon, first identified by Cox [87] and Cartie [88]. Snubbing refers to the significantly large and non-uniform shear stresses at the delaminated interface when a bridged through-thickness reinforcement laterally deflects under mode II conditions. The influence of the snubbing effects can also be induced under mode I conditions when specimens are subjected to large displacements. After deformation of the matrix due to plowing, the through-thickness reinforcement fails primarily due to shear plasticity, internal splitting, and frictional pullout [88].

Similar to mode I conditions, increasing the stitch density can increase the normalized mode II fracture energy. The influence of stitch density on the normalized mode II fracture energy for select linear thread densities and stitching thread materials is shown in Figure 2.5. Increasing the stitch density increases the mode II fracture energy by up to 330% when compared to its unstitched composite counterpart for carbon and Kevlar stitching materials [54].
Unlike mode I fracture behavior, neither the thread material nor thread strength seems to have a significant impact on mode II fracture energy [59]. For example, 675 denier polyester thread at a low stitch density (0.02 stitches/mm$^2$) has the same relative performance as that of 756 denier carbon thread with a greater stitch density (0.08 stitches/mm$^2$). This may be greatly influenced by the relative deformation of the matrix as stated in the first paragraph. Additionally, Jain et al. [54] noted that stitching does not significantly influence mode II fracture energy at crack initiation. Wood et al. [29] reported that the number of stitches along the crack front initially improved the mode II fracture energy, but this improvement was not apparent for long crack growth with significant stitch bridging zone lengths. Furthermore, stitching in carbon/epoxy laminates also reduces unstable crack propagation that is normally associated with end-notch flexure (ENF) testing [54]. Prior to failure, the crack front wraps around the stitch as the stitches bridge the crack plane [29].

Figure 2.5 Influence of stitch density on the normalized mode II fracture energy for select stitching thread materials.
The influence of linear thread density on the steady-state mode II fracture energy for select stitch densities is shown in Figure 2.6. Increasing the linear thread density of Kevlar thread increases the fracture energy of PMCs, whereas the polyester thread does not improve the fracture energy for moderate (675 denier) to large (1350 denier) linear thread densities. Additionally, relatively high strength and stiffness stitching threads, such as carbon thread, do not appear to significantly increase the mode II fracture energy as compared to a polyester stitching thread. Further increases in the mode II fracture energy may be attained by increasing the stitch pre-tension. Jain et al. [54] analytically showed that the mode II fracture energy increases at a greater rate due to larger surface tractions near the crack tip generated by greater stitch pretensions. The steady-state fracture energy can also be improved by altering the through-thickness orientation of the stitches. Stitches that are diagonally oriented against the direction of crack growth, as shown in Figure 2.7, have been shown to have twice the mode II fracture energy as compared to stitches oriented with the direction of crack growth [59]. This behavior is primarily attributed to an increase in the shear stiffness along the delamination plane [59]. Stitches that are oriented with the direction of crack growth result in lower fracture energy due to stitch frictional sliding, but greater sustained load at the delaminated interface. Stitches that are oriented against the direction of crack growth will fail due to high shear stresses that result in unstable crack growth, but greater crack-growth resistance.
2.4 Mixed-Mode Fracture Energy

In many practical situations, structural components are subjected to combined loads during service operation. As a result, the development of delamination is due to a combination of tensile and shear loading near the crack front. The normalized mode I and mode II fracture energy with respect to the modal ratio is shown in Figure 2.8 [89]. The modal ratio is the mode I fracture energy divided by the mode II fracture energy. A 0% and 100% modal ratio correspond to a pure mode II and mode I fracture energy, respectively. For relatively low modal ratios
(< 30%), Trabelsi et al. [89] experimentally observed that stitch failure does not occur in a woven carbon/epoxy composite material system. Increasing modal ratio from 30% to 70% increases the normalized mode I and mode II fracture energies by a factor of 4.0. The increase in the overall fracture energy under mixed-mode conditions has also been observed by other researchers [90]. Further increases in the modal ratio results in a decrease in the mode II fracture energy as the modal ratio approaches 100% (pure mode I fracture).

![Figure 2.8](image_url)  Normalized mode I and mode II fracture energy as a function of mode mixity [89].

2.5 Interlaminar Strength

The primary test method to evaluate the interlaminar shear strength of stitched CMCs is the compressive interlaminar shear test per ASTM standard C1292-12 [91]. Figure 2.9 shows the influence of stitch density (0.01-0.16 stitches/mm²) on the normalized interlaminar shear strength of CMCs for two selected sizes (1K and 3K) of carbon fiber. Increasing the stitch density is
shown to increase interlaminar shear strength by approximately 10% - 30% using 1K carbon fiber tows as stitching thread materials in CMCs. Similar increases in the interlaminar shear strength has also been observed for polymer matrix composites [19]. High strength carbon fiber stitching tows (T300 3K) resulted in lower interlaminar shear strength as compared to T300 1K carbon fiber tows. This behavior is due to the ineffectiveness of the chemical vapor infiltration process to fill the displaced volume of the in-plane fibers near large carbon fiber tows [78]. Therefore, large voids developed near the stitches. The primary failure mode observed in stitched CMCs was delamination with secondary micro-cracks [2]. The stitching fibers completely ruptured after the silicon-carbide delaminated during the interlaminar shear test, unlike traditional PMCs where a plowing action can occur for ductile matrices [2]. Nie et al. [2] also noted that stitching improved the in-plane tensile strength by approximately 27% with no influence on the flexural strength for high stitch densities (SD ≥ 0.04 stitches/mm²).

Measurements of the interlaminar tensile strength of stitched composites are not presently available in the literature. This is typically because the effectiveness of the through-thickness reinforcement is primarily observed after the stitch has bridged the crack. It is difficult to estimate the tensile strength due to a significant reduction in area after the bulk material has delaminated. However, the interlaminar or flatwise tensile test has been used to characterize the load-displacement response (or traction-separation response) after delamination has occurred. A more detailed discussion is provided in the test methods and analysis portion of this manuscript.
Figure 2.9   Influence of stitch density on the normalized interlaminar shear strength for carbon stitching.

2.6  Impact Behavior

During low-velocity impact events, delamination is initiated by the development of translaminar microcracks. These microcracks initially develop due to a property mismatch between the matrix and reinforcement material; however, they can also occur due to particle inclusions, resin-rich regions, or residual stresses. The translaminar microcracks grow to nearby plies and are halted by nearby plies of different orientation as the impact energy approaches the threshold energy at which delamination occurs [92]. Above the threshold energy, large out-of-plane normal stresses (through the thickness) result in mode I delamination between adjacent plies as a result of nearby microcracks [93]. Further away from the origin of impact, the laminate develops high interlaminar shear stresses due to local bending. This deformation results in the formation of mode II delaminations. Large delamination zones can occur during impact and are typically not visible at the surface.
Due to the inherent discretization of the stitching process, the initiation of delamination from microcracks due to impact is not halted by the presence of stitching. However, delamination growth can be significantly arrested or minimized by the through-thickness reinforcement, resulting in a reduction in the delamination area [41, 67, 74, 75]. Researchers have also reported that the improved impact damage resistance was only observed for quasi-isotropic laminates with a thickness greater than approximately 1.9 mm [85]. The same results were not obtained when using a cross-ply configuration of a similar thickness [25]. The normalized delamination area with respect to impact energy for different linear thread densities, stitch densities, and layup configuration is shown in Figure 2.10. Overall, the normalized damage area is relatively independent of the impact energy for select stitch densities and linear thread densities. Very little differences are observed in the damaged area between a cross-ply and a quasi-isotropic laminate at relatively low stitch densities ($< 0.04$ stitches/mm$^2$). This similarity suggests that damage associated with an increase in the interlaminar stresses between laminae, due to a greater property mismatch in the cross-ply laminate, is relatively contained between adjacent stitching regions. From a damage tolerance perspective, this characteristic may increase the tailorability of composite designs that were otherwise unachievable without through-thickness reinforcements.

Stitch density is the primary stitch parameter that arrests and delays delamination for quasi-istropic and cross-ply laminates with a carbon/epoxy material system, as shown in Figure 2.10. Increasing the stitch density from 0.028 stitches/mm$^2$ to 0.111 stitches/mm$^2$ decreases the normalized delamination area from $93\%$ to $53\%$ when compared to its unstitched counterpart using Vectran thread with a carbon/epoxy laminate. This decrease in the delamination area did not result from increasing the linear thread density of the stitching thread,
but rather increasing the linear thread density from 200 denier to 400 denier decreased the damaged area by approximately 10% for relatively low stitch densities (≤ 0.028 stitches/mm²). Greater reductions in the delamination area may be achieved by increasing the linear thread density or other stitch parameters such as thread stiffness or thread pretension. Additionally, there were no observed differences in the damaged area for high stitch densities (≥ 0.111 stitches/mm²) for a quasi-isotropic carbon/epoxy material system with 200-400 denier Vectran threads. Increasing the linear thread density (> 400 denier) increases the fiber waviness near the stitched zones and thus increases the resin-rich regions. These resin-rich regions act as crack initiators, which may promote additional delamination [75]. Tan et al. [69] performed quasi-static indentation tests and found that the incipient damage load for laminates with high linear thread densities is lower than in their unstitched counterpart; this difference was primarily due to matrix cracks near the stitching loops [94, 95]. The formation of microcracks also reduces the fatigue life [93, 96], increases the development of delaminations associated with impact [66, 71, 97], and increases gas permeability [40, 98-101] during service operations.
Significant attention has also been given to reinforcing sandwich composites with stitching to enhance the core-to-facesheet separation resistance under low-velocity impact. Internal cores such as polymeric foams have been primarily used since the through-thickness stitching produces resin pathways that form stitch-resin columns during the resin transfer molding process. Stitching increases the weight by approximately 1% [66]. Furthermore, stitches that are oriented at 45° have been shown to enhance the in-plane flexural rigidity, in-plane shear, and out-of-plane compressive strength [71, 97]. Under impact, traditional damage modes [102, 103] of foam core sandwich composites subjected to impact appear to be absent when the sandwich composite is reinforced with through-thickness stitching [72]. Specifically, the bottom surface of the impacted sandwich composite does not develop delaminations as typically observed in sandwich composites without through-thickness reinforcement. In stitched sandwich composites subjected to low-velocity impact, the primary form of failure is stitch-matrix column...
buckling and delamination of the top-most surface. A high density of stitches has also been shown to stiffen and strengthen composite structures, allowing for greater energy absorption and facesheet delamination suppression during the impact of orthogonally-stitched [70] and obliquely-stitched [72] sandwich composites. However, incipient failure during impact may occur earlier in the stitched component due to the weak interface between the stitching and facesheets [72, 97]. Stitched regions subjected to impact have also been reported to undergo larger regions of core cracking than unstitched panels [72].

2.7 Test Methods and Analysis to Characterize Stitched Composites

Test methods to evaluate the fracture, interlaminar, and impact characteristics of PMCs, sandwich composites, and high-temperature composites (carbon/carbon composites and CMCs) are shown in Table 2.3. Generally, PMCs have been used to evaluate the influence of stitching on their interlaminar and fracture behavior. Few studies have experimentally investigated the influence of stitching using sandwich composite and high-temperature composite material systems. The DCB [104], ENF [105], and low-velocity impact test methods [106] have been the principal experimental methods to evaluate the effectiveness of stitching in a PMC. The DCB and ENF test methods are used to estimate the mode I and mode II fracture energy, respectively, whereas low-velocity impact test methods are used to estimate the energy absorption characteristics.

Estimating the fracture energy of stitched composites has resulted in the development of new test methods [84, 107] and the modification of existing standards [108]. Current DCB test standards use an embedded insert at the specimen midplane to act as a delamination or debond site. Loading blocks or piano hinges are used to apply an external load above and below the debond region, resulting in flexural bending loads that promote crack growth. The propagation of
the debond occurs when the internal tensile stresses at the crack front exceed the interfacial strength between the plies. As the delamination front approaches a stitched seam, the internal tensile stresses must exceed the tensile strength of the stitched seam before further crack propagation can occur. As a result, a high-rotational constraint is developed at the crack front due to the through-thickness reinforcement. This constraint subjects the delaminated arms to greater bending stresses and induces flexural failure. Guenon et al. [109] also observed that crack growth can directionally deviate, or branch to adjacent plies, instead of propagating in a self-similar manner. Significant stitch bridging can also occur during fracture, which may invalidate the small plastic zone assumption within linear elastic fracture mechanics that is used to develop these test methods. Therefore, current test standards are not considered suitable to evaluate stitch composite laminates. The influence of crack length relative to the stitch location has also not been investigated, although a significant amount of fracture data is available for stitched PMCs. Alternatives to the DCB, ENF, and mixed-mode bending (MMB) fracture tests have been developed to address the high rotational constraint by inducing additional tension to delay failure [84, 107]. Other researchers have also reinforced the DCB and ENF fracture specimens with aluminum, steel, or composite doubler plates to prevent failure of the delaminated arms [28, 29, 53, 84]. The thickness of the doublers used in these studies range from 2 mm to 10 mm in thickness. Lastly, adding doublers to the specimens has shown to develop stable crack growth as compared to specimens without doublers [89]. Further research on the sizing of the doublers for select stitch densities and linear thread densities needs to be performed.

The addition of the doubler plates for the DCB or ENF test specimens alters flexural rigidity and the location of the neutral axis of the delaminated arms that are subjected to bending. Modified beam theory may not provide an accurate estimate of the fracture energy due to the
homogeneous specimen stiffness assumption. Reeder [110] used a strength of materials approach to estimate the shift in the neutral axis to correct the flexural rigidity of the delaminated arms with doubler plates to estimate the fracture energy. Other methods, such as the modified compliance calibration method, do not require this correction as the bending stiffness of the specimen is measured directly. The analytical approach proposed by Reeder was validated using MMB fracture tests, and similar results have been obtained from another study [89].

Furthermore, the correction proposed by Reeder is not valid for delaminations that occur in bi-materials, such as core-to-facesheet separation in sandwich composites. Due to an interfacial property mismatch between the facesheet and core, sandwich composites do not develop a pure mode I delamination response during modal fracture tests [111, 112]. Although doubler-reinforced DCB tests have been used to evaluate the effectiveness of stitching on the mode I fracture energy of sandwich composites [108], the unreinforced single cantilever beam test appears to be the most suitable standardized test method to evaluate the mode I fracture energy for stitched sandwich composites [113-115]. This difference is due to a dependency on the thickness of the doubler in estimating the fracture energy [116]. Therefore, it is not recommended to use doublers for bi-materials to evaluate the effectiveness of stitching without further investigation.

To improve the predictive capability of crack progression in stitched composites, experimental tests such as interlaminar tension tests [6, 60, 62, 65, 68] and interlaminar shear tests [117] are often used to characterize the traction-separation response of the through-thickness reinforcement. Although, several analytical micromechanical models have been developed by Jain and Mai [118-120], Cox [87, 121, 122], and Plain [123] to predict the traction-separation response of a through-thickness reinforcement. These analytical methods do
not include pullout behavior after the failure of the through-thickness reinforcement. Tan et al. [65] used interlaminar tension tests to identify essential features of the fracture process of through-thickness stitching. The primary failure mechanisms during mode I fracture were identified as 1) debonding of the thread/matrix interface, 2) slack absorption, 3) thread failure, and 4) pullout friction [65]. Thread failure can occur at the interlocked region near the bobbin thread or near the crack interface, which can lead to variation in the measured load at which frictional sliding occurs [60]. Overall, the stitch traction-separation responses to simulate mode I delaminations were incorporated as a material model for nonlinear spring elements in a cohesive zone finite element model. Excellent correlation of the load-displacement responses between the experimental measurements and the finite element modeling predictions were achieved [62].

A lack of experimental data to verify analytical and computational approaches exists for unique material systems such as sandwich composites and high-temperature composites (carbon-carbon composites and CMCs), as shown in Table 2.3. High-temperature composites have received very little attention with regard to their interlaminar and fracture properties, which may be the result of the restricted access due to governmental regulations. Current available literature shows that in-plane mechanical properties and interlaminar shear strengths can be improved by incorporating stitches in a high-temperature composite [2, 9, 58]. The only test that has been used to evaluate the effectiveness of stitching in high-temperature composites is the compressive interlaminar shear test [124]. Corresponding test methods to evaluate unstitched high-temperature composites are the DCB and single-edge notch bend tests. These studies show that complications can occur during testing due to the complex woven architecture of high temperature composites. These complications include tow bridging [125], material nonlinearity
from pre-existing voids [126], and energy-releasing mechanisms outside of the initial delamination plane [127].

<table>
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<td>Mixed-mode fracture energy</td>
<td>[89, 90, 107]</td>
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<td>[6, 55, 60, 62, 65, 68]</td>
<td>[70] - [129]</td>
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</tr>
<tr>
<td>Single-edge-notch Tension [137]</td>
<td>translaminar fracture toughness</td>
<td>[80]</td>
<td>- - [138-140]</td>
</tr>
<tr>
<td>Interlaminar Shear [117, 124]</td>
<td>Interlaminar shear strength</td>
<td>-</td>
<td>- [2, 78] [141]</td>
</tr>
<tr>
<td>Low Velocity Impact [142]</td>
<td>Energy absorption</td>
<td>[25, 41, 67, 74, 75, 81, 85]</td>
<td>[66, 72, 73, 143] - [144-146]</td>
</tr>
</tbody>
</table>
2.8 Summary

A literature review on the out-of-plane behavior of stitched composites is presented. Studies reveal that stitching generally improves the out-of-plane properties of polymer matrix composites. In particular, mode I and mode II steady-state fracture energies of stitched polymer matrix composites have shown to be dependent on stitch parameters and may increase by a factor up to 15 and 3, respectively, for Vectran, carbon, and Kevlar stitching materials. Stitch density and linear thread density are the two primary stitch parameters that have shown to improve the out-of-plane properties. Under mode I delamination, stitches behave primarily as a bridging mechanism to resist crack growth. Stitches have also been observed to mainly deform and plow through the adjacent matrix during mode II loading conditions.

Current test standards do not appropriately address how to effectively determine the modal fracture energies of stitched composites. This is primarily due to the high rotational constraint developed by the through-thickness stitching that leads to facesheet failure and possible significant stitch bridging, which may lead to inaccuracies in the calculated fracture energy. Doublers and new test methods have been developed to prevent failure of the delaminated arms during testing. Standard procedures for determining doubler sizing and estimating appropriate initial crack lengths relative to the stitch location need to be developed.

Additional investigation is needed to understand the influence of unique stitch parameters, such as stitch pretension, stitch twist, and stitching style. Lastly, the influence of stitch parameters on high-temperature composites, such as carbon-carbon composites, CMCs, and sandwich composites, are not well studied. Research indicates that the out-of-plane properties of CMCs can be improved by approximately 30% without impacting in-plane properties by using untwisted low-filament count (1K) carbon fiber tows for stitching thread.
Overall, proper selection of stitch parameters such as stitch density and linear thread density can further improve the out-of-plane properties of polymer matrix composites.
CHAPTER III

INFLUENCE OF STITCH PARAMETERS ON THE FRACTURE ENERGY OF STITCHED SANDWICH COMPOSITES USING A DESIGN OF EXPERIMENTS APPROACH

3.1 Introduction

Modern aircraft employ the use of lightweight engineering materials such as sandwich composites for their superior flexural rigidity. Sandwich composites are comprised of two rigid, outer facesheets that are bonded to a lightweight internal core. The separation of the facesheets increases the stiffness of the structure and allows the facings to resist in-plane axial and bending stresses, whereas the internal core primarily counteracts the through-thickness shear stresses. However, the applicability of sandwich composites when subjected to out-of-plane loads is limited due to their low interlaminar strengths that arise from a material property mismatch between the two constituents [1]. Separation of the facesheets to the core material reduces the strength and stiffness of the structure [147]. Furthermore, visual inspection of the outer-most surface provides little indication of the internal delamination that may be present due to out-of-plane impact.

The enhancement of interlaminar and fracture properties of polymer matrix composites is achievable by introducing crack-retardant features such as nano-fillers [148] or through-the-thickness reinforcements [9, 10]. In particular, through-the-thickness stitching that is orthogonally [66] or obliquely [97] inserted has been shown to generally improve the mechanical properties of sandwich composites with only a 1% weight increase. The stitching process
involves sewing yarns into dry sandwich preforms using an industrial or robotic sewing machine, followed by resin infusion and cure. Delaminations that develop from incidents during service operations are impeded by bridging traction stresses due to through-thickness reinforcements [88, 122, 149, 150].

