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Optimal control of adaptive wild hogs

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Wild hogs (*sus scrofa*) have caused major damage to agricultural crops in the US due to their lack of natural predators and fast reproduction rates. Wild hogs change their behavior to evade capture. Thus, control methods are thwarted and may not result in sufficient mortality to keep pace with the reproduction of wild hogs. This study extends previous invasive species literature to include increasing costs due to adaptability in two settings: the presence of hogs is deterministic or stochastic. The analysis is limited to one farmer's objective function with varying degrees of adaptability for "smartness". The findings concluded the population and harvest of wild hogs does change when there is a higher level of adaptability to control methods or, "smartness". The net benefit of the farmer decreases as adaptability and the probability of hogs' present increase for deterministic and stochastic case, respectively.

Key words: wild hogs, intelligent animal control, adaptability

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CHAPTER I

INTRODUCTION

Wild hogs (*sus scrofa*) are considered to be a nuisance pest primarily to agriculture and the environment in the US (McClure 2015). With few natural predators, and high reproductive rate, their numbers have quickly increased in agricultural regions, like the Mississippi Delta, and other ecoregions around the U.S. Population estimates range from 2.5 million hogs in 1982 to roughly 6.5 million in 2012 and it is estimated to be around 7 million in 2016 (Lewis et al. 2019). A study conducted in 2007 estimated the total cost of damages from wild hogs in the United States to be 1.5 billion dollars annually. More than half of these damages were to agriculture crops and livestock (Pimental 2007). Figure 1.1 below depicts how drastically the hog population has increased from 1982 to 2012.

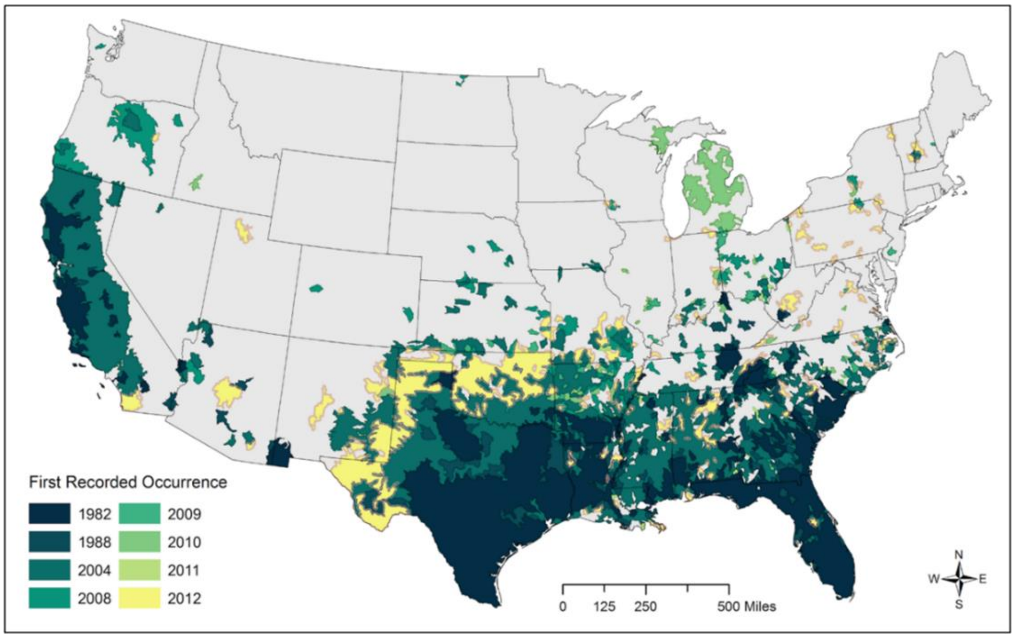


Figure 1.1 Distribution of Wild Hogs in the United States: 1982-2012

McClure et al 2015 p. 3

As wild hog populations and damages increase in the U.S., there has been an increase in efforts to identify and execute effective measures of controlling population growth, and by extension reducing damages. One of these increased efforts includes government action including providing money to help with hog depopulation. The government contributed over 2.8 million dollars in control efforts in Mississippi alone since 2015 (USDA-APHIS 2019). For example, arial shooting in Mississippi has contributed to 59 percent of its total wild hog depopulation efforts, where ground shooting contributes to 19 percent and 22 percent by trapping (USDA-APHIS 2019). Despite these efforts, the hog population continues to increase throughout the U.S. and in particular Mississippi.

One problem is that wild hogs adapt to human interference. Wild hogs are intelligent and have an adaptability which can aid them when being hunted or trapped (Baldwin 1969, Resser 2005, Massei et al. 2011). For example, they could remember not to go to a place where they have experienced danger. Young piglets learn from their mothers where to find food, shelter, and water. It is assumed that as these hogs are growing and adapting from the previous generation. This makes them extremely difficult to hunt and far more difficult to trap.