The capability of stitching to improve mechanical properties and impede crack growth is highly dependent on stitch processing parameters. These parameters include stitch density (stitches/area), linear thread density (mass/length), stitch material, stitch distribution, stitching style, and stitch pretension [1]. The stitch density is characterized by \( \frac{1}{PS} \), where \( S \) and \( P \) are the spacing and pitch, respectively. The spacing is the distance between two adjacent seams of stitching, and the pitch is the distance between two adjacent stitches along the same stitch seam [9]. Under impact, increasing the stitch density increases the total absorbed energy capacity and reduces the damage area [73]. However, traditional damage modes [102, 103] of foam core sandwich composites subjected to impact appear absent when sandwich composites are reinforced with stitching. In particular, the bottom surface of the impacted sandwich composite does not appear to develop delaminations as typically observed in unstitched sandwich composites. The primary form of failure in stitched sandwich composites is stitch-column buckling and stitch-column penetration of the facesheet [72]. Furthermore, higher stitch densities result in lower incipient failure loads due to the development of microcracks within the resin-rich zones that are adjacent to the stitches [69, 72, 151]. Lastly, stitched sandwich composites subjected to impact have also been reported to have larger regions of core cracking than unstitched panels [72].

A linear elastic fracture mechanics approach has been shown to be ineffective in estimating the mode I fracture of stitched sandwich composites due to difficulty in modeling the
stitch failure energy during crack propagation [108]. Therefore, alternative approaches are needed to estimate the fracture energies in stitched sandwich composites. Statistical design of experiments (DoE) can be used to identify and model complex relationships between input factors and output material responses to facilitate the development of a more physics-based model. In this study, a phenomenological response surface model (RSM) is developed using a face-centered central composite design (FC-CCD) of experiments. The RSM will be used to estimate the influence of stitching processing parameters, and their interaction on the mode I fracture energy, which is determined from single cantilever beam (SCB) tests. The selected input factors in this study are stitch density ($X_1$), linear thread density ($X_2$), and facesheet thickness ($X_3$). The fracture energy is calculated by assuming a cubic form of the measured compliance. In the following discussion, the analytical development, fabrication, experimental procedure, and results are presented.

3.2 Design of Experiments Approach

The development of the RSM was performed using an FC-CCD to determine the effects of stitch density ($X_1$), linear thread density ($X_2$), and facesheet thickness ($X_3$) on the steady-state fracture energy of stitched sandwich composites. The FC-CCD is based on a $2^3$ factorial treatment design consisting of 15 design points [152]. The FC-CCD contains eight corner points, one center point, and six axial points. The axial points are at a normalized distance $\alpha$ from the center point. Each point represents a treatment combination of the levels of each factor ($X_1$, $X_2$, $X_3$). At each design point, three replicates were tested to determine the pure error for estimating the lack of fit of the measured fracture energy. A total of 45 experiments were performed.

The range of levels of each factor ($X_1$, stitch density; $X_2$, linear thread density; and $X_3$, facesheet thickness) was determined based on a previous experimental study [115] and from
current literature. A stitch density range of $0.0015 \leq X_1 \leq 0.01$ stitches/mm$^2$ was used. The upper limit of $X_1 = 0.01$ stitches/mm$^2$ was selected based on a study from Wang et al. [153] and corresponds to an equal pitch and spacing of 10 mm. Increasing the stitch density from 0.01 stitches/mm$^2$ to 0.04 stitches/mm$^2$ was shown to degrade the flexural modulus of the carbon/epoxy sandwich composite when compared to its unstitched counterpart. Furthermore, the lower limit $X_1 = 0.0015$ stitches/mm$^2$ was selected from previous studies [97, 108], where the spacing between stitching was approximately 25 mm. The linear thread density ranged from $400 \leq X_2 \leq 1200$ Denier was based on previous studies on stitched composite laminates [60, 69]. The minimum facesheet thickness was selected to prevent facesheet failure during mode I fracture testing in the range of $1.8 \leq X_3 \leq 3.6$ mm. The upper limit of the facesheet thickness $X_3 = 3.6$ mm was based on a sandwich composite redesign of a T-38 strut door [154].

The FC-CCD design space is based upon nondimensional coded levels ($x_i$) of the actual input factors ($X_i$), where $x_i = -\alpha, -1, 0, 1, or \alpha$ [152]. A graphical representation of the FC-CCD with treatment combinations of the coded factors $x_1$, $x_2$, and $x_3$ at each design point is shown in Figure 3.1. A factor is tested at low, mid, and high levels while maintaining two factors at fixed prescribed values dictated by the FC-CCD. The normalized distance $\alpha$ was selected to be 1.0 due to material availability in the fabrication process. The coded levels, and their corresponding actual levels, are shown for each actual input factor ($X_i$) in Table 3.1. The linear relationship between the coded levels ($x_i$) and the actual levels ($X_i$) is given by [152, 155]

$$x_i = \frac{2X_i - (X_{iHigh} + X_{iLow})}{(X_{iHigh} - X_{iLow})}$$ (3.1)

where $X_{iHigh}$ and $X_{iLow}$ correspond to $x_i = 1$ and -1, respectively, for $i = 1, 2, 3$. 38
Figure 3.1  Face-centered central composite design space with non-dimensional coded factors ($x_1 =$ stitch density, $x_2 =$ linear thread density, and $x_3 =$ facesheet thickness).

Table 3.1  Non-dimensional coded levels and actual levels for each FC-CCD factor.

<table>
<thead>
<tr>
<th>Coded Level</th>
<th>Stitch Density, $X_1$ (stitches/mm$^2$)</th>
<th>Linear Thread Density, $X_2$ (Denier)</th>
<th>Ply Thickness, $X_3$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>0.00155</td>
<td>400</td>
<td>1.778</td>
</tr>
<tr>
<td>0</td>
<td>0.005775</td>
<td>800</td>
<td>2.667</td>
</tr>
<tr>
<td>1</td>
<td>0.01</td>
<td>1200</td>
<td>3.556</td>
</tr>
</tbody>
</table>

The fracture energy at initiation and steady-state was determined for each treatment combination in the FC-CCD to develop the RSM. In this study, a full quadratic RSM with a three-factor interaction term was considered and is expressed as

$$Y = \beta_o + \sum_{i=1}^{3} \beta_i x_i + \sum_{i=1}^{3} \beta_{ii} x_i^2 + \sum_{i=1}^{3} \sum_{j=1}^{3} \beta_{ij} x_i x_j + \beta_{123} x_1 x_2 x_3 + \epsilon$$  \hspace{1cm} (3.2)
where $Y$ is the fracture energy response, $x_i$ are the coded levels of each input factor ($x_1 = \text{stitch density}$, $x_2 = \text{linear thread density}$, and $x_3 = \text{facesheet thickness}$). The estimates of the model parameters ($\beta_0, \beta_j, \beta_{ij}, \beta_{ij}$, and $\beta_{123}$) are determined by the method of least squares. The terms $x_i x_j$ represent two-factor interaction effects and $x_1 x_2 x_3$ represents the three-factor interaction effect, where $i, j = 1, 2, 3$, and $i < j$. The significance of each regressed term in Eq. (3.2) is evaluated using an analysis of variance (ANOVA) procedure. The error term $\varepsilon$ is assumed to be normally distributed with a zero mean and constant variance. The error random variables are independent and identically distributed. These assumptions are verified by determining the studentized residuals, the normal probability plot of the predicted response, and are provided as supplemental data for this study. The regression parameters $\beta$ are estimated as [152]

$$
\beta_m = (A_{ml}A_{tm})^{-1}A_{ml}G_l
$$

(3.3)

The nondimensional FC-CCD design space $A_{lm}$ is represented geometrically for each regression term and observation at each treatment combination [152], where $l = 1, 2, \ldots, n$ and $m = 1, 2, \ldots, p$. The total number of regression parameters and measured observations correspond to $p$ and $n$, respectively. The significance of each regressed term in Eq. (3.2) is evaluated using an analysis of variance (ANOVA) procedure. The measured fracture energy $G_l$ is analytically represented as

$$
G_l = \left. \frac{P^2}{2B} \frac{dC}{d\alpha^*} \right|_{l=1,2,\ldots,n}
$$

(3.4)

where $P$ is the reactive load and $B$ is the specimen width. The term $dC/d\alpha^*$ is the derivative of the compliance $C$ with respect to the true crack length $\alpha^*$. A modification of the crack length
\((a^* = a + \Delta)\) is introduced to ensure that the flexural modulus is independent of the measured crack length [156]. The compliance is given as [114]

\[
C = m(a + \Delta)^3
\]  

(3.5)

where the slope \(m\) and length \(\Delta\) are evaluated from the linear relationship between \(C^3\) and \(a\) [114, 156]. Substituting Eq. (3.5) into (3.4), the fracture energy can be obtained as

\[
G_I = \frac{3P}{2B} \frac{\delta}{(a + \Delta)}_{l=1,2,...,n}
\]

(3.6)

where \(\delta\) is the applied displacement during testing.

### 3.3 Fabrication of Stitched Sandwich Composite Specimens

Single cantilever beam (SCB) specimens were fabricated from an infused epoxy/carbon fiber sandwich composite panel with 110 kg/m\(^3\) foam core. The core was perforated manually in a 6.35 mm grid spacing with a 0.79 mm diameter needle to allow the resin to perfuse through the core during infusion. The carbon/epoxy facesheets were comprised of a cross-ply layup configuration \([0^\circ/90^\circ/90^\circ/0^\circ]\)_i, where \(i = 2, 3, 4\) correspond to the facesheet thicknesses \(X_3\) in Table 3.1. A 0.0127 mm Teflon film was used as the crack initiator at a depth of approximately 76.2 mm from the edge of the laminate. The dry preform with foam core was stitched using Vectran™ thread with a stitch density \(X_1\) and linear thread density \(X_2\) dictated by the FC-CCD. In this study, the stitch pitch is the same distance as the stitch spacing and was determined by the relative stitch distance \(D = \sqrt{1/X_1}\). The sandwich composite preforms were stitched using a 2000H Juki industrial sewing machine using a modified lock stitch. A 10 mm distance
was maintained between the initial crack length and the first row. The 2.25 mm diameter needle was used to stitch the dry preforms and was selected based on robotic stitching processes [14]. The dry sandwich composite preforms were infused using a one-sided vacuum assisted resin transfer molding process with an out-of-autoclave Hexflow 1078 epoxy resin system. The dry sandwich structure and epoxy resin were separately heated to 88 °C to reduce the viscosity of the resin system before infusion. After infusion, the temperature was increased at a rate of 1.8 °C/min to a temperature of 177 °C, held for two hours, and reduced to room temperature (24 °C). After cure, the samples were sectioned into 20 cm by 5 cm test coupons, as illustrated in Figure 3.2. The sizing of the SCB coupons was based on preliminary tests and based upon current SCB standards [157]. Piano hinges were bonded to the top facesheet above the initial defect using Loctite Hysol EA 9394 adhesive.

Figure 3.2  Schematic of a single cantilevered beam (SCB) specimen.
3.4 Single Cantilever Beam (SCB) Test Procedure

The SCB specimens were conducted in accordance with ASTM STD 5528-13 and based on testing standards developed by Ratcliffe [157] and Cantwell [158]. Force is applied to the piano hinge at a constant displacement rate of 0.5 mm/min. The bottom surface of the specimen is rigidly constrained with a non-rotating base. An example of this test with a specimen subjected to load is shown in Figure 3.3. A 1-kN load cell was used to measure the reactive load. Crack lengths were recorded using visual measurements with the aid of an ARAMIS digital image correlation system. The SCB tests were performed at each treatment combination ($X_1, X_2, X_3$) as determined by the FC-CCD and a total of three replicates were performed at all design points.

![Single cantilever beam test setup.](image)

Figure 3.3 Single cantilever beam test setup.

3.5 Stitched Sandwich Composite Fracture Behavior

The load and crack length response for typical stitched SCB specimens is shown in Figure 3.4. An initial linear response prior to the start of the propagation of delamination was
observed for each test. At crack initiation, a sudden decrease in the reacted load occurred for each test. A layer of epoxy resin formed underneath the Teflon film during fabrication. As a result, unstable crack growth initially occurred due to an increase in the crack-tip radius. As the crack front approaches the vicinity of the first stitch row, the reacted load linearly increases until stitch failure occurred (Region 2). Upon failure of a stitch row, the crack lengths immediately progressed to the next stitch row. Additionally, multiple stitch rows appear to fail for high linear thread densities ($X_2 = 1200$ Denier) and low facesheet thicknesses ($X_3 = 1.778$ mm). As subsequent stitch rows failed, a “saw-tooth” pattern was observed in the load-displacement response (Points 2-6). The magnitude of the failure load at each stitching row decreases with an increase in the applied displacement. The required load to develop the necessary tensile stresses to advance the crack front decreases. This behavior is due to the greater distance between the location of the reacted load and the crack front, which increases with each subsequent stitch row failure.
Figure 3.4  Load and crack length behavior for stitched SCB specimens ($X_1=0.01$ stitches/mm$^2$, $X_2=1200$ Denier, $X_3=1.776$ mm).

The fracture surface of a representative SCB sample is shown in Figure 3.5. During facesheet-to-core separation, “candle-like” structures formed on the surface of the facesheet that correspond to the epoxy matrix-stitch columns that internally failed inside the core during each test. It was observed that the stitch primarily failed within the core (developing matrix-stitch columns on the facesheet) or within the facesheet. The diameter of the matrix-stitch column is approximately 6% greater than the needle diameter used in the stitching process. This indicates that resin will bleed into damaged foam cells adjacent to the stitch. In summary, three failure mechanisms are observed: 1) matrix-stitch pullout at the facesheet-core interface, 2) matrix-stitch column frictional pullout, and 3) matrix-stitch frictional pullout with ductile core failure. These failure mechanisms are illustrated in Figure 3.6. The facesheet-core interface failure primarily occurs due to tensile failure of the Vectran™ thread within the facesheet and is consistent with previously observed stitch interface failures for composite laminates [65, 88]. Failure at the
facesheet-core interface may also develop secondary in-plane fiber bridging due to carbon fibers that were damaged in the stitching process. Frictional pullout of the matrix-stitch column occurs when the interfacial shear strength between the column and foam core is exceeded due to high linear thread densities \( X_2 \geq 1200 \) Denier.

![Fracture surface of a delaminated specimen](image)

**Figure 3.5** Fracture surface of a delaminated specimen. Region 1 (R1) is the characteristic resin-stitch column after stitch failure. Region 2 (R2) is the interface between a resin-stitch column and facesheet after stitch failure.
Figure 3.6  Characteristic fracture behavior during mode I core-to-facesheet separation showing (a) stitch pullout at facesheet-core interface, (b) matrix-stitch column frictional pullout, and (c) matrix-stitch frictional pullout with ductile failure of the core.

The fracture energy of representative stitched specimens with respect to the applied displacement is shown in Figure 3.7. To assist in the discussion of the relative influence of stitching on the fracture energy, Figure 3.7 is divided into three regions, Zones 1-3. Upon crack initiation (Zone 1), the fracture energy is relatively constant and is consistent with measurements previously reported for unstitched sandwich composites [115]. This is primarily due to the lack of toughening mechanisms to retard crack growth. However, the fracture energy increases (by approximately 600%) linearly as the crack growth approaches the initial stitch row (Zone 2). This behavior is highly dependent on the input factors ($X_1 =$ stitch density, $X_2 =$ linear thread...
density, and \( X_3 = \) facesheet thickness) and will be discussed later. Fracture energies obtained from Zone 2 represent the fracture energy required to propagate crack growth due to the presence of the through-thickness stitching. The maximum fracture energy developed due to each stitch row is relatively constant. The failure of the stitch rows resulted in significant reductions in the fracture energies and produced unstable crack growth between the facesheet and the core (Zone 3). As the stitch density decreases, the fracture energy approaches the unstitched fracture energies.

Figure 3.7 Fracture energy response for stitched sandwich composite specimens \((X_1=0.01 \text{ stitches/mm}^2, X_2=1200 \text{ Denier}, X_3=1.776 \text{ mm})\).

To develop the RSMs, the maximum fracture energy and unstitched fracture energies were recorded for each coded treatment combination shown in Table 3.2. The maximum fracture
energy was normalized by the fracture energies in Zone 1 (unstitched) to study the relative influence of each input factor ($X_1$ = stitch density, $X_2$ = linear thread density, and $X_3$ = facesheet thickness). These experimental results were then used to develop the RSM based on the ANOVA procedure.

<table>
<thead>
<tr>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>Max Fracture Energy* ($J/m^2$)</th>
<th>$\sigma$ (Max)</th>
<th>Fracture Energy at Initiation* ($J/m^2$)</th>
<th>$\sigma$ (Initial) ($J/m^2$)</th>
<th>Normalized Fracture Energy* ($J/m^2$)</th>
<th>$\sigma$ (Normalized) ($J/m^2$)</th>
</tr>
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<tbody>
<tr>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>507.6</td>
<td>46.6</td>
<td>280.9</td>
<td>29.3</td>
<td>1.8</td>
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<td>-1</td>
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<td>22.4</td>
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<td>2.9</td>
<td>0.1</td>
</tr>
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</table>

Table 3.2  Initial, maximum, and normalized fracture energy by coded treatment combinations.

$x_1$: coded levels of stitch density
$x_2$: coded levels of linear thread density
$x_3$: coded levels of facesheet thickness
$\sigma$: standard deviation

*Average from three replicates per design point

3.6  Response Surface Model (RSM) Development

The ANOVA procedure was used to determine the significance of the RSM, each corresponding term, and the overall lack of fit. The ANOVA table for the response surface model is shown in Table 3.3. The sum of squares (SS), mean square (MS), and F-value are
determined for the model, each regression parameter, and the residual. The $SS_{\text{Residual}}$ is decomposed into the lack of fit and pure error components. The lack of fit represents how well the predicted response approximates the experimental values. The pure error signifies the variability in the measured response and is estimated at each design point and across all replicated treatment combinations. In this study, an $\alpha = 0.05$ is used to evaluate the significance of the lack of fit F-test prior to determining the significance of the model. A non-significant lack-of-fit is desired since that indicates the RSM fits well with the measured values. Following the analysis of variance, the regression parameters were determined using Eq. (3.3) to develop the final RSM as a function of stitch density ($x_1$), linear thread density ($x_2$), and facesheet thickness ($x_3$). The RSM was evaluated at a 0.05 level of significance. Furthermore, the model’s corresponding terms (linear, quadratic, and interaction terms) were evaluated using partial F-tests at a 0.10 level of significance. In this study, all terms with an initial P-value $< 0.10$ were kept in the model to ensure that significant terms were not prematurely removed due to error introduced by other terms [159]. A backward elimination approach was used to remove any nonsignificant terms based on the principle of hierarchy [152]. Nonsignificant lower order polynomial terms were retained because they appeared in significant higher-order polynomial terms.
Table 3.3 ANOVA table of the normalized fracture energy model.

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F-value</th>
<th>p-value</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>14.7983</td>
<td>9</td>
<td>1.6443</td>
<td>74.5641</td>
<td>&lt; 0.0001</td>
<td>Significant</td>
</tr>
<tr>
<td>$x_1$</td>
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<td>2.7178</td>
<td>123.2469</td>
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<td>Significant</td>
</tr>
<tr>
<td>$x_2$</td>
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<td>10.0978</td>
<td>457.9206</td>
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<td>Significant</td>
</tr>
<tr>
<td>$x_3$</td>
<td>1.0081</td>
<td>1</td>
<td>1.0081</td>
<td>45.7141</td>
<td>&lt; 0.0001</td>
<td>Significant</td>
</tr>
<tr>
<td>$x_1 \cdot x_2$</td>
<td>0.3100</td>
<td>1</td>
<td>0.3100</td>
<td>14.0569</td>
<td>0.0006</td>
<td>Significant</td>
</tr>
<tr>
<td>$x_1 \cdot x_3$</td>
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<td>1</td>
<td>0.1740</td>
<td>7.8918</td>
<td>0.0081</td>
<td>Significant</td>
</tr>
<tr>
<td>$x_2 \cdot x_3$</td>
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<td>1</td>
<td>0.0089</td>
<td>0.4049</td>
<td>0.5287</td>
<td>Not significant</td>
</tr>
<tr>
<td>$x_1^2$</td>
<td>0.2325</td>
<td>1</td>
<td>0.2325</td>
<td>10.5417</td>
<td>0.0026</td>
<td>Significant</td>
</tr>
<tr>
<td>$x_2^2$</td>
<td>0.3249</td>
<td>1</td>
<td>0.3249</td>
<td>14.7315</td>
<td>0.0005</td>
<td>Significant</td>
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<tr>
<td>$x_1 \cdot x_2 \cdot x_3$</td>
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<td>1</td>
<td>0.0799</td>
<td>3.6225</td>
<td>0.0653</td>
<td>Significant</td>
</tr>
<tr>
<td>Residual</td>
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<td>35</td>
<td>0.0221</td>
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</tr>
<tr>
<td>Lack of Fit</td>
<td>0.2024</td>
<td>5</td>
<td>0.0405</td>
<td>2.1325</td>
<td>0.0887</td>
<td>Not significant</td>
</tr>
<tr>
<td>Pure Error</td>
<td>0.5694</td>
<td>30</td>
<td>0.0190</td>
<td></td>
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<tr>
<td>Total</td>
<td>15.5701</td>
<td>44</td>
<td></td>
<td>2.1325</td>
<td>0.0887</td>
<td></td>
</tr>
</tbody>
</table>

| SS: Sum of squares |
| df: degrees of freedom |
| MS: Mean square |

The normalized fracture energy was transformed to lognormal space to obtain an improved fit to the experimental data. The ANOVA table provided in Table 3.3 shows the development of the F-tests of the model and the corresponding terms. The model was determined to be adequate (p-value < 0.0001) for predicting normalized fracture energies within the design space of the FC-CCD. Furthermore, the lack of fit was determined to be non-significant and shows good agreement to measured data in predicting the normalized fracture energies. Only two terms ($x_3^2$ and $x_2 \cdot x_3$) were determined to be non-significant; however, $x_2 \cdot x_3$ was retained within the model due to the principle of hierarchy. The $R^2 = 0.9504$ indicates that 95.04% of the total variation in the normal fracture energies is explained by the RSM [152].

Using Eq. (3.3) with the selected terms from the ANOVA analysis, the RSM was developed for the normalized fracture energy. The nondimensional form ($x_1$, $x_2$, $x_3$) of the RSM is expressed as
\[
\ln(G_{\text{normal}}) = 1.14 + 0.30x_1 + 0.58x_2 - 0.18x_3 + 0.11x_1x_2 + 0.09x_1x_3 \\
- 0.02x_2x_3 - 0.17x_1^2 + 0.20x_2^2 + 0.06x_1x_2x_3
\]

(3.7)

This model can be transformed from coded variables \((x_1, x_2, x_3)\) to uncoded variables \((X_1, X_2, X_3)\) using Eq. (3.1). The corresponding uncoded RSM is expressed as

\[
\ln(G_{\text{normal}}) = 0.66 + 146.54X_1 + (1.7 \times 10^{-4})X_2 - 0.12X_3 - 0.035X_1X_2 \\
- 8.04X_1X_3 - (2.8 \times 10^{-4})X_2X_3 - 9319.26X_1^2 + (1.23 \times 10^{-6})X_2^2 \\
+ 0.0384X_1X_2X_3
\]

(3.8)

The magnitude and corresponding sign of the coefficient signify the overall contribution of each term to the normalized fracture energy. For instance, increasing the facesheet thickness \((x_3, X_3)\) decreases the normalized fracture energy. A three-dimensional plot of the normalized fracture energy as a function of stitch density \((X_1)\) and linear thread density \((X_2)\) at a facesheet thickness \(X_3 = 1.776\) mm is shown in Figure 3.8. Increases in the normalized fracture energies are primarily associated with increases in linear thread density and stitch density. Linear thread density \((x_2, X_2)\) appears to be the most dominant factor to increase the normalized fracture energy, followed by stitch density \((x_1, X_1)\).
Figure 3.8  Normalized fracture energy as a function of stitch density ($X_1$) and linear thread density ($X_2$) for a thickness of $X_3 = 1.778$ mm ($x_1 = -1$).

3.7  Discussion

The normalized fracture energies as a function of stitch density ($x_1$) for select linear thread densities ($X_2 = 400, 800, \text{and} 1200$ Denier) and facesheet thicknesses ($X_2 = 1.776, 2.667, \text{and} 3.556$ mm) are shown in Figure 3.9(a)-3.9(c). Experimental data used in the FC-CCD and auxiliary data are provided for comparison to the RSM. Overall, the response surface model shows adequate precision in estimating the normalized fracture energy in comparison to the auxiliary data points identified in Figure 3.9(c).

For each facesheet thickness ($X_3$), the fracture energy is improved by up to 400% for low linear thread densities ($X_2 = 400$ and 800 Denier) and is relatively constant with respect to stitch density ($X_1$). At these linear thread densities, the primary traction-separation response is stitch pullout at the facesheet-to-core interface, which is identified in Figure 3.6(a). However, increasing the linear thread density from $X_2 = 400$ Denier to $X_2=1200$ Denier greatly increases
the normalized fracture energy from a factor of two to a factor of ten for each facesheet thickness ($X_3$). The relative improvement is attributed to the change in the fracture morphology during mode I facesheet-to-core separation. At high linear thread densities ($X_2 \geq 1200$ Denier), matrix-stitch column pullout and ductile failure of the core occurs (Figure 3.6(b) and 3.6(c)). Furthermore, the overall contribution to the observed fracture energies is due to differences in the cylindrical surface area of the stitch and matrix-stitch column that develop due to altering the linear thread density. The matrix-stitch column has approximately three times greater surface area than a $X_2 = 1200$ Denier Vectran thread and requires greater fracture energy to promote crack growth.

The contribution of stitch density ($X_1$) to the overall normalized fracture energy is primarily seen at high linear thread densities ($X_2 = 1200$ Denier). For a facesheet thickness between $2.667 \text{ mm} \leq X_3 \leq 3.556 \text{ mm}$, increasing stitch density linearly increases the normalized fracture energy from approximately 250% to 800%. At a facesheet thickness of $X_3 = 1.778 \text{ mm}$ and linear thread density $X_2 = 1200$ Denier, the normalized fracture energy increases linearly between $X_1 = 1.5 \times 10^{-3}$ stitches/mm$^2$ and $7 \times 10^{-3}$ stitches/mm$^2$. At a stitch density of approximately $9.3 \times 10^{-3}$ stitches/mm$^2$, the normalized fracture energy reaches a maximum value of approximately 929%. This stitch density corresponds to an optimum stitch distance $D = 10.38$ mm. The relative optimum stitch distance is consistent with previous literature and appears to be dependent on linear thread density. Further increases in the stitch density may result in the degradation of flexural stiffness of the stitched sandwich composite.

For a constant stitch density ($X_1$) and linear thread density ($X_2$), the normalized fracture energy decreases with increasing facesheet thickness ($1.778 \text{ mm} \leq X_3 \leq 3.556 \text{ mm}$) by approximately 13%. This may be due to bridging of the stitches near the crack front. Farmand-
Ashtiani et al. [160] has shown that fracture energy is dependent on specimen geometry when large-scale bridging occurs at the delaminated interface. Therefore, the fracture energy values in this study should be treated as apparent values due to a geometry dependence. The reduction in the fracture energy has also been observed by Saseendran et al. [161] for unstitched sandwich composite specimens with carbon/epoxy facesheets with a honeycomb core. The single-cantilever beam test is inherently a mixed-mode test, but mode I dominates, due to the material mismatch at the facesheet-to-core interface. The reduction in the fracture energy may be due to a shift in the mode-mixity phase angle, which quantifies the ratio of shear (mode II) to normal (mode I) loading at the crack tip [116]. These shifts in the mode-mixity phase angle may result from differences in the crack root rotation as the crack growth processes near a stitch row for different facesheet thicknesses. High crack root rotations develop additional shear stresses that can reduce fracture energy of the stitching required to promote crack growth, which is counter intuitive for unstitched laminates. This is primarily because stitched laminates have a greater mode I fracture energy than a mode II fracture energy [89, 90, 107]. Therefore, additional shear stresses near the crack front can reduce the effectiveness of the stitching to arrest crack growth.
Figure 3.9  Normalized fracture energy as a function of stitch density ($X_1$) for select linear thread densities ($X_2 = 400, 800,$ and 1200 Denier) and for thicknesses of (a) $X_3 = 1.778 \text{ mm}$, (b) $X_3 = 2.556 \text{ mm}$, and (c) $X_3 = 3.556 \text{ mm}$.
3.8 Conclusion

In this study, a face-centered central composite design (FC-CCD) of experiments is used to evaluate the influence of stitch parameters (stitch density and linear thread density) and specimen geometry (facesheet thickness) on the mode I fracture energy of stitched sandwich composites. Sandwich composite specimens were manufactured with various treatment combinations of stitch densities (0.0015-0.01 stitches/mm\(^2\)), linear thread densities (400-1200 Denier), and facesheet thicknesses (1.8-3.6 mm) as dictated by the FC-CCD. The mode I fracture energy was determined by performing single cantilever beam tests. A response surface model (RSM) was developed using an analysis of variance procedure to predict the normalized fracture energy within the design space.

The incorporation of stitching significantly increases the mode I fracture energy and is highly dependent on stitching parameters. Linear thread density is determined to be the most influential factor to improve steady-state fracture energy of stitched sandwich composites. During testing, crack growth exhibits unstable stick-slip behavior that results in a saw-tooth response in the observed fracture energies. The fracture surfaces reveal three different failure mechanisms: (1) stitch pullout at the delaminated interface, (2) frictional sliding of the matrix-stitch column, and (3) ductile foam core failure. Furthermore, the RSM developed in this study reveals that an optimum stitch density of 0.0093 stitches/mm\(^2\), or a corresponding relative stitch distance of 10.38 mm, is obtained using a linear thread density of 1200 Denier. Increasing the facesheet thickness results in a reduction in the observed fracture energies. This is attributed to fiber bridging and changes in the mode-mixity phase angle, which have been observed to be dependent on the sandwich composite specimen geometry.
This study establishes the overall fracture behavior of stitched sandwich composites due to stitch processing parameters (stitch density, linear thread density, facesheet thickness). The results indicate that stitching is an excellent method to inhibit crack growth within sandwich composites for damage-tolerant composite designs.
CHAPTER IV
ON THE ESTIMATION OF THE MODE I FRACTURE ENERGY OF STITCHED SANDWICH COMPOSITES USING THE J-INTEGRAL APPROACH

4.1 Introduction

Sandwich composites are composed of two outer, rigid facesheets and a lightweight internal core. The increased part thickness with a lightweight core increases the flexural rigidity of the composite structure that may be needed for primary and secondary load applications. Sandwich composites can delaminate between the facesheets and the core at relatively low, out-of-plane loads. The delamination is due to a property mismatch between the facesheet and core and leads to decreased strength and stiffness of the structure. Furthermore, visual inspection of the outermost surfaces provides little indication of the severity of delamination.

The arrestment of delamination is achievable by incorporating through-the-thickness reinforcements such as stitching and z-pinning [5, 7, 10]. These reinforcements act as crack-growth inhibitors by bridging the opposing crack faces and inducing bridging tractions to resist crack growth. Additionally, through-the-thickness reinforcement that is orthogonally or obliquely inserted into foam core sandwich composites has generally shown to improve the mechanical [66, 97] and fracture properties [115, 162] without significant mass gain. Traditional damage modes [102, 103] of foam core sandwich composites subjected to impact appears to be absent when the sandwich composite is reinforced with through-thickness stitching [72]. In stitched sandwich composites subjected to low-velocity impact, the primary form of failure is
stitch-matrix column buckling and delamination of the top-most surface. Stitched regions subjected to impact have also been reported to undergo larger regions of core cracking than unstitched panels [72]. Also, the mechanical performance of the stitched sandwich composite is highly dependent on the stitch parameters, such as stitch density (stitches/area) and linear thread density (mass/length) [115, 162]. Higher stitch densities can result in lower incipient failure loads due to the development of microcracks within the resin-rich zones that are adjacent to the stitches [69, 72, 151].

The modal fracture behavior of composites is characterized by the critical strain energy release rate or fracture toughness, which is the energy required to promote crack growth by an external load or applied displacement. The most common types of fracture tests for composite laminates are the double-cantilever beam (DCB) [104] and the end-notch flexure [105] tests to determine the mode I and mode II fracture toughness, respectively. In recent years, new test methods have been developed to characterize the modal fracture toughness of sandwich composites. The tilted sandwich debond (TSD) test was developed by Li and Carlsson using an elastic foundation theory [163]. In this test, a sandwich composite is rigidly constrained at an inclined slope. Crack progression under mixed-mode conditions can be observed by applying a vertical load to a partially debonded facesheet [164]. Additionally, Sørensen et al. [165] developed a test to characterize mixed-mode fracture behavior of a multilayered double cantilevered beam specimen by applying uneven bending moments to the debonded arms of the composite. This test method was further extended by Berggreen [166] and Saseendran [116, 167] for determining the energy release rate of sandwich composites using a J-integral approach coupled with classical lamination theory.
The previously mentioned test methods were originally developed in conjunction with linear elastic material behavior [156, 168], which assumes small-scale yielding. Under this condition, the size of the damage or plastic zone ahead of the crack tip is relatively small with respect to a characteristic specimen length. These approaches may also yield inaccurate estimates of the fracture energy for through-the-thickness reinforced specimens that can develop large plastic zone sizes due to large-scale bridging. Alternative approaches, such as crack-tip-opening displacement (CTOD) [169] and the J-integral [170], can be used to accommodate larger plastic zones ahead of the crack tip provided that linear elastic material behavior is not assumed. In this study, the J-integral approach is used to develop simplified expressions for mode I fracture energy of the single cantilever beam (SCB) test. The SCB test [114, 157] is a variant of the TSD method, where the incline of the single cantilever beam specimen is zero degrees. A single cantilevered beam test of a stitched sandwich composite specimen is performed. The estimated fracture energies from the J-integral approach are compared to existing linear elastic fracture mechanics approaches and finite element analysis.

4.2 Analytical Development

The J-integral relationship is a contour integral that can be expressed as [170]

\[
J = \int_{\Gamma} \left( W dy - T_i \frac{du_i}{dx} ds \right)
\]

(4.1)

where \( J \) is the nonlinear elastic strain energy rate along the contour \( \Gamma \). The contour \( \Gamma \) is represented by a clockwise segmented path around the boundary of the fractured domain, as shown in Figure 4.1. The strain energy density and traction vector are denoted as \( W \) and \( T_i \), respectively. The traction stresses represent the external stresses acting on the cracked boundary and can be expressed as \( T_i = \sigma_{ij} n_j \). The perimeter of the crack is assumed to be stress free under
monotonic loading conditions. The term \( \frac{du_i}{dx} \) is the rotation vector in the \( x \) direction about the incremental path \( ds \) along the contour.

Consider the single cantilevered beam with discrete traction above a delaminated arm shown in Figure 4.2. The bottom surface of the SCB specimen is rigidly constrained. A discrete traction load \( T_y \) is applied above the partially debonded region. The perimeter of the SCB specimen is delineated by the segmented contours, \( \Gamma_1 - \Gamma_8 \). For clarity, the contours are represented slightly internal to the specimen’s perimeter. The \( J \)-integral is path independent for any closed contour. Therefore, the compatibility relationship between the segmented contours can be expressed as

\[
\sum_{i=1}^{8} J_i = J_1 + J_2 + J_3 + J_4 + J_5 + J_6 + J_7 + J_8 = 0
\]

(4.2)
where \( J_i \) is the strain energy release rate for the \( i^{th} \) segmented contour \( \Gamma_i \) around the perimeter of the fractured domain.

The strain energy density is analytically represented by

\[
W = \int \sigma_{ij} \varepsilon_{ij} \, \text{d}x
\]  

(4.3)

where \( \sigma_{ij} \) and \( \varepsilon_{ij} \) are the stress and strain tensors, respectively. The stresses \( \sigma_{ij} \) are zero along \( \Gamma_1 \), \( \Gamma_3 \), and \( \Gamma_5 \) since those surfaces are traction-free. The perimeter of the crack is also assumed to be traction-free under monotonic loading conditions. There are also no rotations along a clamped boundary condition acting on a horizontal path, i.e.,

\[
\frac{du}{dx} = 0, \quad dy = 0
\]  

(4.4)

Therefore, the following relationships are obtained.

\[
J_1 = J_3 = J_4 = J_5 = J_6 = J_8 = 0
\]  

(4.5)
\[ J_7 = -J_2 \]  

(4.6)

The remaining segmented paths are on horizontal surfaces \((dy = 0)\) with a traction acting only in the \(y\)-direction. Therefore, Eq. (4.1) can be reduced to

\[
J_7 = \int T_y \frac{du_y}{dx} \, dx
\]

(4.7)

For finite displacements \([171]\), the rotations \(\frac{du_y}{dx}\) can be exactly represented as

\[
\frac{du_y}{dx} = \tan(\theta)
\]

(4.8)

where \(\theta\) is the total angle between the rigid beam and the deformed beam at the applied traction \(T_y\). Alternate forms of the rotation have been developed by other researchers \([171-173]\); however, these formulations assume small strains for a finite displacement of a beam subjected to pure bending. This behavior may not be the case for stitched composites which can develop rotational constraint near the vicinity of the stitching and crack front \([84, 107, 108]\). The traction stress \(T_y\) can be expressed as a distributed load acting over an infinitesimal length \((x_2-x_1)\) and width \(b\). The applied loading is illustrated in Figure 4.3. Incorporating Eq. (4.8) into Eq. (4.7) and representing the traction stress as a distributed load, the J-Integral can be expressed as

\[
J = \int_{x_1}^{x_2} \frac{P}{b(x_2 - x_1)} \tan(\theta) \, dx = \frac{P}{b} \tan(\theta)
\]

(4.9)

Herein, the subscript of the nonlinear strain energy release rate \(J_7\) is removed and generally interpreted as the mode I strain energy release rate \(J\) near the crack front.
For small rotations, Eq. (4.9) can be expressed as [174, 175]

\[
J = \frac{P\theta}{b}
\]  \hspace{1cm} (4.10)

The strain energy release rate \( J \) is primarily a function of the applied loading \( P \), the specimen width \( b \), and the angle \( \theta \) at the applied load. The rotation at the applied traction is primarily influenced by the crack root rotation and elastic foundation below the deformed beam. This formulation is independent of crack length and does not assume linear-elastic material behavior. This formulation may be used to determine the traction-separation law of sandwich composites that may have large plastic zone sizes during fracture [176]. Under linear elastic material conditions, Eq. (4.10) is equivalent to the Mode I fracture energy \( G_I \) using modified beam theory (MBT). The corresponding linear elastic rotation \( \theta \) and displacement \( \delta \) can be expressed as [177]

\[
\theta = \frac{6Pa^2}{Ebh^3}
\]  \hspace{1cm} (4.11)

and
The terms $E$ and $h$ are the axial modulus of the deformed laminate and thickness, respectively.

The effective crack length is denoted as $a^*$. In beam theory, the root of the crack front is assumed to have zero slope. Therefore, an effective crack length $a^* = a + \Delta f$ is used to compensate for the foundation and rotation effects near the crack front [156]. The $\Delta f$ term can be estimated empirically by evaluating the x-intercept from the cube root of the compliance ($\delta/P$) with respect to the crack length $a$ [156]. Equations (4.11) and (4.12) can be used in Eq. (4.10) to obtain

$$G_I = J = \frac{3PD}{2b(a + \Delta f)}$$

(4.13)

The mode I fracture energy can be calculated using the modified compliance calibration (MCC) method as

$$G_I = \frac{3P^2C^{2/3}}{2A_1bh}$$

(4.14)

where $A_1$ is the slope of the normalized crack length $(a/h)$ as a function of the cube root of the compliance ($C^{1/3}$) and $h$ is the total thickness of the specimen.

4.3 Materials and Fabrication

Single cantilever beam (SCB) specimens were fabricated from an infused epoxy/carbon fiber sandwich composite panel with 110 kg/m$^3$ foam core. The core was perforated manually in a 6.35 mm grid spacing with a 0.79 mm diameter needle to allow the resin to perfuse through the core during infusion. The carbon/epoxy facesheets were comprised of a cross-ply layup
configuration $[0^\circ/90^\circ/90^\circ/0^\circ]_3$. A Teflon™ film of thickness 0.0127 mm and length 76.2 mm was placed between the facesheet and the core to initiate the crack. The dry sandwich composite preforms were stitched using a 2000H Juki industrial sewing machine using a modified lock stitch. A 10 mm distance was maintained between the initial crack length and the first row. A 2.25 mm diameter needle was used to stitch the dry preforms and was selected based on robotic stitching processes [14]. A stitch spacing (D) of approximately 10 mm was used with a 1200 Denier Vectran™ thread. The stitch density and linear thread density were based on previous studies [115, 162]. The sandwich composite preforms were infused using a one-sided vacuum assisted resin transfer molding process with an out-of-autoclave Hexflow 1078 epoxy resin system. Prior to infusion, the dry sandwich structure and epoxy resin were separately heated to 88 °C to reduce the viscosity of the resin. Following infusion, the temperature was increased at a rate of 1.8 °C/min to a temperature of 177 °C, held for two hours, and reduced to room temperature (24 °C). After cure, the samples were sectioned into 20 cm by 5 cm test coupons. Piano hinges were bonded to the top facesheet above the initial defect using Loctite Hysol EA 9394 adhesive.
4.4 Test Procedure

The SCB tests were conducted in accordance with ASTM STD 5528-13 and based on testing standards developed by Ratcliffe [157] and Cantwell [158]. Force is applied to the piano hinge at a constant displacement rate of 0.5 mm/min. The bottom surface of the specimen is rigidly constrained with a non-rotating base [178]. A 1-kN load cell was used to measure the reactive load. Crack lengths were recorded using visual measurements with the aid of an ARAMIS digital image correlation system. The rotation at the load point was measured with the aid of an inclinometer with an accuracy of ±0.05 degrees.