There are gaps in the literature which include the way hogs adapt to negative stimuli such as a control method and for how long they can retain the memory of the negative stimuli. This research is intended to provide evidence as to a contributing factor that likely mitigates control efforts, primarily that feral hogs adapt to control measures by learning to evade capture and ultimately keeping an increasing population growth.

1.1 Background

A case study in Louisiana demonstrates how detrimental hogs can be to farmland. During the period from winter 2014 to spring 2015, a mail questionnaire was sent to over 4000 commodity farmers in Louisiana based on tax records for 2013 (Tanger et al. 2015).

The study was to help determine feral hog damage to farming operations in the state, examining both commodity losses as well as other damages (Tanger et al. 2015). Total production losses were calculated at approximately $4.1 million dollars from the study respondents. Wild hogs can cause upsetting economic damages to farms, with damages ranging up to 53 million dollars in statewide damages (Tanger et al. 2015). Figure 1.2 depicts an example of the percentage of farms in a given area that have had monetary damages from wild hogs.

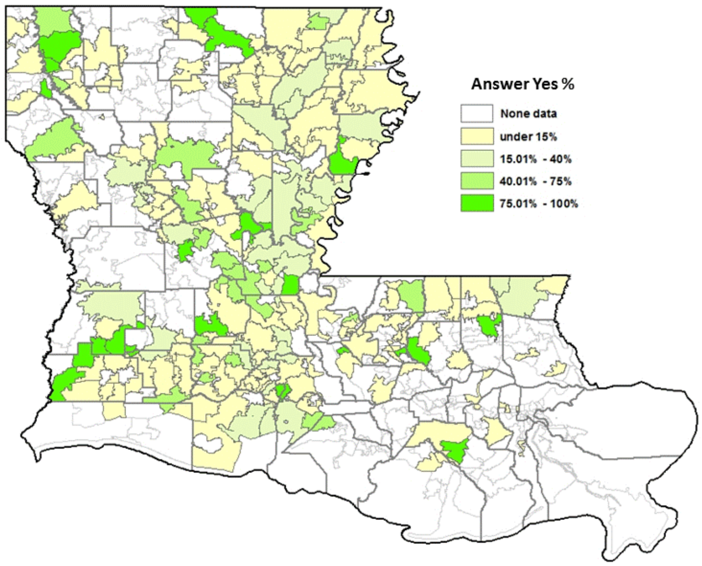


Figure 1.2 Percentage of Farms that have had monetary damages in Louisiana (Tanger et al. 2015)

Texas has the largest wild hog population in the US. Though Texas has access to more total funding for population control measures than Mississippi, their wild hog populations continue to rise. The methods of removal included various forms of hunting and trapping. From survey results of 679 landowners in Texas, 35,000 hogs were harvested, 58% by trapping (trap and destroy, trap and sell, trap and use, snares) and 42% by hunting (owner shooting, government aerial shooting, dogs) (Timmons et al. 2012). Despite these efforts, the state reported in 2012 an annual average population growth of 21 percent with the highest year of 25 percent. (Timmons et al. 2012)

Estimates provided by the Texas Department of Agriculture indicate that roughly 29% of the wild hog population was harvested in 2010; however, over 66% of the current population would have to be harvested for at least 5 years to maintain the current population and more so to decrease the population (Timmons et al. 2012).

A 2009 Alabama survey estimates that feral swine caused $75 million in damage to agricultural crops (Shi et al. 2010). In 2011, Georgia estimated feral swine caused $81 million in agricultural and property damage (Mengak 2012). Over 60 million dollars a year of wild hog damage was estimated for Mississippi crops and 1.5 billion in the US. One report from Mississippi in 2014 calculated the cost of controlling to an average cost of $230 for hunting per hog and $160 for trapping per hog (Anderson 2016).

1.2 Impacts of the Wild Hog

Wild hogs are opportunistic omnivorous (Graves 1984). They tend to migrate towards areas with plentiful crops to supplement their dietary requirements throughout the year. Crops have different growing seasons and thus targeted crops vary throughout the year. Wild hogs also damage crop and other prairie lands by rooting (McClure 2015). This is the behavior when hogs use their snout and push up the ground to explore for edible roots and other subterranean food items (Graves 1984). Rooting can also create large holes in the field that damage equipment and are costly to repair. Finally, wild hogs have been known to not only prey on small rodents, but livestock as well such as sheep and small calves (Graves 1984).

Wild hogs may also be disruptive to virgin biosystems (Graves 1984). Through a behavior called rubbing, wild hogs can cause damage to other forms of agriculture other than crops including fruit and nut trees. Trees are damaged when the animal rubs its body against a tree in order to rid itself of parasites and may result in killing the tree (Graves 1984). When hogs get too hot, they cannot thermoregulate by sweating so they will commonly cool down by getting in water. When there is no body of water present, they make their own by wallowing. This is when they create a big hole near a water supply and roll in it to coat their hair in mud and water. This can cause large stagnant puddles of water that can be a prime location for bacteria and parasites to grow (Graves 1984).