4.5 Computational Approach

A two-dimensional (2D) and three-dimensional (3D) finite element analyses of an SCB specimen were performed using ABAQUS Standard 2019 commercial software. Material properties were obtained from prior studies [6, 179-181] and are summarized in Table 4.1. The assigned loads and boundary conditions for each model are shown in Figures 4.5 and 4.6. The
bottom surface of the 2D model was rigidly constrained. A vertical load was applied above a partially debonded region of the sandwich composite. The compliance of the SCB test fixture was simulated within the symmetric 3D model to determine its influence on the calculated fracture energy. Surface-to-surface hard contact between the bottom face of the SCB specimen and the test fixture was applied. The test fixture was translationally fixed using a kinematic coupling. The 2D and 3D finite element models were comprised of plane strain shell (CPE4I) and hexagonal elements (C3D8I), respectively, with an approximate element length of 0.2 mm. The J-integral was evaluated across the entire crack front. Due to the influence of Poisson’s effect on the estimated fracture energy [182, 183], an averaged value of the fracture energy was used for comparisons to other experimental and computational methods.

Figure 4.5 Two-dimensional finite element model of an SCB specimen.
**Table 4.1** Bulk material properties for the facesheet and core [6, 179-181].

<table>
<thead>
<tr>
<th></th>
<th>Facesheet</th>
<th>Core</th>
</tr>
</thead>
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<td>$E_{11} = E_{22}$ (Pa)</td>
<td>5.66E+10</td>
<td>$E$ (Pa) 1.86E+08</td>
</tr>
<tr>
<td>$E_{33}$ (Pa)</td>
<td>8.64E+09</td>
<td>$v$         3.30E-01</td>
</tr>
<tr>
<td>$G_{12} = G_{13}$ (Pa)</td>
<td>4.66E+09</td>
<td></td>
</tr>
<tr>
<td>$G_{23}$ (Pa)</td>
<td>4.95E+09</td>
<td></td>
</tr>
<tr>
<td>$v_{12}$</td>
<td>0.0619</td>
<td></td>
</tr>
<tr>
<td>$v_{13} = v_{23}$</td>
<td>0.25</td>
<td></td>
</tr>
</tbody>
</table>

**4.6 Results and Discussion**

The load and crack length as a function of the applied displacement are shown in Figure 4.7(a) for an SCB test. An initial linear response prior to the start of the delamination propagation was observed for the SCB test. At crack initiation, unstable crack growth was observed in the unstitched portion of the sandwich composite and a decrease in the reacted load occurred. The load then increased as the crack front approached the initial stitch row. Failure of
the initial stitch row occurred at the maximum measured load and resulted in an immediate progression of the crack to the adjacent stitch row. Subsequent rows of stitching were observed to bridge the crack-plane near the vicinity of the crack front. The stitching primarily pulled out from the facesheet near the facesheet-to-core interface. During each subsequent failure of a stitch row, a saw-tooth pattern in the measured load was observed. Additionally, the crack front did not propagate between failures of stitch rows.

![Figure 4.7](image)

Figure 4.7  (a) Load, crack length, and (b) rotation with respect to the applied displacement.

During testing, the rotations were measured at initiation and at the maximum loads prior to stitch failure. The measured and predicted rotations with respect to the applied displacement are shown in Figure 4.7(b). The rotation increases with an increase in the applied displacement. Additionally, good agreement (~1% percent) was obtained between the predicted rotations from the 2D finite element model and experimental measurements. Greater deviations (~7%) between the experimental measurements and the 3D finite element model were obtained; this may be
attributed to differences in material properties and boundary conditions. The cube root of the 
compliance with respect to the crack length is shown in Figure 4.8. As the crack length increases, 
a linear variation in the measured compliance is observed. Excellent agreement was obtained for 
both the 2D and 3D finite element results when compared to the experimental compliance.

![Figure 4.8](image)

Figure 4.8 Cube root of the compliance with respect to crack length.

The fracture energy with respect to the measured crack length for different experimental 
(J-Integral, MBT, and MCC) and computational methods (2D and 3D finite element analysis) is 
shown in Figure 4.9. During crack initiation, the fracture energy is relatively constant due to the 
lack of toughening mechanisms to resist crack growth. Experimental and computational results 
are in excellent agreement in the unstitched region of the sandwich composite. As the crack front 
approaches the initial stitch row, the fracture energy increases linearly by a factor of 6, and 
remains relatively constant during the initial and subsequent failures of stitch rows. As noted 
from previous studies [115, 162, 184], the relative improvement in the crack-growth resistance is
highly dependent upon stitch parameters. The percent differences between each method are shown in Table 4.2. Results from the 2D and 3D finite element models are in close agreement (7%-11%) with the experimentally obtained J-integral estimate (Eq. 4.9). However, the MBT and MCC methods significantly unpredict (~20%) the fracture energy required to propagate cracks for the selected stitch parameters and material design. The relative differences are primarily attributed to the small-scale yielding that is assumed when using the MBT and MCC methods. The fracture energies calculated from the MBT and MCC methods provide relatively conservative estimates of the fracture energy in the stitched region of the sandwich composite, which may be useful in preliminary designs due to its availability in the open literature.

Figure 4.9  Fracture energy as a function of crack length for different methods.
Table 4.2  Percent difference in the fracture energy for different methods.

<table>
<thead>
<tr>
<th>Method</th>
<th>Unstitched</th>
<th>Stitched</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fracture Energy (J/m²)</td>
<td>Percent Difference (%)</td>
</tr>
<tr>
<td>Modified Beam Theory</td>
<td>372.2</td>
<td>4.98</td>
</tr>
<tr>
<td>Modified Compliance Calibration Method</td>
<td>363.2</td>
<td>7.30</td>
</tr>
<tr>
<td>J-Integral (2D FEA)</td>
<td>369.8</td>
<td>5.61</td>
</tr>
<tr>
<td>J-Integral (3D FEA, Average)</td>
<td>395.8</td>
<td>1.02</td>
</tr>
<tr>
<td>J-Integral (Large Rotations)</td>
<td>391.7</td>
<td>-</td>
</tr>
</tbody>
</table>

4.7  Conclusion

In this study, simple analytical expressions for the nonlinear elastic fracture energy of a stitched single cantilevered beam specimen are developed. Under linear elastic material assumptions, the proposed solution is equivalent to existing solutions such as the modified beam theory (MBT) and the modified compliance calibration (MCC) method. Additionally, 2D and 3D finite element analyses were performed to estimate the fracture energy using the J-integral approach. A single cantilevered beam test of a stitched foam core sandwich composite was performed to quantify the percent differences between experimental and computational methods in the unstitched and stitched regions. Stitching was shown to improve the fracture resistance by a factor of 6. Conservative estimates were obtained using the MBT and MCC approaches. Reasonable agreement of the fracture energy was obtained using the 2D and 3D finite element models and the analytical J-integral method.
CHAPTER V

FINITE ELEMENT ANALYSIS OF MODE I DELAMINATION OF STITCHED SANDWICH COMPOSITES

5.1 Introduction

Interlaminar strength of sandwich composites can be increased by incorporating through-the-thickness reinforcements such as stitching or z-pinning. Sandwich composites are widely used for their superior flexural rigidity due to their material architecture, which is composed of outer, rigid facesheets and a lightweight internal core. However, these composites are limited by their low interlaminar strength between the two constituents. Through-the-thickness stitching of sandwich composites with an internal foam core has shown to minimize facesheet-core debonding [108, 162] and improve load-carrying capability [71, 72, 97, 185, 186].

The stitching process involves sewing polymeric threads through the thickness of a dry carbon preform at orthogonal or oblique angles [97, 153, 186] using an industrial or robotic sewing machine. The processing parameters, such as the number of stitches per unit area (stitch density), the mass per unit thread length (linear thread density), thread material and finish, stitch distribution, pattern, pretension, and the stitch architecture, influence the properties and mechanical performance of these composites. The two primary parameters that have been shown to influence the out-of-plane performance of composite materials are stitch density and linear thread density [162]. It has also been shown that stitching does not significantly contribute to the overall part mass (~1% increase) of a sandwich composite with a perforated foam core [66].
Traditional damage modes of foam core sandwich composites subjected to impact also appear to be absent when these structures are reinforced with through-the-thickness stitching [72]. In stitched sandwich composites subjected to low-velocity impact, the primary form of failure is stitch-matrix column buckling and delamination of the topmost surface. Stitched regions subjected to impact have also been reported to undergo larger regions of core cracking when compared to their unstitched counterparts [70].

Simulating delamination of unstitched composite laminates is typically performed by using a cohesive zone modeling approach. In unstitched composites, cohesive elements or contact surfaces are incorporated between plies to simulate delamination. The cohesive material behavior is defined by a traction-separation law, which describes the micromechanical damage process that occurs at the interface. For an unstitched composite laminate, a bilinear traction-separation law (Figure 5.1) is commonly assumed and is primarily associated with small-scale bridging conditions. This model is described by three parameters: the critical strain energy release rate, the maximum traction stress ($\sigma_1$), and a penalty stiffness ($K_{in}$). Damage occurs during the linear softening region ($\overline{AC}$) and is represented by a reduction of the penalty stiffness, where cohesive failure occurs when the effective displacement ($\delta_c$) is reached (Point C). Due to the quasi-brittle nature of composite materials, an arbitrarily high penalty stiffness ($10^{13}$ Pa/m to $10^{14}$ Pa/m) is commonly assumed. Thus, convergence of load-displacement response during fracture testing can be achieved by only modifying the maximum traction stress with a known strain energy release rate. Additionally, the overall shape of the cohesive law is considered to be insignificant under small-scale bridging conditions [187].
In stitched composites, an increase in the strain energy release rate during crack propagation may occur due to the through-the-thickness reinforcement. Under mode I conditions, the stitching bridges the opposing crack faces and induces traction stresses to resist crack growth. Therefore, a bilinear traction-separation law is not appropriate due to large-scale bridging conditions that may be present. Under large-scale bridging conditions, the shape of the cohesive law for the through-thickness reinforcement is needed to capture the failure mechanisms near the delaminated interface [187]. Ranatunga and Clay [6] assumed a linear softening law (bilinear traction) to represent the failure process of a z-pin cohesive zone under mode I conditions. The evolution of damage was predominately due to frictional sliding of the z-pin near the delaminated interface, which is characteristic of a bilinear traction-separation law. The pullout process of the through-thickness reinforcement is mechanically stable. However, this traction-separation response may not be true for z-pins subjected to in-plane shear near the delaminated interface or for other types of through-the-thickness reinforcements [88, 121]. For stitched samples, instability may occur in the traction-separation response during mode I and mode II separation and can yield sharp decreases in the traction stresses [65].
The selection of the shape of the cohesive law for stitched composites is determined by performing interlaminar tension [6, 60, 62, 63, 65] and interlaminar shear tests [117] of a single through-thickness reinforcement. The traction-separation law can also be determined using a J-integral approach [188] and by superposing bilinear cohesive laws to represent multiple damage mechanisms [187]. Moreover, interlaminar tension tests of stitched composite laminates have revealed that the traction-separation law is a trilinear shape [55, 62, 65], as shown in Figure 5.1. The trilinear traction-separation law represents the interaction between different damage mechanisms of the through-thickness reinforcement [189], and is dictated by a relatively linear material response, which consists of a maximum traction stress (Point A), followed by a sudden decrease in the penalty stiffness that represents fiber failure. Subsequently, large-scale fiber bridging is obtained and represented by a linear softening phase ($BC$) from an estimated maximum bridging stress (Point B).

Linear or nonlinear spring elements have been widely used by researchers to represent the failure process of the through-the-thickness reinforcements during composite delamination [55, 60, 190-193]. The material behavior of the spring element is typically dictated by the load-displacement response obtained from interlaminar tensile or shear tests. For example, Tan et al. [62] incorporated experimental load-displacement measurements from interlaminar tension tests as the constitutive behavior for nonlinear spring elements in a cohesive zone FEM of a stitched double cantilevered beam (DCB) specimen. Reasonable agreement between experimental measurements and predicted results was achieved. An alternative approach is to represent through-the-thickness reinforcements as discrete cohesive zones, where two cohesive zone laws are employed to represent the delamination resistance of the unstitched and stitched regions. This allows the microscale damage mechanisms to be better represented near damage sites, rather than
being localized within beam elements that are not connected to internal plies. Encouraging results from several researchers have been obtained [6, 194, 195].

In this study, two-dimensional (2D) and three-dimensional (3D) finite element analyses (FEAs), simulating the facesheet-to-core separation process in stitched single cantilever beam specimens, was performed. The facesheet-to-core separation and failure of the through-the-thickness stitching are represented using discrete cohesive zones using a bilinear and trilinear traction-separation law, respectively. A sensitivity analysis is conducted on the components of a unique trilinear traction-separation law that represents the bridging behavior of through-the-thickness reinforcements using 2D FEA. The facesheet-to-core separation is then modeled using 3D FEMs using a discrete cohesive zone modeling approach. Validation of the models is performed by comparing load and crack growth predictions to experimental measurements. The fabrication, computational, and experimental approaches are discussed in the following sections.

5.2 Materials and Fabrication

Single cantilever beam (SCB) specimens were fabricated from an infused epoxy/carbon fiber sandwich composite panel with 110 kg/m³ foam core. The core was perforated manually in a 6.35 mm grid spacing with a 0.79 mm diameter needle to allow the resin to perfuse through the core during infusion. The carbon/epoxy facesheets were comprised of a cross-ply layup configuration [0º/90º/90º/0º]₃. A crack initiator made from Teflon™ film of thickness 0.0127 mm and length 76.2 mm was placed between the facesheet and the core. The dry sandwich composite preforms were stitched using a 2000H Juki industrial sewing machine using a modified lock stitch architecture. A Vectran™ thread was selected based on previous studies [14, 55] and based on material availability. A 10 mm distance was maintained between the initial crack length and the first row. The 2.25 mm diameter needle was used to stitch the dry preforms and was selected based on robotic stitching processes [14]. A range of stitch densities (X₁ =...
0.0016, 0.0058, and 0.01 stitches/mm²) and linear thread densities ($X_2 = 400, 800, 1200$ Denier) were investigated. The selection of stitch densities and linear thread densities is based on previous studies [115, 162]. The distance between adjacent stitching can be determined by the relative stitch distance $D = \sqrt{1/X_1}$, where $X_1$ is a measure of the stitch density for a single stitched laminate. The sandwich composite preforms were infused using a one-sided vacuum assisted resin transfer molding process with the out-of-autoclave Hexflow 1078 epoxy resin. To reduce the viscosity of the resin, the dry sandwich structure and epoxy resin were separately heated to 88 °C before infusion. Following infusion, the temperature was increased at a rate of 1.8 °C/min to a temperature of 177 °C, held for two hours, and reduced to room temperature (24 °C). The cured laminates were sectioned into 200 mm by 50 mm test coupons, as shown in Figure 5.2. Piano hinges were bonded to the top facesheet above the initial defect using Loctite Hysol EA 9394 adhesive.

Figure 5.2  Schematic of a single cantilever beam (SCB) specimen.
5.3 Single Cantilevered Beam (SCB) Test Procedure

The SCB tests were conducted in accordance with the specimen sizing and test standards proposed by Ratcliffe and Reeder [114]. A displacement rate of 0.5 mm/min was applied to the SCB specimens [104]. The bottom surface of the SCB specimen was rigidly constrained with a non-rotating base. The test fixture with a specimen under load is shown in Figure 5.3. A 1-kN load cell and a linear variable displacement transducer were used to measure the reactive load and applied displacement, respectively, at 1 Hz sampling frequency. Visual measurements of the crack length were quantified using an ARAMIS digital image correlation system. A total of three replicates were performed for each specimen configuration.

![Single cantilevered beam test setup](image)

Figure 5.3 Single cantilevered beam test setup.

5.4 Computational Modeling Approach

Implicit FEA of stitched SCB specimens was performed using ABAQUS 2019 commercial software. A 2D analysis was performed to assess the influence of a trilinear traction-separation law on the
predicted load-displacement response. These results were then used to inform a 3D FEM using a discrete cohesive zone methodology to evaluate the influence of stitch parameters on the crack growth behavior. The facesheet and core interface was discretized to independently simulate the debonding of the foam core from the facesheet and failure of the through-the-thickness reinforcement near the delaminated interface. The material properties used in the FEMs are summarized in Table 5.1. In the following sections, the 2D and 3D finite element analyses are discussed.

<table>
<thead>
<tr>
<th>Facesheet Material Properties [6, 180, 182]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{11} = E_{22}$ (Pa)</td>
</tr>
<tr>
<td>$E_{33}$ (Pa)</td>
</tr>
<tr>
<td>$G_{12} = G_{13}$ (Pa)</td>
</tr>
<tr>
<td>$G_{23}$ (Pa)</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
</tr>
<tr>
<td>$\nu_{13} = \nu_{23}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Core Properties [181]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$ (Pa)</td>
</tr>
<tr>
<td>$\nu$</td>
</tr>
</tbody>
</table>

5.4.2 Two-Dimensional Finite Element Analysis

The 2D FEM is shown in Figure 5.4. The bottom surface of the SCB model is rigidly constrained for all degrees of freedom and a displacement of 50 mm was directly applied above the partially debonded region. An element mesh size of approximately 0.15 mm was used near the cohesive interface. The cohesive surface is discretized into three zones [6]: 1) initiation, 2) facesheet-to-core interface, and 3) an area to represent stitching, as shown in Figure 5.4. The relative cohesive zone of the stitch row is approximately 2 mm and based on physical measurements of the stitch column diameter. At the facesheet-to-core interface (unstitched regions), a bilinear traction separation law was assumed to
represent the unstable behavior at crack initiation. A fracture energy of 220 J/m² was used in the unstitched regions and were obtained from a previous study [162].

![Diagram of two-dimensional finite element model](image)

**Figure 5.4** Two-dimensional finite element model to simulate crack growth in stitched sandwich composites.

In this study, a trilinear traction-separation law is used to represent the bridging behavior of the through-thickness reinforcements in stitched sandwich composites based on an assumed shape from interlaminar tests [65]. A sensitivity analysis of the stitch trilinear traction-separation law on the load-displacement response of an SCB specimen was performed. The influence of the cohesive stiffness (K), maximum elastic stress (σ₁), effective bridging displacement (δₑ), and bridging stress (σ₂) are shown in Figures 5.5(a)-5.5(d), respectively. In Table 5.2, the nominal cohesive parameters used to represent each interface is shown. The trilinear traction-separation law for the stitched region has an initial linear-elastic material behavior, followed by a sharp reduction after the maximum elastic stress to a known bridging stress (Figure 5.1). The traction
stress then decreases linearly from the bridging stress, where cohesive failure occurs when the traction stresses are zero at the maximum effective displacement input. The ratio of the maximum elastic stress to known bridging stress is denoted as the elastic-plastic ratio ($\sigma_1/\sigma_2$). It is important to note that the area under of the curve of the stitch traction-separation law was not maintained to be constant in order to evaluate the influence of each parameter independently. This is unorthodox as compared to traditional cohesive zone modeling approaches, where the area under of a bilinear traction-separation law is assumed to be constant in order to determine the effective displacement by varying the maximum elastic stress.

Table 5.2  Nominal cohesive parameters of the 2D FEM.

<table>
<thead>
<tr>
<th>Interface</th>
<th>TSL* Shape</th>
<th>Penalty Stiffness $K_{mn}$ (Pa/m)</th>
<th>Maximum Elastic Stress, $\sigma_1$ (Pa)</th>
<th>Elastic-Plastic Ratio ($\sigma_2/\sigma_1$)</th>
<th>Effective Displacement, $\delta_e$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initiation</td>
<td>Bilinear</td>
<td>1.00E+13</td>
<td>1.58E+06</td>
<td>-</td>
<td>2.78E-04</td>
</tr>
<tr>
<td>Facesheet to core</td>
<td>Bilinear</td>
<td>1.00E+13</td>
<td>3.00E+04</td>
<td>-</td>
<td>1.47E-02</td>
</tr>
<tr>
<td>Stitch</td>
<td>Trilinear</td>
<td>5.00E+11</td>
<td>2.50E+08</td>
<td>0.1</td>
<td>1.00E-03</td>
</tr>
</tbody>
</table>

TSL: Traction-Separation Law

The load as a function of the applied displacement for select cohesive stiffnesses ($K=1, 2, 5,$ and $10 \times 10^{11}$ Pa/m) is shown in Figure 5.5(a). Decreasing the mode I stiffness of the cohesive traction-separation law increases the magnitude of the maximum load required to fail the stitch rows. For high cohesive stiffnesses ($5 \times 10^{11} < K \leq 5 \times 10^{12}$ Pa/m), the measured load increases linearly until failure of the initial stitch row. As a note, very high cohesive stiffnesses ($>10^{13}$) may yield spurious results and inaccurately represent the traction-separation behavior during the delamination process. Additionally, premature separation between the facesheet and core may occur beyond the initial stitch row if the cohesive stiffness of the through-thickness
reinforcement is too low (K<2×10^{11} \text{ Pa/m}). As a result, greater applied displacements are necessary to reach the failure load of the more ductile cohesive interface. This behavior indicates that ductile through-the-thickness reinforcements with high tensile strengths can be beneficial in resisting delamination in polymer composites.

The influence of the maximum elastic stress on the load-displacement response of a stitched sandwich composite is shown in Figure 5.5(b). Increasing the maximum elastic stress from 250 MPa to 450 MPa linearly increases the maximum load required to fail initial and subsequent stitch rows. Additionally, no significant change in the stiffness (slope) of the load-displacement response is observed. The load as a function of the applied displacement for select effective displacements is shown in Figure 5.5(d). As expected, the bridging behavior of the cohesive interface only influences subsequent stitch rows after the initial stitch row has failed, as shown in Figure 5.5(c). Increasing the effective bridging displacement of the trilinear traction separation law increases the applied displacement and load magnitude required to fail subsequent stitch rows. Furthermore, increasing the effective displacement from 1 mm to 2 mm does not show any difference in the predicted response. This is primarily attributed to the spacing of the through-the-thickness reinforcements. Decreasing the spacing may likely develop greater large-scale bridging and increases in the load-magnitude response. Lastly, increasing the ratio of the bridging stress to the maximum elastic stress (\sigma_1/\sigma_2) after damage initiation does not globally affect the magnitude of compliance or maximum stitch failure loads, as shown in Figure 5.5(d). However, the primary influence is associated with the difference in the magnitude of load before and after stitch failure. Increasing the elastic-plastic ratio decreases the load difference during stitch failure, prior to frictional sliding of the through-thickness reinforcement.
Figure 5.5  The influence of the (a) cohesive stiffness, (b) maximum stress $\sigma_1$, (c) effective displacement $\delta_e$, and (d) bridging strength $\sigma_2$ on the load-displacement response.

Based on the observations revealed by altering the four parameters (cohesive stiffness, maximum elastic stress, effective displacement, and bridging stress), a systematic approach for determining the traction-separation law of a through-thickness reinforcement can be determined. The two primary parameters that influence the load-displacement response are the maximum elastic stress and the cohesive stiffness, which can be altered to appropriately predict crack growth within stitched sandwich composites. In this study, the elastic-plastic ratio is assumed to be approximately 0.1. An effective bridging displacement of 1 mm is used in this study, which is based on interlaminar tensile tests of a single stitch [65]. The cohesive stiffness is increased or decreased to match the stiffness (slope) of the load-displacement response. Further increases or
decreases in the effective displacement can alter the magnitude of the load to predict the desired load-displacement response and crack growth behavior.

5.4.3 Three-Dimensional Finite Element Analysis

The 3D FEM used to simulate separation between the facesheet and core is shown in Figure 5.6. Three different stitch densities (0.0016, 0.0057, and 0.01 stitches/mm²) corresponding to 2, 3, and 4 stitches, respectively, across the specimen width were considered. Incompatible hexagonal elements were used to improve the deformation gradients within the domain of the element when the test article is subjected to bending. Symmetry boundary conditions were imposed to decrease the computational time. A small-time increment, damage stabilization parameter, and a dissipated energy fraction of 10⁻³⁰ seconds, 10⁻⁴, and 0.004, respectively, were used to achieve convergence. To improve computational efficiency, the unstable crack growth between the facesheet and core at initiation was not simulated. A bilinear traction-separation law representing the facesheet-to-core interface was used, and a fracture energy of 220 J/m² with a maximum traction stress of 2.75 MPa was applied. The traction-separation laws used to represent the micromechanical damage process of stitching with different linear thread densities (400, 800, and 1200 Denier) are shown in Figure 5.7. The cohesive parameters used to represent each interface is shown in Table 5.3. A cohesive stiffness of 200 GPa/m for the through-the-thickness reinforcements was assumed and iteratively determined based on a sensitivity analysis, which is discussed in the following section. The maximum elastic stress was iteratively determined by comparing it to load-displacement measurements obtained from the SCB tests. The bridging stress and effective displacement after stitch failure were assumed to be 0.1 of the maximum elastic stress and 1 mm, respectively, based on interlaminar tensile tests obtained from [55, 60, 65]. Since the bridging stress and effective displacements were assumed, the area under the curve of the traction-separation law was not maintained to be constant.
Figure 5.6  Three-dimensional finite element model to simulate crack growth in stitched sandwich composites.

Table 5.3  Cohesive parameters of the 3D FEM.

<table>
<thead>
<tr>
<th>Times</th>
<th>TSL* Shape</th>
<th>Penalty Stiffness $K_m$ (Pa/m)</th>
<th>Maximum Elastic Stress, $\sigma_s$ (Pa)</th>
<th>Elastic-Plastic Ratio $(\sigma_2/\sigma_1)$</th>
<th>Effective Displacement, $\delta_e$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Facesheet to core</td>
<td>Bilinear</td>
<td>1.00E+13</td>
<td>2.75E+06</td>
<td>-</td>
<td>1.60E-04</td>
</tr>
<tr>
<td>400 Denier Stitch</td>
<td>Trilinear</td>
<td>2.00E+11</td>
<td>3.00E+07</td>
<td>0.1</td>
<td>1.00E-03</td>
</tr>
<tr>
<td>800 Denier Stitch</td>
<td>Trilinear</td>
<td>2.00E+11</td>
<td>5.00E+07</td>
<td>0.1</td>
<td>1.00E-03</td>
</tr>
<tr>
<td>1200 Denier Stitch</td>
<td>Trilinear</td>
<td>2.00E+11</td>
<td>8.10E+07</td>
<td>0.1</td>
<td>1.00E-03</td>
</tr>
</tbody>
</table>

TSL: Traction-Separation Law
A systematic approach for determining the traction-separation law for through-thickness reinforcements is used for stitched sandwich composite laminates that contain different stitch densities and linear thread densities. A comparison of the predicted and experimental values of the load and crack length for SCB specimens containing select stitch densities (0.0016, 0.0057, and 0.01 stitches/mm²) is shown in Figures 5.8(a) and 5.8(b). As mentioned previously, unstable crack propagation is observed at crack initiation, which results in a sharp decrease in the reacted load. Correspondingly, an increase in the measured crack length is observed. The reacted load then increases linearly as the crack front approaches the initial stitch row. Upon failure of a stitch row, the crack front immediately progresses to the adjacent stitch row. Sharp decreases in the measured load were observed as subsequent stitch rows failed. The magnitude of the failure load at each stitch row decreased with an increase in the applied displacement. This behavior is due to a greater distance between the location of the reacted load and crack front, which decreases with each subsequent failure of a stitch row. Increasing the stitch density from 0.0016 stitches/mm² to
0.01 stitches/mm² proportionally increases the maximum load prior to failure by approximately 48% and decreases the measured crack growth by approximately 16%.

Figure 5.8  Influence of stitch density on the (a) load and (b) crack growth response.

The influence of select linear thread densities (400, 800, and 1200 Denier) on the load and crack growth of an SCB specimen is shown in Figures 5.9(a) and 5.9(b). Increasing the linear thread density from 400 Denier to 800 Denier increases the maximum load at which stitch failure occurs by approximately 18%, but only a 6% decrease is realized in the measured crack lengths. At high linear thread densities (>1200 Denier), the crack lengths decreased by approximately 14% to 30% was observed. The greater reduction in crack growth is mainly attributed to a change in the fracture morphology, as shown in Figures 5.10(a) and 5.10(b). Primarily stitch pullout from the facesheet was observed for low linear thread densities (400 Denier). At high linear thread densities (1200 Denier), stitch-column pullout and foam core failure were observed [162]. This change in fracture morphology results in significantly greater
capability to resist crack growth. Lastly, multiple stitch rows were observed to fail for high and low linear thread densities.

![Figure 5.9 Influence of linear thread density on the (a) load and (b) crack growth response.](image)

The discrete cohesive FEM approach shows excellent agreement with the load and crack length measurements for each select stitch density and linear thread density. In this approach, the traction-separation law for the through-thickness reinforcement was assumed to be the same for each stitch row. During some of the tests, the simultaneous pullout of the through-thickness reinforcements along a stitch row did not occur, which caused some discrepancy between the predicted and experimental results, as shown in Figure 5.9(a). This behavior may occur due to the misalignment of the through-thickness reinforcement. Additionally, compaction of the structural threading can occur during the vacuum bagging process, which can distort the reinforcement in the through-the-thickness direction and result in a variation of the maximum elastic strength. These aforementioned effects induce changes in the applied displacement at which a stitch row failure occurs. Therefore, the load-displacement and crack-growth responses
can appear shifted due to a variation in the traction-separation law of each through-the-thickness reinforcement. It may be more appropriate to use a stochastic finite element process to take in account the variation in the traction-separation response for various linear thread densities and stitch densities used in stitched sandwich composites.

Figure 5.10  Fracture morphology during the facesheet-to-core separation of a sandwich composite stitched with (a) 400 Denier and (b) 1200 Denier Vectran™ thread.

The predicted crack front curvature along the specimen’s width for select loads and applied displacements is shown in Figure 5.11. The initial crack front curvature is straight along the specimen’s width prior to crack initiation (Region 1). As the crack propagates (Region 2), a variation in the growing crack front is observed due to the anticlastic curvature of the facesheet subjected to bending [183, 196-199]. This behavior is due to Poisson’s effect, which results in opposing concave and convex curvature along each side of the composite facesheet. Additional curvature in the crack front is observed as the crack front approaches the initial stitch row (Region 3). The through-thickness reinforcements locally constrain the facesheet to resist crack
growth and result in some curvature in the crack front near the stitching. Crack front curvature appears to be dominated by the anticlastic deformation of the facesheet away from the through-thickness reinforcements. At the maximum applied load, the crack front moves beyond the initial stitch row (Region 4), which is followed by failure of the initial stitch row. Afterward, the crack front immediately propagates to the second stitch row and stitch bridging is observed (Regions 5-6).
5.5 Conclusion

A discrete cohesive zone modeling approach was used to simulate the mode I load and crack length response of stitched sandwich composites with select stitch densities (0.0016-0.01 stitches/mm²) and linear thread densities (400-1200 Denier). Single cantilever beam (SCB) tests were performed to determine the load and crack growth response as a function of the applied
displacement. Experimental tests reveal unique fracture morphologies that are dependent on stitch parameters. Using low linear thread densities (400 Denier), stitch pullout from the facesheet near the facesheet-to-core interface was observed. However, matrix-stitch column pullout from the core was observed at the facesheet-to-core interface for stitched sandwich composites with high linear thread densities (1200 Denier) of Vectran™ thread.

In this study, a trilinear traction-separation law was used to represent the failure process of the through-the-thickness reinforcement within 2D and 3D FEMs. The 2D FEA revealed that the cohesive stitch stiffness and elastic maximum traction stress of the stitch are the primary parameters that influence the initial load-displacement response of an SCB test, whereas the bridging stress and effective displacement influenced the load-displacement response after the initial stitch row failure. The predicted load and crack length responses using a 3D FEM have good agreement with experimental measurements. However, the current approach does not consider the variation of the traction-separation law used to represent the through-thickness reinforcement. Variation in the maximum elastic stress, bridging stress, and effective displacement after stitch failure may occur due to misalignment, angle of the reinforcement, and stitch compaction that could occur during fabrication. The 3D FEM approach showed that crack fronts may be influenced by the through-the-thickness reinforcements and can lead to crack curvature along the specimen width. Lastly, the methodology presented in this study provides a pathway to guide researchers on the selection of a trilinear traction-separation law that accurately represents the failure process of stitching during delamination events.
CHAPTER VI
ON THE ESTIMATION OF INTERNAL CRACK GROWTH IN POLYMER COMPOSITES USING OPTICAL FIBERS

6.1 Introduction

Advanced composite laminates are highly susceptible to delaminations because of their low interlaminar shear and tensile strengths. As a result, numerous studies [200-203] have investigated the required energy to induce fracture, known as the critical strain energy release rate (SERR) or fracture toughness, to characterize the initiation of delamination within composite laminates. Standardized tests such as the double cantilever beam (DCB) or end-notch flexural (ENF) tests are the primary methods to measure fracture toughness [104, 105]. However, ply orientation and coupon size are limited because of the relative variation in the SERR across the width of DCB or ENF coupons.

The variation of the critical SERR along the delamination front is non-uniform and is primarily due to a boundary layer phenomenon. However, the critical SERR can also fluctuate due to local differences in the fiber-to-resin bond strength, fracture surface morphology, and porosity [204]. The boundary layer is developed from an anticlastic curvature due to Poisson’s effect [196-199]. A bending-bending coupling is formed between the in-plane cardinal directions, which results in an opposing concave and convex curvature along each side of the laminate. As a result, the variation in the critical SERR is developed and can exist in both isotropic and anisotropic materials [183].
The critical SERR distributions for an unidirectional [0°/0°] specimen undergoing a mode I delamination [183] is shown in Figure 6.1. For mode I crack growth, the SERR away from the boundary layer (or laminate edge) approaches a constant value and decreases near the laminate edge due to the anticlastic curvature of the delaminated arms [183]. Thus, the SERR will develop a “thumbnail” variation across the delamination front within DCB specimens. As a consequence of the facesheet undergoing anticlastic curvature, the crack length will be greater near the center of the laminate even though the delamination front is initially straight [205]. For ENF specimens (mode II), the maximum critical SERR occurs near the laminate edges and a minimum near the center width of the laminate [183, 206].

![Figure 6.1 Energy release rate distributions for a [0°/0°] composite specimen [183]. Permission was obtained to republish this work by Elservier.](image-url)
The fracture toughness is also highly influenced by the ply orientation [183, 199, 204, 207, 208] and specimen width [209]. Anderson [201] determined that depending on the direction of delamination relative to the ply orientation, fracture toughness characterization using only unidirectional plies can result in an underestimation (mode I) or overestimation (mode II) of fracture toughness values. Davidson [204] has shown that the visual crack length from the laminate edge is approximately 7.3% and 1.6% less than the actual crack length for [±45°] and [0]T, respectively, and is symmetric along the center axis along the specimen’s length. Sun and Zheng [183] have shown that the normalized fracture toughness with respect to its average can vary by as much as 150% for angle-ply laminates. Furthermore, the variation along the specimen width can be skewed for angle-ply laminates that contain bend-twist coupling. Increasing the characteristic skewness \(D_{16}/D_{11}\) results in a greater asymmetry of the critical SERR, where \(D_{16}\) and \(D_{11}\) are the laminate’s bending stiffness components. Lastly, studies [183, 209] have shown that the variation in the critical SERR can depend on the specimen width and result in a 50% difference in the calculated critical SERR as compared to visual surface crack measurements.

Much effort has been made in designing DCB and ENF specimens to reduce the variation across the delamination front by modifying the ply configuration. Sun [183], Davidson [206], and Hudson [210] have recommended including a large amount 0° plies near the mid-plane to measure fracture toughness for delaminations bounded by angled plies. The addition of 0° plies near the laminate mid-plane results in a reduction in the critical SERR variation and produces a behavior similar to a [0]T ply configuration. However, the inclusion of additional 0° plies may not be entirely representative of the damage that can occur in adjacent non-zero degree plies, which is necessary for accurate prediction of delamination growth [201]. Furthermore, residual stresses can develop due to differences in the CTE mismatch between plies [211] and due to a
spatial thermal variation within thicker laminates, which subsequently contribute to the total energy released during delamination [201, 212-215].

Due to the significant variation in the critical SERR and the considerable influence of ply orientation and specimen width, it is necessary to obtain accurate internal measurements to simulate delamination accurately. Thus far, only one study has been found regarding the measurement of the internal delamination length. De Kalbermatten [216] used an x-ray and acoustic emission technique to quantify the shape of the delamination experimentally. The crack front was characterized by stopping the test intermittently, slightly opening the DCB specimen, and injecting liquid dye penetrant during the delamination growth. This test results in a time-consuming and costly procedures to visualize the crack front.

Other methods, such as the electrical potential drop (EPD) method [217] or detecting damage using optical fiber (OF) etched with fiber Bragg gratings (FBG) [218], have shown promise to locate delamination and determine damage size. In particular, Ueda et al. [219], investigates uses a two-stage EPD method with an externally bonded array of electrodes to estimate the location of delamination. However, the method is highly dependent on the formation of delamination. Bocherens et al. [220] used an embedded array of optical fiber etched with FBGs to detect permanent setting in composite laminates subjected to impact loading. Furthermore, optical fibers have been shown to not significantly influence interlaminar properties provided that they are oriented in the reinforcement direction [218]. Both methods, however, are limited to a relatively low spatial resolution.

Recently, unmodified OFs embedded within composites, and subjected to end-notch flexure [221] and impact [222], were used to map the delamination front with a high spatial resolution (<1.25 mm). In addition, several studies have demonstrated that unmodified OFs are
excellent candidates to detect crack growth [218, 223-225]. However, these methods presented require the use of the OF to bridge the crack plane and induce additional stiffness that may alter the crack growth behavior.

The objective of this paper is to demonstrate a Lagrangian cross-correlation approach to estimate the delamination front location without influencing crack growth behavior and to determine the critical SERR variation experimentally using high-spatial resolution optical fibers. As a note, cross-correlation has been used to detect damage based on strain modes under ambient excitation [226] and from mode-converted lamb waves [227]. However, the use of cross-correlation has been adopted in this paper to identify the location of the delamination front to measure the internal delamination in situ within DCB composite specimens. A single DCB specimen is fabricated with multiple passes of embedded OF is used to demonstrate the cross-correlation numerical approach and its efficacy. In the following sections, the interrogation method and numerical procedure are presented. The fabrication process and test procedure are presented. Lastly, a comparison of crack lengths, fracture toughness, and flexural moduli between the numerical approach and visual edge measurements is discussed.

6.2 Distributed Optical Fiber Sensing

High spatial resolution (< 1 mm) strain and temperature measurements can be obtained using distributed OF sensing, which uses traditional non-inscribed single mode fibers. In distributed fiber optic sensing, both strain and temperature are obtained by measuring the Rayleigh backscatter along the length of the fiber using swept-wavelength coherent interferometry. Rayleigh backscatter is caused by the reflection of light due to heterogeneities that are naturally present in the OF [228-230]. Mechanical or thermal loading can induce shifts in
the Rayleigh backscatter spectrum. These shifts can be correlated with the mechanical strain and change in temperature acting on the OF, as [229]

\[
\frac{\Delta \lambda}{\lambda} = K_T \cdot \Delta T + K_\varepsilon \cdot \varepsilon
\]  
(6.1)

where \(\Delta \lambda\) is the wavelength shift, \(\lambda\) is the wavelength, \(\Delta T\) is the temperature change, and \(\varepsilon\) is the strain along the fiber length. \(K_T\) is a thermal coefficient (~0.634) that relates the thermal expansion coefficient and thermo-optic coefficient of the OF [229]. \(K_\varepsilon\) is a strain coefficient (~6.67) based on the material properties of the fiber optic sensor [229].

### 6.3 Materials and Fabrication

The double cantilever beam specimens used in this study were fabricated from an infused epoxy/ carbon fiber laminate of \([0/90/90/0]_{3s}\) configuration. The layup configuration was selected to minimize the variation of the SERR across the delamination front [183, 206, 210]. This configuration allows for a relatively good comparison to the visually-obtained edge measurements and to assist in establishing the efficacy of this approach. The epoxy matrix is an out-of-autoclave API-1078 VARTM resin system. The resin was infused into a dry carbon biaxial \([0/90]\) noncrimped fabric (NCF) by SAERTEX, Inc. A Teflon™ film of 0.0127 mm thickness was used as the crack initiator at the midplane of the laminate. The initial starting crack length was approximately 50.8 mm. The laminate was cured using the cure cycle shown in Figure 6.2. The temperature of the oven, resin, and carbon fiber were increased to 88 °C to reduce the viscosity of the resin before infusion. The temperature was then increased at a rate of 1.8 °C/min to a temperature of 149 °C and held for six hours. The temperature was then raised to 177 °C for a two-hour soak and then reduced to ambient temperature (~24 °C).
Figure 6.2  Two-step cure cycle.