Human health is also affected as wild hogs may carry diseases that can be spread to humans, such as foot and mouth disease and influenza. Because hogs carry bacteria in their hair, they can also contaminate drinking water supplies from wallowing and certain crops can become contaminated if irrigated with the contaminated water (McClure 2015).

Another problem posed from the current eradication process is economical. An externality is a consequence reflected onto a nonparticipating party given an action from the participating party. For example, if two farmers are next to each other geographically and one farmer is actively controlling hogs on their land, the hogs will learn that moving there means harm and instead moves toward the neighboring farm, causing more damage to them. This is an externality to the nonparticipating party as they are doing nothing different but now have more damage on their farm due to the neighboring farmer exerting control measures. This is believed to influence whether hogs will be controlled at a population level or not.

1.3 Introduction to Model

The natural renewable resource theorem will be extended to work on the control of wild hogs to include their adaptability. This research aims to add a behavioral learning aspect, or adaptability, to the base model equation to make it more precise. A general model for invasive species can be found in numerous papers. Specific to this research, Zivin et al. (2000) developed a time discrete dynamic optimization model based on the optimization of profits from farmers given harvest function and a growth function of wild hogs. Other models do not account for adaptability which is key in this research to discovering the optimal control of this smart species.

To improve these models, behavior will be included and then compared to results found in previous models. In hopes that this method will provide a more effective way to control the population, some policy implications may be needed to see real change in the Mississippi area and in the future, the US.

Using the equations and modeling framework from Zivin et al. (2000), the models used will be replicated and updated by comparing the baseline scenario with the multiple scenarios created. In these scenarios, one field with one Farmer who is controlling or not controlling the hogs will be the subject. The different scenarios include control in the field with varying smartness levels and probability presence parameters. This will help discern the difference smartness levels make in general and the difference between probabilities of hogs present. Because there is only one farmer, externalities or possible social planners' problems will not be taken into account. This new model and cost function will also help explain why the hog population continues to grow despite efforts being made to reduce it.

CHAPTER II

LITERATURE REVIEW

There is an extensive amount of research regarding wild hog damage to agricultural land, the ecosystem, and to humans. However, this research has been conducted regarding the behavioral and learning ability of wild hogs and how it contributes to their increasing population.

Research has shown that wild hogs are a thinking animal. Baldwin (1969) stated that "Hogs are cooperative animals and learn classical and operant conditioning tasks rapidly." It is important to include these experiments because it only furthers the idea that hogs have a learning or memory forming ability that must be included in the model to appropriately calculate the best management style for controlling the population. Social learning is defined as when a hog acquires information from either the behavior of other individuals or the products of their behavior (Galef and Laland 2005). This can be anything from foraging, to travel routes, to how an animal responds to predators, etc. Hogs entertain all of these, and it is suspected that they hold on to this social learning for a period of time giving them a learning memory.

2.1 Wild Hog Social Behavior and Intelligence Experiments

Hogs are very knowledgeable about their surroundings. Many tests and experiments have been studied on the ability and range of a wild hogs' memory including foraging behavior. Foraging behavior includes not only finding food but remembering where that food was placed so the hog may later go back to the spot if needed. (Held et al. 2005). There is a social hierarchy for wild hogs. Generally, withing a herd or family unit the larger, heavier animals are dominant over smaller ones. These larger, dominant animals tend to be the foragers for the group. Food intake is highly suggestive to younger mammals by older mammals. Sometimes young mammals will not even go near food unless one of their elders has either eaten it first or shown the food to be edible. (Galef and Whisken 1998). These social hierarchies can be disrupted when it comes to food because hogs are smart and will always find a way to forage, even if it means reversing roles. For example, a wild hog that may seem superior in the herd because of weight or age will follow a lesser (meaning younger or smaller in size) hog if it has a greater ability to find food when in an unknown territory (Held et al. 2000). Held even went as far to say that hogs will exploit each other for food if one hogs knows more than the other.

An experiment was conducted to test these concepts in hogs (Held et al. 2000). Buckets were placed in a pen with only one having bait and a smaller female pig was allowed in the pen to search for the food. They were then led out of the pen and did the whole thing again an hour later. The larger, naïve, pig was only allowed in the pen once, so they would not know where the bucket of food was the second time. Then, both hogs were allowed in the pen. The smaller pig in the paired trials would remember where the food was located and visit the bucket with the food, showing the heavier pig which bucket was baited. The larger hogs were able to exploit the lesser hogs by following them to the baited buckets realizing they would find it faster and then moving them out of the way to the food so they could eat. This is generally only observed in more intelligent species. (Baldwin and Meese 1979). This shows that they not only have spatial memory but can also change their foraging tactics to better suit themselves (Held et al. 1999).