Prior to embedding the OF sensors in the NCF preform, polytetrafluoroethylene (PTFE) tubes were temporarily woven through the local preform stitching of the NCF fabric as shown in Figure 6.3(a). The optical fibers were then passed through these tubes as shown in Figure 6.3(b). Once the optical fiber was interlaced into the NCF fabric, the PTFE tubes were removed. Each optical fiber pass was placed within a one degree tolerance relative to the carbon fiber tow direction. Additional PTFE tubing was used at the ingress of the laminate to prevent OF breakage during handling of the cured part. The OFs outside of the laminate were encased in a separate bag to prevent resin buildup on the sensors. The layout of an OF sensor within the DCB specimen is shown in Figure 6.4. Three equally-spaced fiber passes were used to characterize the delamination front.
Figure 6.3  (a) Temporary placement of PTFE tubes through local preform stitching and (b) OF threaded through an NCF.

Figure 6.4  The layout of the OF within the DCB specimen.

6.4  Experimental Procedure

A single DCB specimen was clevis-mounted in an Instron model 8872 hydraulic test frame with a 1 kN load cell at a displacement rate of 0.5 mm/min. One side of each test article was marked in 12.7 mm intervals to obtain visual edge measurements; additionally, surface measurements of crack
lengths were measured using an ARAMIS camera system. Internal strains were measured using the Luna Technologies ODiSI-B fiber optic system. The optical fiber used in this study is a polyimide-coated, low bend loss optical glass fiber (GEOSIL®-SM) with an operating wavelength of 1550 nm. A schematic of the test setup is shown in Figure 6.5. The strain distributions were measured along three fiber passes that were embedded within the DCB specimen. Due to the limitations of the testing apparatus, tests were terminated when the crack-tip opening displacement reached approximately 50.8 mm.

![Figure 6.5 Schematic of the DCB test setup.](image)

6.5 Internal Strain Characterization of DCB Specimen

The profile of strain along the length of a double cantilever beam can be characterized by a “wave-like” strain distribution that propagates with the progression of delamination, as shown in Figure 6.6. This strain profile can be categorized into three strain regions: 1) Initial flexural
strain region, 2) perturbed strain region due to the crack front, and 3) a zero-strain region ahead of the delamination front. Region 1 is due to the separation of the DCB test article along the mid-plane of the laminate. As the crack front is initiated, the strain increases linearly due to the bending moment reacted near the crack front. As a result, high tensile stresses develop slightly ahead of the crack. Beyond the crack tip, the magnitude of the strain decreases to zero. This region of strain measurement is perturbed by the radius of the crack front and its associated stress intensity. Region 2 also represents the process zone length, the length of the cohesive zone ahead of the crack that undergoes a stiffness degradation prior to crack progression [231].

![Characteristic strain distribution along the specimen length.](image)

Figure 6.6 Characteristic strain distribution along the specimen length.

The strain distributions along the OF length and for select applied displacements (0 mm, 15 mm, and 30 mm) are shown in Figure 6.7. Unique strain distributions are developed to identify each fiber pass as the delamination progresses with an increase in displacement. Due to the looping pattern of the OF, the symmetry between the characteristic “wavy” distributions between each adjacent fiber pass is seen. As expected, the magnitude of strain in each fiber pass
increases with increasing displacement and, thus, shifts the strain distribution uniformly as the delamination progresses. Interestingly, as will be shown, the slope of the strain curves (differential strain) in Region 2 for each fiber pass remains relatively uniform with increasing displacement.

Figure 6.7 Strain distribution along each fiber pass and select crack-tip opening displacements (0 mm, 15 mm, 30 mm, and 45 mm).

The differential strain as a function of location along the OF length for select applied displacements is shown in Figure 6.8. The differential strain region near the crack front (Region 2) reveals unique peaks that correspond to the location of the crack front. The shift in these peaks is a measure of the change in crack length (Δa) for each fiber pass. The differential strain (slope) was calculated along each fiber pass in Region 2 (perturbed strain region) for each
applied displacement and is shown in Figure 6.9. The differential strain was normalized by the
differential strain at the maximum applied load upon the initial occurrence of delamination. The
start of delamination was identified by the initial reduction in slope of normalized differential
strain with respect to the applied displacement. For each fiber pass, an initial elastic response is
developed followed by a relatively uniform distribution of differential strain. This uniformity of
the differential strain occurs throughout the propagation of delamination during the DCB test and
is relatively independent of the applied loading conditions. Furthermore, the consistent
differential strain after delamination indicates that the process zone length is relatively constant
after initial propagation. This is due to very little toughening mechanisms, such as fiber bridging,
that could increase the strain energy release rate during crack propagation [232]. This behavior
allows for the development of a numerical method to identify the delamination front, its
progression, and termination point. Furthermore, the crack front shape and distribution of the
SERR across the delamination front can be determined.
Figure 6.8  Differential strain distribution along each fiber pass and select crack-tip opening displacements (0 mm, 15 mm, 30 mm, and 45 mm).

Figure 6.9  Normalized differential strain in Region 2 as a function of the applied displacement for each fiber pass.
6.6 Numerical Procedure to Determine Crack Length and Fracture Toughness

Identification of the delamination front is achieved by measuring the shift, as shown in Figure 6.8, in the differential strain due to an increase in crack length. The differential strain is evaluated as

\[
\frac{d\varepsilon}{dx} \approx \frac{\varepsilon_i - \varepsilon_{i-1}}{x_i - x_{i-1}}, (i = 1, 2, ..., N)
\] (6.2)

where \(\varepsilon_i\) and \(\varepsilon_{i-1}\) correspond to the \(i^{th}\) and \((i-1)^{th}\) measurement of strain for a total of \(N\) samples. Similarly, \(x_i\) and \(x_{i-1}\) correspond to the \(i^{th}\) and \((i-1)^{th}\) location along the OF. These differential strain peaks correspond to the location of the delamination front along the OF. Furthermore, the distance between the differential strain peaks obtained at the max load and corresponding peaks at subsequent loads can be used to estimate the change in crack length as the delamination progresses.

The crack length is estimated by using a Lagrangian cross-correlation approach, as shown in the flow chart in Figure 6.10. In this Lagrangian scheme, the measured differential strain during delamination propagation are correlated to the differential strain at the onset of delamination, or max applied load. Cross-correlation \(\rho_{12}\) measures the similarity between two signals [233] and can be expressed as

\[
\rho_{12}(\Delta a) = \int_{-\infty}^{\infty} \frac{d\varepsilon_1(x)}{dx} \cdot \frac{d\varepsilon_2(x + \Delta a)}{dx} dx
\] (6.3)

where \(\frac{d\varepsilon_1(\Delta a)}{dx}\) and \(\frac{d\varepsilon_2(x+\Delta a)}{dx}\) corresponds to the differential strain when the maximum load occurs and for subsequent loadings, respectively, at a location \(x\) along the optical fiber. The normalized correlation \(\rho_{12}/\rho_{max}\) with respect to the normalized distance \((\Delta a/\Delta a_{max})\) is shown in Figure 6.11(a). The correlation value \(\rho_{12}\) becomes a maximum \(\rho_{max}\) when the differential
strain $\frac{d\varepsilon_1}{dx}$ at $x$ has the same gradient in $\frac{d\varepsilon_2}{dx}$ at $x+\Delta a$. The distance $\Delta a$ corresponds to the change in crack length when $\rho_{12} = \rho_{\text{max}}$. The distance $\Delta a_{\text{max}}$ corresponds to the largest distance to obtain a measure of correlation between $\frac{d\varepsilon_1}{dx}$ and $\frac{d\varepsilon_2}{dx}$. To minimize the noise present within the measurements, a beta distribution of the cross-correlation coefficients is used to locate the maximum correlation coefficient, as shown in Figure 6.11(b). The probability beta distribution function is given by [234]

$$f(\Delta a) = \frac{\rho_{12}^{a-1}[1 - \rho_{12}]^{b-1}}{\beta(a, b)}$$

(6.4)

where $\beta(a, b)$ can be expressed as

$$\beta(a, b) = \int_0^1 \rho_{12}^{a-1}[1 - \rho_{12}]^{b-1} d\rho_{12}$$

(6.5)

The shape parameters $a$ and $b$ were estimated using maximum-likelihood estimation, i.e.,

$$b = \frac{(1 - \bar{x})}{s^2}(\bar{x}(1 - \bar{x}) - s^2)$$

(6.6)

$$a = \frac{\bar{x}b}{1 - \bar{x}}$$

(6.7)

where $s^2$ and $\bar{x}$ correspond to the variance and mean of normalized correlations, respectively.
Once the shape parameters for the beta distribution are determined, the maximum correlation can be estimated as the average of all correlation values that exceed a 5% probability that the true correlation can exceed the mean correlation value. Once the true maximum correlation is estimated, the change in crack length can be calculated as

$$\Delta a = \frac{\Delta a}{\Delta a_{max}} \cdot FPL$$

(6.8)

where $\frac{\Delta a}{\Delta a_{max}}$ is the normalized crack length and $FPL$ is the fiber pass length. The total crack length $a$ can then be estimated as

$$a = a_0 + \Delta a$$

(6.9)

where $a_0$ is the initial crack length. The corresponding mode I fracture toughness $G_I$ is estimated using the modified compliance calibration method using
\[ G_I = \frac{3 P^2 C^{2/3}}{2 A_1 b h} \] (6.10)

where \( P \) and \( C \) are the applied load and compliance, respectively. The compliance is determined from the ratio of the applied displacement \( (\delta) \) to the applied load \( (p) \). The specimen width and thickness are denoted as \( b \) and \( h \), respectively. The slope \( A_1 \) of the normalized crack length \( (a/h) \) is a linear function of \( C^{1/3} \).

The flexural stiffness \( E_{of} \) is estimated from strain data using a linear elastic approach as

\[ E_{of} = \frac{M \cdot y}{I^* \cdot \varepsilon_{max}} = \frac{P \cdot (a - L_{app}) \cdot y}{I^* \cdot \varepsilon_{max}} \] (6.11)

where \( M \) is the bending moment, \( I^* \) is the weighted moment of inertia using classical lamination theory, \( y \) is the distance from the neutral axis of the cross-section to the location of the OF, and \( \varepsilon_{max} \) is the maximum axial strain near the crack tip. The term \( (a - L_{app}) \) represents the distance from the applied load to the crack front. The distance \( y \) is estimated to be approximately 0.8 mm. Comparisons are made to the Modified Beam Theory (MBT) estimate of the flexural modulus \( E_{1f} \) as

\[ E_{1f} = \frac{64 \cdot P \cdot (a + |\Delta|)^3}{\delta \cdot b \cdot h^3} \] (6.12)

where \( a + |\Delta| \) is the corrected delamination length determined in accordance with ASTM STD D5528.
6.7 Results and Discussion

The load and crack lengths, from the visual edge measurements and from OF strain measurements are shown in Figure 6.12. Excellent agreement is obtained between the surface measurement data and OF strain measurements from Fiber Pass 3. A somewhat greater deviation is obtained from OF passes 1 and 2. These greater variations are attributed to the non-uniformity in the crack length at the delamination front.

The fracture toughness values with respect to the crack lengths that are visually-obtained and computed from OF strain measurements are shown in Figure 6.13. As before, the fracture toughness from Fiber Pass 3 has excellent agreement with the surface measurements as Fiber
Pass 3 is the nearest to the edge from which the data was taken. Fiber passes 1 and 2 show a greater variation in the estimated internal crack lengths due to the variation in the SERR across the delamination front. The variation of the SERR for select displacements is shown in Figure 6.14. As expected, the SERR has a concave curvature along the delamination front. This variation is primarily due to the anticlastic behavior of composite laminates undergoing bending. Furthermore, increasing the applied displacement does not significantly influence the relative variation of the SERR across the delamination front.

The relative percent differences of fracture toughness estimates obtained from the OF estimates and visual edge measurements range from 2.7% to 4.7% and are given in Table 6.1. These differences agree with analytical predictions obtained from a previous study [204]. It is noted that a maximum of 17.7% difference is obtained at an applied displacement of 11.5 mm. This large error is attributed to noise in the measurements that were used to calculate the differential strain. The statistical significance of these optical fiber fracture toughness estimates, as compared to the visual measurements, depends greatly on the high spatial resolution of strain measurements to determine the internal crack length. The spatial resolution of the crack length measurements using optical fibers is approximately ±0.625 mm, which is much lower than the crack lengths measured (~1.25% at initial crack length). For relatively small crack lengths (< 2.5 mm), the optical fiber estimates are statistically insignificant as compared to visual measurements. In addition, optical fiber waviness or inclination along the specimen may influence the crack lengths estimates. The relative error $\varepsilon$ due to the optical fiber waviness to can be estimated as

$$
\varepsilon = \left| \frac{a - a \cdot \cos(\varphi)}{a \cdot \cos(\varphi)} \right| \cdot 100
$$

(6.13)
where $\varphi$ is the estimated waviness or inclination angle along the length of the DCB specimen. The term $a - a \cdot \cos(\varphi)$ represents the difference in the true crack length as compared to the optical fiber estimates. In Figure 6.15, the estimated error as a function of the inclination angle. For relatively low angles ($< 2^\circ$), the estimated error is approximately 1.5%. Furthermore, the relative error associated with the measurements is considered negligible due to the total crack growth developed during test. However, optical fibers that have relatively large inclination angles ($> 2^\circ$) will require a correction factor to properly determine the true crack length. Otherwise, an excellent correlation between the OF measurements and visually-obtained measurements is obtained. This measurement technique allows for the characterization of the internal SERRs for composites with unique ply configurations, where the SERR variation may be much greater.

Flexural modulus was also determined from the OF strain data using Eq. 6.11 and from MBT using Eq. 6.12 and is shown in Figure 6.16. Both methods indicate a decrease in the flexural modulus with an increase in the applied displacement after the start of delamination. The decrease in flexural moduli is attributed to the plastic effects associated with matrix cracking and filament failure near the delamination zone. In addition, MBT greatly over predicts the flexural modulus even with the corrections proposed by Hashemi [156]. Furthermore, the OF data shows an increase in the flexural modulus before delamination occurs and for displacements less than approximately 0.6 mm as shown in an enlarged view within Figure 6.16 (Region of Interest, ROI). The flexural moduli from MBT and OFs are compared to experimentally obtained values from reference [235] and are given in Table 6.2. Flexural modulus predictions from Fiber Pass 2 show a relatively good agreement (approximately 25.5%) with experimentally obtained values before the start of delamination. However, the flexural stiffness predictions from Fiber Passes 1
and 3 are influenced by the variation in the SERR along the delamination front and show greater deviations from experimentally obtained measurements (approximately 20% difference).

Table 6.1 Percent differences between visual edge measurements and optical fiber predictions.

<table>
<thead>
<tr>
<th>Displacement (mm)</th>
<th>Load (N)</th>
<th>Fracture Toughness (J/m²)</th>
<th>Percent Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>186</td>
<td>330</td>
<td>325</td>
</tr>
<tr>
<td>2.1</td>
<td>171</td>
<td>389</td>
<td>400</td>
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<td>2.9</td>
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<td>6.7</td>
<td>119</td>
<td>512</td>
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</tr>
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<td>8.4</td>
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<td>515</td>
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<td>9.6</td>
<td>102</td>
<td>534</td>
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<td>11.5</td>
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<td>43.3</td>
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<tr>
<td>Average</td>
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<td>-</td>
<td>-</td>
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Table 6.2  Comparison of measured and predicted fracture toughness.

<table>
<thead>
<tr>
<th>Method to estimate Flexural Modulus</th>
<th>AVG (GPa)</th>
<th>STD (GPa)</th>
<th>%Diff from Exp. Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber Pass 1* (Eq. 6.11)</td>
<td>214.4</td>
<td>118.0</td>
<td>360.45</td>
</tr>
<tr>
<td>Fiber Pass 2* (Eq. 6.11)</td>
<td>32.1</td>
<td>3.2</td>
<td>31.03</td>
</tr>
<tr>
<td>Fiber Pass 3* (Eq. 6.11)</td>
<td>25.6</td>
<td>3.5</td>
<td>45.03</td>
</tr>
<tr>
<td>Modified Beam Theory (Eq. 6.12)</td>
<td>34.7</td>
<td>4.6</td>
<td>25.52</td>
</tr>
<tr>
<td>Experimental values from Ref. [235]</td>
<td>46.6</td>
<td>2.5</td>
<td>-</td>
</tr>
</tbody>
</table>

*Average values obtained prior to delamination

Figure 6.12  Load and crack length as a function of displacement for each fiber pass and edge measurement.
Figure 6.13  Load and crack length as a function of displacement for Fiber Passes 1, 2, and 3, and edge (visual) measurement.

Figure 6.14  Fracture toughness variation across the specimen width for select applied displacements.
Figure 6.15  Estimation of the prediction error and relative difference in the true crack length as a function of inclination angle of the optical fiber along the DCB specimen.

Figure 6.16  Flexural modulus (Optical fiber predictions and measurements using MBT) and load as a function of the applied displacement.
6.8 Conclusions

In this study, strain distributions were obtained from OF sensors embedded within DCB test articles and correlated to the propagation of delamination. The sensors were woven through the local preform stitching before resin infusion. A unique “wave-like” strain distribution that shifts with the propagation of delamination was obtained. The differential strain along the length of the OF was computed, resulting in unique peaks that correspond to the location of the delamination front.

A cross-correlation based approach was developed to estimate the crack length using the shift in the differential strain gradient. Excellent agreement between SERR surface measurements and OF estimates (<4.7% difference) were obtained. Measuring the fracture toughness across the specimen width revealed a concave curvature that is associated with the anticlastic behavior of laminates undergoing bending. In addition, the flexural moduli (Table 6.2) before the start of delamination can be estimated with reasonable accuracy (6.93%).

Using OF strain data, the measurement of internal delamination propagation in composite laminates of unique ply configurations can be achieved. This approach alleviates the need to require unidirectional plies near the midplane of composite laminates to determine the SERR of angle-ply laminates. This approach also allows engineers and researchers to improve their predictive capability and design composites from a crack-progression perspective by using internal SERR measurements. Lastly, this approach can be used to monitor crack growth progression, leading to a more condition-based maintenance procedure. Future work includes the investigation of the SERR variation of angled-ply laminates using the Lagrangian cross-correlation method with validation using ultrasonic methods.
CHAPTER VII
ON THE ESTIMATION OF CRACK GROWTH DURING THE FRACTURE OF STITCHED SANDWICH COMPOSITES

7.1 Introduction

Sandwich composites are composed of two outer, rigid facesheets and a lightweight internal core. The increased part thickness with a lightweight core increases the flexural rigidity of the composite structure that may be needed for primary and secondary load applications. Sandwich composites can delaminate between the outer facesheets and the internal core at relatively low out-of-plane loads. The delamination is due to a property mismatch between the facesheet and core and leads to decreased strength and stiffness of the structure. Furthermore, visual inspection of the outermost surfaces provides little indication of the severity of delamination.

The incorporation of through-the-thickness reinforcements, such as stitching, can greatly enhance the interlaminar properties of the sandwich composite and resist crack growth [9, 10]. Stitching has also been shown to increase the in-plane mechanical properties of sandwich composites with only a 1% weight increase [66, 97]. The separation between the core and facesheets is impeded by bridging stresses that are developed by through-the-thickness reinforcements, which limits the opening and sliding displacements along the delaminated plane [88, 122, 149, 150]. Stitch processing parameters, such as the number of stitches per unit area (stitch density), the stitch distribution, pretension, stitch angle, and mass per unit length (linear
thread density), greatly influence the in-plane and out-of-plane performance of composite materials. For example, stitches oriented at 45° have been shown to enhance the in-plane flexural rigidity, in-plane shear, and out-of-plane compressive strength [71, 97]. Additionally, a high density of stitches also allows for greater energy absorption and facesheet delamination suppression [178].

In this study, mode I characteristics of stitched sandwich composites are examined. Single cantilevered beam (SCB) tests were performed to determine the influence of stitching on the mode I fracture energy. The SCB specimens were embedded with optical fibers (OF) to determine internal crack length measurements using a Lagrangian cross-correlation approach [182]. Implicit finite element analysis is performed and compared to OF strain measurements. In the following sections, the OF interrogation method, FEA computational approach, and numerical approach to determine internal crack lengths are presented.

7.2 Distributed Optical Fiber Sensing

High spatial resolution (< 1 mm) strain measurements can be obtained using distributed OF sensing, which uses traditional non-inscribed single-mode fibers. In distributed fiber optic sensing, strain is obtained by measuring the Rayleigh backscatter along the length of the fiber using swept-wavelength coherent interferometry. Rayleigh backscatter is caused by the reflection of light due to heterogeneities that are naturally present in the OF [228-230]. Mechanical loads can induce shifts in the Rayleigh backscatter spectrum, which can be expressed as [229]

\[
\frac{\Delta \lambda}{\lambda} = K_T \cdot \Delta T + K_\varepsilon \cdot \varepsilon
\]  

(7.1)

where \( \Delta \lambda \) is the wavelength shift, \( \lambda \) is the wavelength, \( \Delta T \) is the temperature change, and \( \varepsilon \) is the mechanical strain along the fiber length. \( K_T \) is the thermal coefficient (~0.634) that relates the
thermal expansion coefficient and thermo-optic coefficient of the OF, and $K_e$ is the strain coefficient (~6.67) based on the material properties of the fiber optic sensor [229].

7.3 Materials and Fabrication

The SCB specimens were fabricated from an infused epoxy/carbon fiber laminate of [((0/90/90/0)2/OF/0/90/90/0/2] configuration. The layup configuration was selected to minimize the variation of the SERR across the delamination front [183, 206, 210]. Additionally, this configuration allows for a relatively good comparison to the visually-obtained edge measurements and to assist in establishing the efficacy of this approach. The dry sandwich composite preforms were stitched using a 2000H Juki industrial sewing machine using a modified lock stitch. An 800 Denier Vectran™ thread was selected based on previous studies [14, 55] and material availability. A 10 mm distance was maintained between the initial crack length and the first row of stitches. The 2.25 mm diameter needle was used to stitch the dry preforms and was selected based on robotic stitching processes [14]. The SCB specimens were stitched with a stitch density of 0.0058 stitches/mm². The epoxy matrix is an out-of-autoclave API-1078 VARTM resin system. The resin was infused into a dry carbon biaxial [0/90] noncrimped fabric (NCF) by SAERTEX, Inc. A Teflon™ film of 0.0127 mm thickness was used as the crack initiator at the midplane of the laminate. The initial starting crack length was approximately 50.8 mm from the applied load. The laminate was cured using the cure cycle shown in Figure 7.1. Prior to infusion, the temperature of the oven, resin, and carbon fiber were increased to 88 °C to reduce the viscosity of the resin. The temperature was then increased at a rate of 1.8 °C/min to a temperature of 149 °C and held for six hours. The temperature was then raised to 177 °C for a two-hour soak and then reduced to ambient temperature (~24 °C).
Prior to embedding the OF sensors in the NCF preform, polytetrafluoroethylene (PTFE) tubes were temporarily woven through the local preform stitching of the NCF fabric. Afterward, the samples were stitched, and OFs were passed through the PTFE tubes. Once the OF was interlaced into the NCF fabric, the PTFE tubes were removed. Each OF pass was placed within a one-degree tolerance relative to the carbon fiber tow direction. Additional PTFE tubing was used at the ingress of the laminate to prevent OF breakage during handling of the cured part. Silicone padding was used to support the PTFE tube at the ingress of the laminate. The OFs outside of the laminate were encased in a separate bag to prevent resin buildup on the sensors. The layout of an OF sensor within the SCB specimen is shown in Figure 7.2. Two equally spaced fiber passes were used to characterize the delamination front.

![Figure 7.1](image.png)

**Figure 7.1** Cure cycle used to manufacture the composite sandwich laminates.
7.4 Experimental Procedure

Single cantilever beam specimens were clevis-mounted in an Instron model 8872 hydraulic test frame with a 1 kN load cell at a displacement rate of 0.5 mm/min. Five replicate tests were performed. One side of each test article was marked in 12.7 mm intervals to obtain visual edge measurements using an ARAMIS system. Internal strains were measured using the Luna Technologies ODiSI-B fiber-optic system. The OF used in this study is a polyimide-coated, low bend loss optical glass fiber (GEOSIL®-SM) with an operating wavelength of 1550 nm. The SCB test setup is shown in Figure 7.3. Due to the limitations of the testing apparatus, tests were terminated when the crack-tip opening displacement reached approximately 50.8 mm.
7.5 Computational Procedure

Implicit FEA of stitched SCB specimens was performed using ABAQUS commercial software. The material and fracture properties used in the FEMs are summarized in Table 7.1. The unstitched and stitched fracture energies were determined from single cantilever beam testing and were obtained from a previous study [162]. The 3D FEM used to simulate separation between the facesheet and core is shown in Figure 7.4. Incompatible hexagonal elements were used to improve the deformation gradients within the domain of the element when the test article is subjected to bending. Symmetry boundary conditions were imposed to decrease computational time. A time increment, damage stabilization parameter, and a dissipated energy fraction of $10^{-30}$ seconds, $10^{-4}$, and 0.004, respectively, were used to achieve convergence. To improve computational efficiency, the unstable crack growth between the facesheet and core at initiation
was not simulated. A bilinear traction-separation law representing the facesheet-to-core interface was used, and fracture energy of 220 J/m$^2$ with maximum traction stress of 2.75 MPa was applied. A trilinear traction-separation law was used to represent the failure process of the through-thickness reinforcement and is shown in Figure 7.5. A cohesive stiffness of 2E11 Pa/m for the through-thickness reinforcements was assumed and based on a sensitivity analysis obtained from Chapter 5. The maximum elastic stress was iteratively determined by comparing it to load-displacement measurements obtained from the SCB tests. The bridging stress and effective displacement after stitch failure were assumed to be 0.1 of the maximum elastic stress and 1 mm, respectively, based on interlaminar tensile tests obtained from [55, 60, 65].

<table>
<thead>
<tr>
<th>Facesheet [6, 180, 182]</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{11} = E_{22}$ (Pa)</td>
<td>5.66E+10</td>
</tr>
<tr>
<td>$E_{33}$ (Pa)</td>
<td>8.64E+09</td>
</tr>
<tr>
<td>$G_{12} = G_{13}$ (Pa)</td>
<td>4.66E+09</td>
</tr>
<tr>
<td>$G_{23}$ (Pa)</td>
<td>4.95E+09</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td>0.0619</td>
</tr>
<tr>
<td>$\nu_{13} = \nu_{23}$</td>
<td>0.25</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Core [181]</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$ (Pa)</td>
<td>1.86E+08</td>
</tr>
<tr>
<td>$\nu$</td>
<td>3.30E-01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Facesheet to Core Fracture Properties [162, 184]</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_I$ (J/m$^2$; Facesheet to core)</td>
<td>220</td>
</tr>
<tr>
<td>$K_{nn}$ (Pa; Mode I Penalty Stiffness)</td>
<td>10E+13</td>
</tr>
<tr>
<td>$\sigma_{\text{max}}$ (Pa; Facesheet to core – 3D)</td>
<td>2.75E+6</td>
</tr>
</tbody>
</table>
7.6 Numerical Procedure to Determine Internal Crack Length

The profile of strain along the length of a double cantilever beam can be characterized by a “wave-like” strain distribution that propagates with the progression of delamination, as shown in Figure 7.6 [182]. At crack initiation, the flexure of the cantilevered arm exhibits a linear strain
variation along the specimen length (0.98 m < x < 1.02 m). Near the crack front, the strain distribution is perturbed by the stress intensity of the crack front (0.95 m < x < 0.98 m). The magnitude of the strain decreases to zero as the location of the measurement of strain decreases (x < 0.95 m). As the crack propagates to the nearest stitch row, an increase in the strain magnitude and a shift in the strain distribution is observed. As the initial stitch row fails, the magnitude of the strain decreases with a further shift in the strain distribution. Good agreement is observed between the OF strain measurements and predicted FEA strains.

Figure 7.6 Predicted and measured strain distributions at initiation, before stitch failure, and post stitch failure.

Identification of the delamination front is achieved by measuring the shift in the differential strain due to an increase in crack length [182]. The differential strain is evaluated as
\[
\frac{d\varepsilon}{dx} \approx \frac{\varepsilon_i - \varepsilon_{i-1}}{x_i - x_{i-1}}, \quad (i = 1, 2, ..., N)
\] (7.2)

where \(\varepsilon_i\) and \(\varepsilon_{i-1}\) correspond to the \(i^{th}\) and \((i-1)^{th}\) measurement of strain for a total of \(N\) samples.

Similarly, \(x_i\) and \(x_{i-1}\) correspond to the \(i^{th}\) and \((i-1)^{th}\) location along the OF length. The crack length is estimated by using a Lagrangian cross-correlation approach. In this Lagrangian scheme, the measured differential strain during delamination propagation are correlated to the differential strain at the onset of delamination, or max applied load. Cross-correlation \(\rho_{12}\) measures the similarity between two signals [233] and can be expressed as

\[
\rho_{12}(\Delta a) = \int_{-\infty}^{\infty} \frac{d\varepsilon_1(x)}{dx} \cdot \frac{d\varepsilon_2(x + \Delta a)}{dx} dx
\] (7.3)

where \(\frac{d\varepsilon_1(\Delta a)}{dx}\) and \(\frac{d\varepsilon_2(x+\Delta a)}{dx}\) corresponds to the differential strain when the maximum load occurs and for subsequent loadings, respectively, at a location \(x\) along the OF. The correlation value \(\rho_{12}\) becomes a maximum \((\rho_{max})\) when the differential strain \(\frac{d\varepsilon_1}{dx}\) at \(x\) has the same gradient in \(\frac{d\varepsilon_2}{dx}\) at \(x+\Delta a\). The distance \(\Delta a\) corresponds to the change in crack length when \(\rho_{12} = \rho_{max}\). The distance \(\Delta a_{max}\) corresponds to the largest distance to obtain a measure of the correlation between \(\frac{d\varepsilon_1}{dx}\) and \(\frac{d\varepsilon_2}{dx}\). Once the change in the crack length is determined, the total crack length can then be estimated as

\[
a = a_0 + \Delta a
\] (7.4)

where \(a_0\) is the initial crack length. The corresponding mode I fracture toughness \(G_I\) is estimated using the modified compliance calibration method using
\[ G_I = \frac{3}{2} \frac{P^2 C^{2/3}}{A_1 b h} \]

where \( P \) and \( C \) are the applied load and compliance, respectively. The compliance is determined from the ratio of the applied displacement \( (\delta) \) to the applied load \( (p) \). The specimen width and thickness are denoted as \( b \) and \( h \), respectively. The slope \( A_1 \) of the normalized crack length \( (a/h) \) is a linear function of \( C^{1/3} \).

### 7.7 Results and Discussion

A comparison of the predicted and experimental values of the load and crack growth for each test is shown in Figures 7.7 and 7.8, respectively. Unstable crack propagation is observed at crack initiation, which results in a sharp decrease in the reacted load and a corresponding increase in the crack length. The reacted load then increases linearly as the crack front approaches the initial stitch row. Stitch row failure occurs at each maximum load peak during crack propagation, where the magnitude of the maximum load decreases with an increase in crack length. This behavior is due to a greater distance between the location of the reacted load and crack front, which increases with each subsequent failure of a stitch row. Good agreement was obtained between the experimental measurements and predicted values using the discrete cohesive zone approach.
Figure 7.7  Predicted and measured load-displacement response.

Figure 7.8  Predicted and measured crack growth response.
The OF crack length estimates showed approximately 10% greater crack growth magnitude as compared to visually obtained measurements (Figure 7.8). This behavior is attributed to the variation of the fracture energy along the delamination front. The crack front is typically non-uniform and exhibits greater crack growth near the centerline of the specimen. This behavior is developed from an anticlastic curvature of the facesheets due to Poisson’s effect. However, the fracture energy can also fluctuate due to local differences in fiber-to-resin bond strength, fracture morphology, and porosity [204]. In particular, variation in the stitch failure strength may occur in stitches along the same stitch row due to misalignment and differences in stitch pretension. Therefore, greater crack lengths may be observed along the width of the SCB specimen due to premature failure of stitches along the same stitch row.

The fracture energy of the stitched sandwich composite specimens with respect to the crack length is shown in Figure 7.9. At crack initiation, a relatively low fracture energy value is observed (250 J/m²). This fracture energy depicts the required energy of separation between the facesheet and core without the presence of through-the-thickness reinforcements. As the crack front approaches the initial stitch row, the fracture energy is increased by approximately 330%. The maximum energy for each stitch row is relatively constant prior to the stitch row failure. Failure of the stitch row results in a significant reduction in the fracture energy and produces unstable crack growth between the facesheet and core. However, the nearby stitch rows arrests the crack growth after each subsequent failure of a stitch row.
7.8 Conclusions

In this study, the crack growth behavior of stitched sandwich composites is investigated by performing single cantilevered beam tests with embedded optical fibers. Unstable crack growth was primarily observed during each test. Strain distributions obtained from optical fibers were correlated to the crack propagation of delamination. Internal crack growth measurements were obtained using a Lagrangian cross-correlation approach. Internal crack growth was approximately 10% greater in magnitude as compared to visual measurements. Additionally, a finite element analysis of the SCB specimen showed good agreement with load and crack growth estimates obtained from the optical fiber measurements. The calculated fracture energy near the through-the-thickness reinforcement was approximately three times the unstitched fracture energy.
energy. These results show the excellent capability of stitching to arrest core-to-facing separation in sandwich composites.
CHAPTER VIII
CONCLUSIONS

The research presented in this dissertation includes an in-depth literature review on the fracture behavior of stitched composite materials, fracture characterization of stitched sandwich composites, optimization of stitch processing parameters, and computational simulation of the mode I fracture of stitched sandwich composites. Additionally, a unique experimental method is developed to determine an internal crack length using optical fibers embedded within composites. The primary objective of this study is to assess the influence of stitching on the mode I fracture behavior of stitched sandwich composites. Therefore, this research develops a stitch parameter design space to characterize the fracture energy using a face-centered central composite design approach. A response surface model (RSM) is developed to determine optimum stitch processing parameters to inform a finite element model to predict crack-growth arrestment.

Lightweight composite materials are essential in the aerospace industry to reduce emissions, decrease fuel cost, and increase aircraft range and payload. Furthermore, composites have highly tailorable in-plane mechanical properties, which can be designed by altering individual ply orientations. However, this results in relatively low interlaminar properties between plies of different orientation when compared to metallic alternatives. Through-the-thickness reinforcement, such as stitching, z-pinning, needling, tufting, and three-dimensional weaving, have been developed in recent decades to enhance composites' interlaminar properties.
Stitching is considered to be an efficient and cost-effective method to reinforce composites in the through-the-thickness direction. However, the in-plane and out-of-plane properties of stitched composite materials are highly dependent on stitch processing parameters. These processing parameters include linear thread density, stitch density, stitch pattern, stitch distribution, pretension, stitching style, twist, ply orientation, and the stitch material. In particular, the mode I and mode II fracture energies of stitched polymer matrix composites are dependent on key stitching parameters such as stitch density and linear thread density. The fracture energy is observed to increase by a factor up to 15 depending on the selection of stitch density and linear thread density. However, current test standards do not appropriately address how to effectively determine composite materials' modal fracture energies. This is primarily due to the large-scale bridging that occurs during test; therefore, the use of linear elastic fracture mechanics methods may not be accurate. Additionally, a high rotational constraint is created by the through-the-thickness reinforcement that leads to failure of the specimen away from the delaminating zone. Therefore, new test methods or modification of existing test standards have been developed to estimate the fracture energy of composites. The failure of the specimen due to the high rotational constraint is prevented by bonding doublers to the outermost surfaces prior to testing.

Based on a literature review summarizing over a hundred papers, much of the research on the out-of-plane and fracture behavior of stitched composites has been focused on polymer composite laminates. Fewer studies have investigated the impact of stitch parameters on high temperature or sandwich composites. Research indicates that the properties of high-temperature composites and sandwich composites can be improved without impacting in-plane properties by incorporating through-the-thickness reinforcements. Therefore, this research investigates the
influence of stitching processing parameters on the mode I fracture of stitched sandwich composites.

An initial study was performed to explore the use of the J-integral approach to better approximate the fracture energy in stitched sandwich composites that can develop large-scale bridging. Single cantilevered beam tests were performed to estimate nonlinear and linear elastic fracture energies based on the J-integral method and modified beam theory, respectively. This study indicates that the J-integral approach is a promising method to estimate the fracture energy of stitched sandwich composites.

The influence of stitch processing parameters, such as linear thread density and stitch density, were investigated using a face-centered central composite design approach. The mode I fracture energy was experimentally determined for each treatment combination of stitch processing parameters by performing single cantilevered beam tests. A response surface model was developed to predict the fracture energy of stitched sandwich composites. Unique fracture surface morphologies were observed that are dependent on stitch processing parameters. The results indicate that stitching is an excellent candidate to inhibit crack growth in sandwich composites. Furthermore, an optimum stitch density of 0.0093 stitches/mm² can be determined based on the maximum fracture energy observed during tests.

The stitched sandwich composites' fracture behavior subjected to mode I loading was computationally examined using a discrete cohesive zone modeling approach. A trilinear traction-separation law was used to represent the failure process of the through-the-thickness reinforcement during delamination. Two-dimensional finite element analysis revealed that the cohesive stitch stiffness and elastic maximum traction stress are the two primary parameters that influence the load-displacement response measured during SCB testing. The bridging and
effective displacement after stitch failure did not significantly impact the overall load-displacement response. The predicted load and crack length responses obtained from a three-dimensional finite element analysis were observed to have good agreement with experimental measurements.

During modal fracture testing, the crack front is not uniform across the width of the specimen. This is primarily due to the anticlastic curvature of the facesheets when subjected to bending and is attributed mainly to Poisson’s effect. The research was performed to determine internal crack lengths obtained from embedded optical fibers within double cantilevered beam and single cantilevered beam test articles. The internal strain distributions from the optical fibers were correlated to the propagation of delamination based on their unique “wave-like” strain distributions that proportionally shift with increasing crack lengths. A cross-correlation approach was developed to estimate the crack length using the shift in the differential strain gradient. Excellent agreement of crack growth was obtained from optical fibers and visual measurements. Measuring the fracture toughness across the specimen’s width revealed a concave curvature that is associated with the anticlastic behavior of laminates undergoing bending. A reasonable correlation between visual measurements was also achieved.

8.1 Significance and Contributions to the State of the Art

The major contributions of this research work are listed as follows:

1. Investigated over 140 papers on the mechanistic behavior of stitched composites
   a. Stitch density and linear thread density are the two primary papers that influence the out-of-plane performance of composite materials.
   b. Very little research has been performed on sandwich composites and high-temperature composites.
2. Developed methodology to use embedded optical fibers to estimate internal crack front within single and double cantilevered beam specimens.

3. Developed a statistical model to predict the influence of stitched parameters and their interactions on the out-of-plane behavior of stitched sandwich composites.

4. Developed a discrete cohesive zone modeling approach with unique traction-separation laws to predict crack growth in stitched sandwich composites.
   a. Performed a 2D sensitivity analysis on the trilinear traction-separation law and provided recommendations for determining the cohesive stiffness, maximum elastic stress, bridging stress, and effective displacement.
   b. Characterized the crack front curvature near the through-the-thickness reinforcement.

8.2 Recommendations for Future Work

A methodology for determining the cohesive law of through-the-thickness stitching subjected to mode I crack growth is established. A more generalized form of this cohesive law needs to be developed for mode II and mixed-mode conditions, which is not completely understood. The current standardized approaches to determine the mode II and mixed-mode fracture energies use a linear elastic fracture mechanics approach that assumes small-scale yielding. This is not the case for stitched composites and can be affected by large-scale bridging that occurs due to the through-the-thickness reinforcement. Furthermore, a high rotational constraint may be generated during modal tests, which can affect the localized plastic zone size assumption. The use of doublers will be necessary to prevent failure of the delaminated arms due to the high rotational constraint that may be present due to the stitching. Therefore, an
Experimental investigation is needed to determine the influence of doubler thickness on the modal fracture energy of stitched sandwich composites.
REFERENCES


APPENDIX A

VERIFICATION OF RESPONSE SURFACE MODEL
A statistical evaluation of the response surface model provided in Chapter 5 was determined to verify the normality and constant variance assumptions. For determining equivalence between data sets, the experimental data was compared to the predicted results. The normal probability plots of the RSMs’ residuals were determined to evaluate the normality assumption. The residual is determined by subtracting the experimental value from the predicted value. A residual is “studentized” when the residual is normalized by the square root of its variance. The studentized residuals, as a function of the predicted response and run order were calculated to test for the constant variance assumption.

![Graph showing Measured Response vs. predicted response](image)

Figure A.1  Measured Response vs. predicted response.
Figure A.2  Normal probability plot.

Error distribution is likely a normal distribution since the plot resembles a straight line.
Figure A.3  Studentized residual vs predicted response.
Samples are centered around zero indicate independency and are relatively constant.

Figure A.4  Studentized residual vs. run order.
APPENDIX B

MATLAB CODE TO DETERMINE AND EVALUATE RESPONSE SURFACE MODEL
In this appendix, the MATLAB code for determining the final form of the response surface model is provided. The MATLAB code is composed of two parts, part one and part two. The first part of this code establishes the estimated form of the response model. The second part is a function file called “SS_Statistics” that is used to determine the statistical quantities needed to determine the response surface model. This function file is used within part one of the matlab code.