This can be potentially interpreted into remembering where traps and kills have taken place and not returning to the area. Because of this, the more a farmer or group of people hunt and trap wild hogs, the harder it gets. In biology, known as risk allocation hypothesis, if risk of harvesting is high, prey (in this case hogs) tend to forage less frequently and are more cautious of their surroundings making it harder to trap or hunt (Ferrari et al. 2009).

Other tests have been conducted that show they have an actual preference for food (Held et al. 2005). In this study, it showed that whether wild hogs have a preference over food, and they do not recognize all food as the same. Female hogs were place in a pen with eight buckets two of which contained food. One had a smaller amount of food with a brick to make it harder and less available than the bucket with the larger amount of food. Two experiments were conducted, one where the hogs could choose both buckets and one where they could only choose one. Hogs showed that they preferred the bucket with the larger amount of food when restricted to one bucket. It proves that hogs have a preference for food type and can remember where the food is located (Held 2005). Through inference we can expect them to keep returning to sites that have either higher quality food or a greater quantity of food which could be used to bait and trap hogs for a more effective control of the population. A trapper or famer could use this to exploit the wild hogs but with their adaptive behavior could prove difficult after the first time.

An experiment conducted by Reimerta et al. (2017) concluded that hogs emotional state is not limited to the time and place of a negative or positive stimuli. A batch of hogs were separated in a treatment group and a nontreatment group or “naïve” as called in the experiment. The treatment groups were subjected to either a positive or negative stimuli and returned to the “naïve” group to observe how the “naïve hogs reacted to the treated hogs. The hogs were observed on test day 2 and test day 18 to see how they reacted. The results concluded that hogs treated with a negative stimulus return to the pen of “naïve” hogs with less energy (walking and moving less frequently) but also with less curled tails indicating more stress. The hogs treated with the positive stimuli returned to the pen with more social energy including nosing other hogs more often and walking more frequently. The “naïve” hogs showed the same tendencies when the treated hogs returned to the pen. This meant that the hogs emotional state affected the untreated hogs as well. If a hog were to experience a negative stimulus, it will continue to feel the stress of that stimuli even after the event has happened and can cause hogs that did not experience the stimuli to feel the negative emotional state as well. This could be very important when studying how trapping and hunting affect not only the hogs that are trapped and hunted but those that escape and return to a heard.

Overall, these tests show that hogs are smart species that can completely change their behavior to maximize food intake and minimize risk, even so much as to go from foraging during the day to foraging at night if needed (Jensen 2003). They can also keep an emotional state of stress after a negative stimulus and affect the mood of other hogs around them. As hunters and trappers are working to control the population, all of these need to be considered to have the most efficient outcome being the least number of hogs in an area as possible.

2.2 Population Control Models

Different methods have been used to control the population. This includes hunting, trapping, or a combination of the two. There have been multiple studies showing that for invasive species, trapping is the more efficient method due to the ability to catch more hogs at once. Trapping is also a preferred method as it is more “humane” and does not affect other species as much as other methods and traps are generally checked often. Trapping is more expensive however, since you must have a trap, bait, and have someone to check the traps and then remove hogs if applicable (Massei et al. 2011). Depending on the area and slope of terrain, trapping may be hard to use especially if there is a high density of underbrush in forests. They may also lead to translocation of hogs when they are caught, but this is generally seen in European countries and very seldom seen in the US. Trapping has seen both positive and negative effectiveness due to these limits. In Pinnacles National Monument, California there was a seventy percent decrease in the hog population in the first 3 months of the eradication process (McCann and Garcelon 2008), but for Hawaii there was little to no effect of trapping due to lack of trapping skill landscape, and trap weary hogs as they could jump over fences hide in dense cover, and actively evade control efforts (Massei et al. 2011, Reeser and Harry 2005). As mentioned earlier, hogs are smart and will remember where things are located.

As for hunting, it can be beneficial in areas with large amounts of hogs. Hunters may decide to hunt alone or in groups and can even bring along hunting dogs for help (Massei et al. 2011). This requires a lot of time, manpower, and supplies as guns and ammunition are needed. In studies by Baker et al. (2017) and Hodgson et al. (2014), ariel shooting/baiting was used for the early stages of depopulation of invasive species while the population was severely high. These papers noted that this cannot be used for the entire project because of the high overhead costs. Hunting also give controversial answers when it comes to effectiveness. It has actual shown to increase populations of wild hogs in France, Germany, and the US (Choquenot et al. 1996). As for hunting dogs, they have negative effects on hunting boars both because they can hunt other animals and kill wildlife disrupting ecosystems and they can also get killed themselves by the boar, rendering themselves useless (Cruz et al. 2005). Overall, hunting has many negative effects that outweigh the positive, but it is still a more commonplace than trapping.