8.3 Matlab Code: Part 1

```matlab
clc
clear all
format longG

%% Coded Point Data
G_norm = xlsread('DOE_import_Data_V1.xlsx', 'Sheet1', 'A:F');

%% Response
Y=log(G_norm(:,5));

%% Determination of Levels of the independent variables, beta
% b0 b1 b2 b3 b12 b13 b23 b11 b22 b33 b123
CM=[1; 1; 1; 1; 1; 1; 1; 1; 0; 1];
[SS_model,SS_E_Model,Beta,LNG_Pred,Residual,X,XpX,Xpy,P,N]=SS_Statistics(G_norm, Y, CM);
Model_Coeff=Beta

%% Determination of Sum of Squares Error
SS_E=SS_E_Model;

%% Residual DOF
DOF_r=N-P;

%% Determination of unbiased estimator:
Sigma_Hat=SS_E/(N-P);

%% Estimation of the Hat Matrix:
H=X*inv(XpX)*transpose(X);

%% Studentized Residual
```

168
for i=1:1:length(G_norm(:,1));
    Studentized_Residual(i,1)=Residual(i)/sqrt(Sigma_Hat*(1-H(i,i)));
end

%% Determining total mean
Total_mean=sum(Y)/length(Y);

%% Model Sum of Squares
SS_M=SS_model;
DOF_model=P-1;

%% Mean Square Model
MS_model=SS_M/DOF_model;

%% Determining Mean Square Residual
MS_R=SS_E/DOF_r;

%% Model F-value
F_model=MS_model/MS_R;

%% Model P-value
P_model=1-fcdf(F_model,DOF_model,DOF_r);

%% Determination of Partial Sum of Squares
% Coefficient matrix to determine which terms to remove to estimate the SS
% b0 b1 b2 b3 b12 b13 b23 b11 b22 b33 b123

% b1
Coeff_b1=[CM(1)*1; CM(2)*0; CM(3)*1; CM(4)*1; CM(5)*1; CM(6)*1; CM(7)*1; CM(8)*1; CM(9)*1; CM(10)*1; CM(11)*1];
[SS_b1a,SS_E_b1a]=SS_Statistics(G_norm, Y, Coeff_b1);
SS_b1=SS_M-SS_b1a;

% b2
Coeff_b2=[CM(1)*1; CM(2)*1; CM(3)*0; CM(4)*1; CM(5)*1; CM(6)*1; CM(7)*1; CM(8)*1; CM(9)*1; CM(10)*1; CM(11)*1];
[SS_b2a,SS_E_b2a]=SS_Statistics(G_norm, Y, Coeff_b2);
SS_b2=SS_M-SS_b2a;

% b3
Coeff_b3=[CM(1)*1; CM(2)*1; CM(3)*1; CM(4)*0; CM(5)*1; CM(6)*1; CM(7)*1; CM(8)*1; CM(9)*1; CM(10)*1; CM(11)*1];
[SS_b3a,SS_E_b3a]=SS_Statistics(G_norm, Y, Coeff_b3);
SS_b3=SS_M-SS_b3a;
%b12
Coeff_b12=[CM(1)*1; CM(2)*1; CM(3)*1; CM(4)*1; CM(5)*0; CM(6)*1; CM(7)*1; CM(8)*1;
CM(9)*1; CM(10)*1; CM(11)*1];
[SS_b12a,SS_E_b12a]=SS_Statistics(G_norm, Y, Coeff_b12);
SS_b12=SS_M-SS_b12a;

%b13
Coeff_b13=[CM(1)*1; CM(2)*1; CM(3)*1; CM(4)*1; CM(5)*1; CM(6)*0; CM(7)*1; CM(8)*1;
CM(9)*1; CM(10)*1; CM(11)*1];
[SS_b13a,SS_E_b13a]=SS_Statistics(G_norm, Y, Coeff_b13);
SS_b13=SS_M-SS_b13a;

%b23
Coeff_b23=[CM(1)*1; CM(2)*1; CM(3)*1; CM(4)*1; CM(5)*1; CM(6)*1; CM(7)*0; CM(8)*1;
CM(9)*1; CM(10)*1; CM(11)*1];
[SS_b23a,SS_E_b23a]=SS_Statistics(G_norm, Y, Coeff_b23);
SS_b23=SS_M-SS_b23a;

%b11
Coeff_b11=[CM(1)*1; CM(2)*1; CM(3)*1; CM(4)*1; CM(5)*1; CM(6)*1; CM(7)*1; CM(8)*0;
CM(9)*1; CM(10)*1; CM(11)*1];
[SS_b11a,SS_E_b11a]=SS_Statistics(G_norm, Y, Coeff_b11);
SS_b11=SS_M-SS_b11a;

%b22
Coeff_b22=[CM(1)*1; CM(2)*1; CM(3)*1; CM(4)*1; CM(5)*1; CM(6)*1; CM(7)*1; CM(8)*1;
CM(9)*0; CM(10)*1; CM(11)*1];
[SS_b22a,SS_E_b22a]=SS_Statistics(G_norm, Y, Coeff_b22);
SS_b22=SS_M-SS_b22a;

%b33
Coeff_b33=[CM(1)*1; CM(2)*1; CM(3)*1; CM(4)*1; CM(5)*1; CM(6)*1; CM(7)*1; CM(8)*1;
CM(9)*1; CM(10)*0; CM(11)*1];
[SS_b33a,SS_E_b33a]=SS_Statistics(G_norm, Y, Coeff_b33);
SS_b33=SS_M-SS_b33a;

%b123
Coeff_b123=[CM(1)*1; CM(2)*1; CM(3)*1; CM(4)*1; CM(5)*1; CM(6)*1; CM(7)*1;
CM(8)*1; CM(9)*1; CM(10)*1; CM(11)*0];
[SS_b123a,SS_E_b123a]=SS_Statistics(G_norm, Y, Coeff_b123);
SS_b123=SS_M-SS_b123a;

%Summary of DF
SS_partial=[SS_b1; SS_b2; SS_b3; SS_b12; SS_b13; SS_b23; SS_b11; SS_b22; SS_b33; SS_b123];
DOF_partial=ones(length(SS_partial),1);
MS_partial=SS_partial./DOF_partial;
F_partial=MS_partial./MS_R;
P_partial=1-fcdf(F_partial,DOF_partial,DOF_r);

% Partial_Matrix_Summary
PMS=cat(2,SS_partial,DOF_partial,MS_partial,F_partial,P_partial);

%% Model Matrix Summary
MMS=cat(2,SS_M,DOF_model,MS_model,F_model,P_model);

%% Total Model Summary
TMS=cat(1,MMS,PMS);
Source1={'Model';'x1';'x2';'x3';'x1*x2';'x1*x3';'x2*x3';'x1*x1';'x2*x2';'x3*x3';'x1*x2*x3'};
TMS_Table=table(Source1(1:11),TMS(:,1),TMS(:,2),TMS(:,3),TMS(:,4),round(TMS(:,5),3),'VariableNames',{'Source','SS','DOF','MS','F_calc','P_value'});

%% Residual Table
Residual_Table=table(SS_E,DOF_r,MS_R,'VariableNames',{'SS','DOF','MS'});

%% Determination of Lack of Fit

% Pure error Sum of Squares
NumGroupsReplicated=15;
NumofReplicates=3;
DOF_pe=NumGroupsReplicated*(NumofReplicates-1);

for j=1:NumofReplicates:length(G_norm)
    LNG_Exp(j,1)=Y(j,1);
    LNG_Exp(j+1,1)=Y(j+1,1);
    LNG_Exp(j+2,1)=Y(j+2,1);
    Group_Mean(j,1)=mean(LNG_Exp(j:j+2,1));
    SqRe_Est(j,1)=(LNG_Exp(j,1)-Group_Mean(j)).^2;
    SqRe_Est(j+1,1)=(LNG_Exp(j+1,1)-Group_Mean(j)).^2;
    SqRe_Est(j+2,1)=(LNG_Exp(j+2,1)-Group_Mean(j)).^2;
end

SS_PE=sum(SqRe_Est);
MS_PE=SS_PE/DOF_pe;

% Sum of Squares Lack of Fit
SS_LOF=SS_E-SS_PE;
DOF_LOF=DOF_r-DOF_pe;
%% Mean Square Lack of Fit
MS_LOF=SS_LOF/DOF_LOF;

%% F_calculated
F_MS_LOF=MS_LOF/MS_PE;

%% P-value Lack of Fit
P_LOF=1-fcdf(F_MS_LOF,DOF_LOF,DOF_pe);

%% Corrected Total Sum of Squares
SS_CT=SS_M+SS_E;
DOF_CT=DOF_model+DOF_r;

%% Lack of Fit Table
LOF_Table=table({'Lack of Fit'},SS_LOF,DOF_LOF,MS_LOF,F_MS_LOF,round(P_LOF,3),'VariableNames',{'Source','SS','DOF','MS','F_calc','P_value'})

%% Pure Error
%% Lack of Fit Table
PE_Table=table({'Pure Error'},SS_PE,DOF_pe,MS_PE,'VariableNames',{'Source','SS','DOF','MS'})

%% Coefficient of Multiple Determination
R2=1-SS_E/SS_CT;

%% Adjusted Coefficient of Multiple Determination
R2_Adj=1-((N-1)/(N-P))*(1-R2);

%% Precision R-Squared
for i=1:length(G_norm(:,1));
    PRESS_SUM(i,1)=Residual(i)/(1-H(i,i));
end
PRESS=sum(PRESS_SUM.^2);
R2_Pred = 1 - PRESS/SS_CT;

Statistics=table(R2,R2_Adj,PRESS,R2_Pred,'VariableNames',{'R2','R2_Adj','PRESS','R2_Precision'})

%% Diagonistic Plots
%% Studentized Residual vs Predicted Response
figure(1)
plot(LNG_Pred,Studentized_Residual,'Or')
axis([min(LNG_Pred) max(LNG_Pred) -4 4])
xlabel('Predicted Response, ln(G)')
ylabel('Studentized Residual, R')

% Studentized Residual vs Run Order
figure(2)
plot(G_norm(:,6),Studentized_Residual,'Or')
axis([0 length(G_norm(:,1)) -4 4])
xlabel('Run Order')
ylabel('Studentized Residual, R')

% Predicted vs Measured Response
figure(3)
plot(LNG_Pred, LNG_Exp,'Or')
xlabel('Predicted Response, ln(G)')
ylabel('Measured Response, ln(G)')

% Probability plot with respect to Studentized Residual
figure(4)
probplot(Studentized_Residual)
ylabel('Probability')
xlabel('Studentized Residual, R')
box on

8.4 Matlab Code: Part 2

function [SS_Model,SS_E,Beta,LNG_Pred,Residual,X,XpX,Xpy,P,N]=SS_Statistics(G_norm,
Y, Coeff)

%% Determination of Levels of the independent variables, beta
L=length(G_norm);
for i=1:1:L
    X1(:,1)=1;                         %b0
    X1(i,2)=G_norm(i,2);               %b1
    X1(i,3)=G_norm(i,3);               %b2
    X1(i,4)=G_norm(i,4);               %b3
    X1(i,5)=X1(i,2)*X1(i,3);           %b12
    X1(i,6)=X1(i,2)*X1(i,4);           %b13
    X1(i,7)=X1(i,3)*X1(i,4);           %b23
    X1(i,8)=X1(i,2)*X1(i,2);           %b11
    X1(i,9)=X1(i,3)*X1(i,3);           %b22
    X1(i,10)=X1(i,4)*X1(i,4);          %b33
    X1(i,11)=X1(i,2)*X1(i,3)*X1(i,4);  %b123
end
X_new=[];
Rank_new=[];
Rank1=1:length(X1);
for i=1:1:length(Coeff)
    if Coeff(i)==1
        X_old=X1(:,i);
        Rank_old=Rank1(i);
        X_new=cat(2,X_new,X_old);
        Rank_new=cat(1,Rank_new,Rank_old);
    end
end
Rank=Rank_new;
X=X_new;

%% Determination of Beta Coefficients
XpX=transpose(X)*X;
Xpy=transpose(X)*Y;
Beta_Old=(XpX^(-1))*Xpy;

%Reassigning beta coefficients within b vector
Beta=zeros(length(Coeff),1);
for i=1:1:length(Coeff)
    for j=1:1:length(Beta_Old)
        if Rank(j)==i
            Beta(i)=Beta_Old(j);
        end
    end
end

%% Determining total mean
Totalmean=sum(Y)/length(Y);

%% Determining Residual Sum of Squares
for i=1:1:length(G_norm(:,1))
    b=Beta;
    x1(i,1)=G_norm(i,2);
    x2(i,1)=G_norm(i,3);
    x3(i,1)=G_norm(i,4);
    LNG_Pred(i,1)= b(1)+...
                   b(2).*x1(i)+    b(3).*x2(i)+    b(4).*x3(i)+...
                   b(5).*x1(i).*x2(i)+ b(6).*x1(i).*x3(i)+ b(7).*x2(i).*x3(i)+...
                   b(8).*x1(i).*x1(i)+ b(9).*x2(i).*x2(i)+ b(10).*x3(i).*x3(i)+...
                   b(11).*x1(i).*x2(i).*x3(i);
    LNG_Est(i,1)=Y(i);  
    Residual(i,1)=LNG_Est(i)-LNG_Pred(i);
end
Sq_Residual(i,1)=(Residual(i)).^2;
Squares_Model(i,1)=(LNG_Pred(i,1)-Totalmean).^2;
end

SS_Model=sum(Squares_Model(:,1));
SS_E=sum(Sq_Residual);

%% Determination of P and N
P=length(Rank);
N=length(G_norm(:,1));
DOF_Model=P-1;

%% Model Sum of Squares
for i=1:length(Y)
    Squares_Model(i,1)=(LNG_Pred(i,1)-Totalmean).^2;
end
## 8.5 Data Input

Table B.1 Input data for matlab code.

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### 8.6 Data Output

![Table and Diagram](image)

Figure B.1 Sample output from Matlab Command Window
APPENDIX C

MATLAB CODE TO DETERMINE INTERNAL CRACK LENGTH USING LAGRANGIAN CROSS-CORRELATION METHOD
In this appendix, the MATLAB code for determining the internal crack length based on strain measurements is provided. The MATLAB code is composed of two parts, part one and part two. The first part of this code is the primary code and the second part is a function file called “x_correlation_v1” that is used to estimate the crack length at a particular time increment.

8.7 Matlab Code: Part 1

clc
clear all

% Format Data String
format longEng

% Gathering data

% %T2_800_13_OF_1
% Time=xlsread('OF_FE_Calcs_V2.xlsx','T2_800_13_OF_1','AB2:AB4382');
% Length1=xlsread('OF_FE_Calcs_V2.xlsx','T2_800_13_OF_1','AM1:BPT1');
% Data_Old=xlsread('OF_FE_Calcs_V2.xlsx','T2_800_13_OF_1','AM2:BPT4382');
% Load=xlsread('OF_FE_Calcs_V2.xlsx','T2_800_13_OF_1','AD2:AD4382');
% Disp=xlsread('OF_FE_Calcs_V2.xlsx','T2_800_13_OF_1','AF2:AF4382');

% %Fiber Locations (0.6, 1.033)
% %Spacing: 0.000653054
% %ICL:  0.0475488

% %T2_800_13_OF_2
% Time=xlsread('OF_FE_Calcs_V2.xlsx','T2_800_13_OF_2','AB2:AB5444');
% Length1=xlsread('OF_FE_Calcs_V2.xlsx','T2_800_13_OF_2','AM1:CST1');
% Data_Old=xlsread('OF_FE_Calcs_V2.xlsx','T2_800_13_OF_2','AM2:CST5444');
% Load=xlsread('OF_FE_Calcs_V2.xlsx','T2_800_13_OF_2','AD2:AD5444');
% Disp=xlsread('OF_FE_Calcs_V2.xlsx','T2_800_13_OF_2','AF2:AF5444');

% %Fiber Locations (0.6, 1.5)
% %Spacing: 0.000653054
% %ICL:  0.0475488

% %T2_800_13_OF_3
% Time=xlsread('OF_FE_Calcs_V2.xlsx','T2_800_13_OF_3','AB2:AB4223');
Length1=xlsread('OF_FE_Calcs_V2.xlsx','T2_800_13_OF_3','AM1:BRK1');
Data_Old=xlsread('OF_FE_Calcs_V2.xlsx','T2_800_13_OF_3','AM2:BRK4223');
Load=xlsread('OF_FE_Calcs_V2.xlsx','T2_800_13_OF_3','AD2:AD4223');
Disp=xlsread('OF_FE_Calcs_V2.xlsx','T2_800_13_OF_3','AF2:AF4223');

% %Fiber Locations (0.6, 1.07)
% Spacing: 0.000653054
% ICL: 0.0486283

% T2_800_13_OF_4
% Time=xlsread('OF_FE_Calcs_V2.xlsx','T2_800_13_OF_4','AB2:AB4175');
% Length1=xlsread('OF_FE_Calcs_V2.xlsx','T2_800_13_OF_4','AG1:BLH1').';
% Data_Old=xlsread('OF_FE_Calcs_V2.xlsx','T2_800_13_OF_4','AG2:BLH4175');
% Load=xlsread('OF_FE_Calcs_V2.xlsx','T2_800_13_OF_4','AD2:AD4175');
% Disp=xlsread('OF_FE_Calcs_V2.xlsx','T2_800_13_OF_4','AF2:AF4175');
% Fiber Locations (0.6, 0.94)
% Spacing: 0.000653054
% ICL: 0.0488315

for i = 1:length(Time)
    Data(i,:)=sgolayfilt(Data_Old(i,:),2,51);
end

% File Location
filename='OF_FE_Calcs_V2.xlsx';
sheetname='T2_800_13_OF_3';
column='AG2:AG4223';
save_file=0;

% Initial Parameters
FP_B=0.6;  % Fiber Pass Begin
FP_E=1.07; % Fiber Pass End
P=0.95;    % Probability
S=0.000653054; % Average Spacing between data points along optical fiber
a= 0.0486283; % Initial Crack Length

% Fiber Pass Beginning
for i = 1:length(Length1)
    L1=Length1(i).';
    if FP_B<=L1;

180
i_fpb=i;
    Length1(i_fpb);
    break
end
end

%Fiber Pass End
for i = 1:length(Length1);
    L1=Length1(i).';
    if FP_E<=L1;
        i_fpe=i;
        Length1(i_fpe);
        break
    else i_fp1e=length(Length1);
    end
end

%Determining Max Load Location
[LoadPeaks,LoadPeaks_Index]=findpeaks(Load,'MinPeakProminence',20);
L_max=LoadPeaks(1);
I_lmax=LoadPeaks_Index(1)-1;
% [L_max,I_lmax]=max(Load);

%Determination of slope per length at Pmax
for i = i_fpb:1:i_fpe
    Slope_Init(i,1)=((Data(I_lmax,i)-Data(I_lmax,i-1))/(Length1(i)-Length1(i-1)));
end

%Crack Location starting at Pmax index
[NRow,NCol]=size(Data);
for i = 1:I_lmax-1
    CL(i,1)=a;
end

figure(2)
% plot(CL,Time,'k-',[CL_New,Time,'r-','LineWidth',2)
hold on
plot(Disp,CL)
axis([0 0.035 0 0.20])
grid on
% ylabel('Time (hr)')
% xlabel('Crack Location (m)')

if save_file == 1
    xlsxwrite(filename,CL,sheetname,column);
end

8.8 Matlab Code: Part 2

function [Crack_Length,a_beta,b_beta]=x_correlation_v1(Length1,Strain_T, Slope_Init,i_fpb,i_fpe,P,S)
    for i = i_fpb:1:i_fpe
        Slope(i,1)=(Strain_T(i)-Strain_T(i-1))/(Length1(i)-Length1(i-1));
    end

% Cross-Correlation
[r,lags]=xcorr(Slope(i_fpb:1:i_fpe),Slope_Init(i_fpb:1:i_fpe));

% normalize the lag
lag_normal=transpose(abs(lags)/max(abs(lags)));

% take the absolute of the r value
r_beta=abs(r)/max(abs(r));
r_beta(r_beta==0)=[];

% Fit the distribution to the data
fit=betafit(r_beta);
a_beta=fit(1);
b_beta=fit(2);

% Determining the inverse of the cumulative distribution function
x=icdf('Beta',P,a_beta,b_beta);

% Estimating lag
lag_1=[];
for i=1:1:length(r_beta)
    if r_beta(i)>x;
        lag_2=abs(lags(i));
        lag_1=cat(1,lag_1,lag_2);
    end
end
lag_mean=mean(lag_1);

%Estimated Crack Length
Crack_Length=lag_mean*S;