2.3 Invasive Species Models

Numerous modeling studies have been conducted to determine how to efficiently control wild hogs' population. The baseline model in this research is provided by Zivin et al. (2000), but others have tried different approaches, in particular is Hone (1995). In this paper, the agricultural damages are assumed to be non-linear unlike the one in Zivin et al. (2000). This allows for a differing damage analysis and provides a more real-world example of how hogs are continuously moving and damaging different properties and crops differently.

In Holderieath (2017), an agent-based model was used with nonlinear programming to show the damages caused by hogs and how costly it would be to remove the hogs from the land. In this case, the agent is the landowner, and their objective is to maximize profits constrained with the cost of hogs present and not present. If hogs are not present, then some cost is added for the removal of the hogs whether it be by hunting or trapping. It also calculates the opportunity cost of keeping the hogs versus removing them. The overall conclusion is similar to the one found in Zivin et al. (2000) in that over time, the population and removal efforts converge after ten years (Holderieath 2017).

Other studies have used similar models, including one from Bute et al. (2003) which used a bioeconomic model to research the declining population of tigers and what carrying capacity could be reached with an endangered species given hunting. In this study, both profits from hunting the tiger and a growth function were used similar to both the model in this paper and the model form Zivin et al. (2000). The growth function is logistic, similar to the one in this model. It provided the carrying capacity along with a level of hunting that would still allow profits from hunting and allow the population to grow.

A similar study in Rondeau (2001) uses the Natural theorem of renewable resources including a growth constraint. This time it was used on wildlife nonmarket valuation. By using dynamic optimization, Rondeau was able to create an effective management plan to reincorporate species back to the wild without destroying the habitats. For this, he used his models to calculate the carrying capacity that would maximize net benefits and find the optimal level of pest control when adding in the new species.

2.4 Dynamic Optimization Model Development

Multiple models include a stochastic probability parameter for the probability of the spread of the invasive species (Higgins et al. 2000). In Hauser et al. (2009), a stochastic parameter was used to predict the probability of an invader. A search effort level is connected to this parameter similar to the one used in the model for this paper. As more effort is expended, the likelihood to find or in this case hunt or trap increases. In Hauser et al. (2009), the focus is on minimizing management costs and expected surveillance costs of the invasive species Orange Hawkweed (*Pilosella aurantiaca*) unlike the model in this paper which focuses on maximizing net profit for a farmer who experiences the invasive wild hog. Hauser et al. also used a linear cost function as the cost for undetected invader and a detected invader are summed together across different investment levels for the cost management.

In another model seen in Chalak et al. (2017), the state of invasion or presence of the invasive species is noted at 1 and if not invaded or there is no presence of the invasive species the state is equal to 0. This is similar to how the model is set up in section 3. Chalak et al. (2017) uses a matrix style for their model where this model only has on field that is used for the study. This model also uses eradication as a possible control method where the model in this study only focuses on one farmer so eradication would see impossible. There is a stochastic presence and a time dependent series from this model in the spread of the invasive species similar to the model shown in this paper. Unlike this paper, the model from Chalak et al. (2017) uses three different control methods such as border control, success of spread, and cell control where the model in this paper uses just hunting and trapping as control methods. This is similar to Hauser et al. (2009) where minimizing control costs was the main equation, and this paper's model uses maximization of the farmer's profit. These two are generally seen as opposite equations but are very similar in both their calculation density and their results. As a farmer tries to minimize costs, he is therefore working to maximize profits whether deliberate or not.

As seen in Olson (2006), the cost function depends on the quantity of the invasive species being controlled similar to the cost function in the model presented in this paper. This gives is an increasing cost function. This models' growth function is represented by a logistic growth function, as the one seen in Olson (2006). Similarly, Olson (2006) and Huffaker et al. (1992) develop a time dependent dynamic model of the invasive species. There is no stochastic parameter in the paper by Olson (2006).

The control method used for this research is based off the average trapping cost for farmers but can easily be calculates for hunting by change the value of fixed and variable costs. A different method was used in Blackwood et al. (2012) and Baker et al. (2017) where multiple control strategies were used on invasive species and how described how it affects the cost function. Hunting and trapping allow for different cost levels of control as well as different effort levels in order to compare the efficiency of methods. Baker et al. (2017) consider eradication of the invasive species using two different control methods. The cost function is comparable to this paper in that it includes a logistic growth with intrinsic growth rate and a carrying capacity. One control method used has a high overhead cost analogous to trapping and the second control method incurs a handling time similar to hunting. Both control methods were not used in the research for this paper because the cost function is general enough to calculate for both by changing the variable and fixed costs akin to those in Baker et al. (2017). The variable and fixed costs match those similar to trapping.

One method to be used for the stochastic element in the dynamic optimization is the brute force method. This method allows for multiple efforts of calculating the equation to find the maximizing point for the model. It offers an advantage of finding multiple outcomes quickly (Gonzalez et al. 2014). Using this method will allow for multiple optimizations problems to be run sequentially and give multiple outcomes that can be compared, and a maximizing level can be found. This provides the ability to find the value of the steady state achieved in this model but uses random numbers instead of a mathematical equation.

By combining many factors from other papers, this model will allow for the use of probability parameter and a constant effort coefficient that will provide an increasing hyperbolic cost function. This will set this paper apart from other invasive species models seen before. Along with the prospect of maximizing profit instead of minimizing costs, the use of this equation may provide insight for how to maximize the profits for a farmer while dealing with the invasive hog species.

CHAPTER III

MODEL

The main points of novelty in this research model includes and adaptability measure, “smartness”, in the cost function and presence probability parameter, which allows the hogs to have a probability of being on the field or not. Here, “smartness” or can be interpreted as the ability hogs have to adapt to their surrounding and in this case, it is their ability to evade capture more readily. As hogs are “smarter”, the technology for the control method used by the Farmer becomes less effective against their ability to adapt. The adaptability is not dynamic, does not change in the period, but it can have a varying level amongst hogs. For example, if the Farmer used trapping as his control method, the “smarter” hog will recognize the trap and avoid it even when baited. This does not mean that the smartness is “growing” but that the smarter hog has a higher effort level attached to it. To counteract the hogs’ adaptability, the Farmer will have to put more money into the bait and therefore have a higher effort level or possibly check the traps more often which would also cause a higher effort level. The hogs’ “smartness” has an effect on the cost of harvesting and the overall population of hogs present on the field.

The adaptability of wild hogs will be included in the cost, where cost is dependent on whether hogs are present on the field or not. The cost increases depending on the smartness level of the hogs and the probability they will be present on the field. When hogs have been exposed to a threat such as hunting or trapping, the percentage of hogs present on the farm will decrease and thus effort levels will increase. As hogs adapt to evade capture while remaining on the field, they will become more difficult to catch and will cause the rise in effort, i.e., increase of costs led by increased harvesting effort.

The other element that is included is the probability parameter. This allows the model to include a probability for the presence of hogs present on the field. There are two states of the field; hogs present, or hogs absent, determined by whether there is a control method in the previous period or not. When there is a control method in the previous period, the hogs will adapt to evade capture and become “smart”. Hogs are less likely to be present on the field in the next time period due to the control method. Similarly, if there is not a control method in the previous period, the hogs do not become smart, and have a higher probability of returning to the field. Adding these elements could help explain why wild hog population continues to increase despite control efforts. In the next section, the suggested conceptual framework is explained.

After, the deterministic model is described, including the empirical model setup and notation explanation. Included in this section is a descriptive outlay of the process created for the model, how smartness affects the net benefit for the Farmer and how population dynamics may change over different levels of smartness. The stochastic version of the model will be explained in the section after. It will provide an explanation of how the probability hogs will be on the field affects the maximum net benefit for the Farmer as smartness level is increased and allows for a comparison of both the net benefit and population dynamics between the stochastic and deterministic cases.

3.1 Dynamic Optimization Model with Adaptability

Including a smartness or adaptability behavior to the model will provide a way to see how the net benefit is affected by a cost function, which includes adaptability. There are different places that behavior could be added into the general formula but for simplicity were not. One difference includes putting a smart parameter into the growth function. Wild hogs may increase their production rate after they have experienced one of their members being harvested. As the hogs are harvested, resources become more readily available since there is a lower population. More resources can be a stimulant to reproduction. Population growth rate will decrease over time as they will reach their carrying capacity and die off naturally due to lower amounts of available space and food. This will result in a periodic growth function instead of the logistic growth function used in this model. The use of a success rate as a function of adaptability could also be used. For example, if the success rate is high, the adaptability level can be used in the likelihood hogs return to the field. If they return after a successful harvest, then the adaptability is low. Similarly, if the hogs do not return after a successful harvest, the adaptability is high.

In this paper, these options were not used as they are biological in nature and tend to require more data and a more complicated model. Instead, the smartness or adaptability is included in the cost function as effort level of the Farmer to harvest the hogs and enables the use of economic parameters. A probability parameter is used in the stochastic case to determine whether hogs are present or not on the field. Using the smartness and probability in this way allows for an economic model to be used.

Suppose there is one agricultural field (or farmer) with an average sized corn field in the Mississippi Delta, roughly 450 acres (USDA-NASS 2021). It is assumed that there is a natural habitat for wild hogs (e.g., conservation area) next to the agricultural field. The proximity of the natural habitat will allow for the probability of hogs to be present on the field. The probability hogs will be present on the field is dependent on whether the hogs were present on the field in the previous time period. Because this is a one Farmer model, there is no opportunity for the hogs to move to the neighboring field and thus the hogs are either present or not on this single

Farmer field. The parameter, p, will show the state-dependent probability the hogs will be on the field at time t depending upon the presence of hogs in the previous time period at $t-1$, i.e., a state transition probability. Figure 3.1 provides a representation for the probability of hogs to be present on the field.

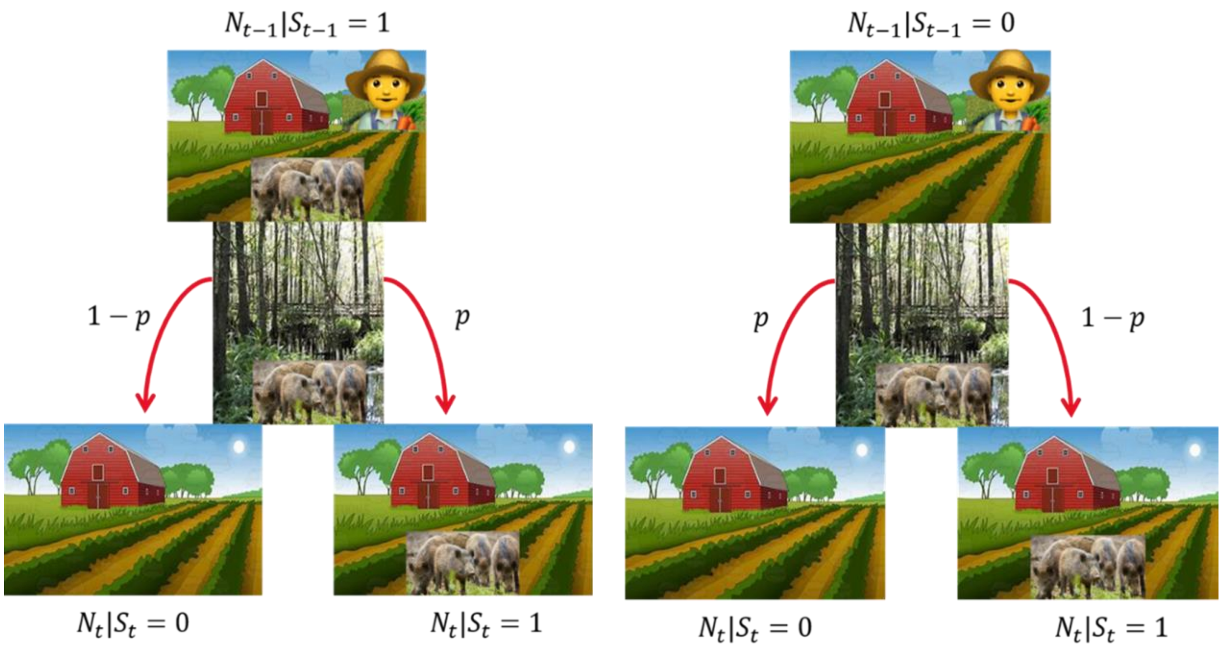


Figure 3.1 Graphical depiction of Farmer's field next to a natrual habitat of wild hogs with a visual of probability, p, for movement from the natural habitat to the field depending on whether hogs were present in the previous period, S_{t-1}.

To calculate the net benefit for the farmer, the cost of hogs being harvested must be subtracted from the revenue generated through the crops. Revenue will be subject to the damages caused by the hogs on the field. Thus, profit will include revenue, subject to the cost of damages from hogs being present on the field, and a cost function which includes an adaptability level of the hogs. It is assumed that the Farmer's revenue does not include any return from hunting the hogs and any control method is seen as a cost for this model. There are also no profits received from consuming the hogs once they have been harvested.

The Farmer's net benefit maximization problem can be generally defined as:

$$\max_{K_t} \sum_{t=0}^{\infty} \rho^t [\pi_t(N_t, K_t, \alpha, p | S_{t-1})]$$

$$s.t.\ N_{t+1} - N_t = g(N_t) - K_t \tag{3.1}$$

where ρ is the discount factor. Farmer A has a net profit, denoted by π, that depends on the population density of hogs (i.e., hogs/acre), denoted as N_t. $g(N_t)$ represents the growth function, where K_t is subtracted as it represents the number of hogs harvested. The effort coefficient α depicts the effort level to harvest the hogs. As hogs become smarter, they can adapt to the control methods used by the Farmer and become costlier to harvest. The effort coefficient will be able to calculate how the smartness of hogs affects the harvest and population. S_t is the presence state of the hogs on the field. S_t follows Markov transition probabilities with a probability p and can be futher explained using Table 3.1 below.

Table 3.1 Transition Probabilities

		Present at t $S_t = 1$	Absent at t $S_t = 0$
Present at $t-1$	$S_{t-1} = 1$	p	$1-p$
Absent at $t-1$	$S_{t-1} = 0$	$1-p$	p

Markov transition probabilities allows for the outcome of the model to depend on the outcome of the previous period denoted above as S_{t-1} (Kemey 1976). The properties of a transition makeup include squareness, as all states must be included as both rows and columns, all numbers, here it is p and $1-p$, in the matrix are between 0 and 1 as they represent probabilities, and all rows should add to equal 1 as the numbers represent the change from the state on the left to the state on the right (Kemey 1976).

To start, a simple conceptual model represents the optimal suppression of wild hogs in the agricultural landscape. Let S_t represent the state of hogs' presence in agricultural field, i.e., $S_t = 1$ for the presence of wild hogs at time t while $S_t = 0$ for the absence of them. It is assumed that the Farmer has perfect information regarding the presence of wild hogs on the field. Thus, the farmer will not engage in harvesting hogs when they are not present in any given period, with the period being the season of crop harvest happening once a year.

Because there is just one Farmer in this model, the possibility of an externality or a social planner's problem is not taken into account. This model can eventually be extended to include these but to keep it simple they are not considered in this model.

For further explanation of how the model works conceptually, if the Farmer has hogs present on the field, the hog can cause damage to the crops. The Farmer will choose some form of harvesting to rid the field of hogs and will incur some costs from it. The effectiveness of the harvest will depend on which form of hog harvesting the farmer chooses (trapping or hunting). If in the previous period a harvesting technique was used, the hogs will adapt and become smarter and may be less likely to return in the next period. This can be shown through the transition matrix as the probability hogs are present in the period after a harvesting of hogs was conducted is lower than the probability hogs would be absent after a period of harvesting. The opposite is true for when hogs are not smart. If hogs are not smart, they may not remember anything from the previous period and will be equally probable to return to the field despite any control method used. This model shows the maximum net benefit over time a farmer can achieve depending on hog harvesting technique given there is a probability of the hogs being present on the farm and the smartness gained from a previous harvest.

Equation (3.1) is the conceptual version of the model, and many functional forms can be made from it. The deterministic model will provide the functional form for the equation used in the next section. The stochastic model and its functional form will be covered in the section after.

It is important to note, adaptability, and smartness can be difficult to include explicitly without making the model very complicated. There are numerous ways to include either of these which would alter the functional form of the model. For example, smartness can be included as a success rate for each harvest, but it was not necessary for this simple extension of the model. For this paper, smartness is only included through an effort level in the cost function. Adaptability was added in as the presence probability in this model but could be added into the growth function. This may require more data and can be difficult to calculate since it is a biological factor instead of economical.

3.2 Deterministic model

The deterministic model will provide a specific functional for the net benefit maximization and outlines the cost and growth function. The benchmark model in Zivin et al. (2000) describes a net benefit maximization where profit is calculated through revenue minus cost subject to an unrestricted growth rate. Hunting and trapping have varying cost functions to incorporate different variable and fixed costs per method. Both cost functions include the number of hogs killed as a variable. This is an extension of Zivin et al. (2000) whose model includes the same revenue and growth function, but the cost function used in this model will allow for a smartness parameter. Because this is in deterministic form, the probability parameter will not be included and thus spatial awareness of the hogs to the farmer will not be calculated in the profit function. Therefore, the deterministic model can be described as:

$$\max_{K_t} \sum_{t=0}^{\infty} \rho^t \left[R(1 - dN_t) - \left(C_0 + \frac{\alpha C_1 K_t}{N_t} \right) \right]$$

$$s.t. N_{t+1} - N_t = r\left(1 - \frac{N_t}{X}\right) - K_t \tag{3.2}$$

$$\pi_t \geq 0 \text{ , } N_t \geq 0 \text{ , and } K_t \geq 0,$$

where ρ is the discount factor described in years, R is the agricultural revenue described in dollars/acre, d is less than 1 as it is the proportional parameter of hog damages described in a percentage of damage/acre. To keep revenue positive, d must be chosen at a sufficiently low level. N_t is the wild hogs' population in hogs/acre, K_t is the harvested wild hogs' population described in hogs/acre. K_t is subtracted from the growth function because the number of hogs harvested decreases the number of hogs in the population. C_0 and C_1 represent the fixed and variable cost of harvesting described in dollars/acre. These are subtracted from the revenue as cost decreases revenue available. Crop production costs are not included in this model and therefore are set to zero as they are sunk costs. If included, they could impact the results and interpretation of the Farmers' net present value. X is carrying capacity and r is the intrinsic growth rate. There are nonnegativity constraints to ensure that population and harvest do not fall below zero. Equation (3.2) is set to maximize the net benefit (of the individual farmer) over time.

The model in Equation (3.2) describes a general cost function of harvesting that could be either trapping or hunting. Fixed cost (C_0) and variable cost (C_1) can vary depending on the method of harvesting used. Hunting requires manpower and ammunition but not much overhead cost as many farmers tend already have the equipment and weapons on their property for other pests (Massei et al. 2011). This means it has high variable cost and low fixed costs, comparatively. For example, for hunting, a fixed cost would be the price of a gun or a stand and variable costs would include opportunity cost of hunting, ammunition, and manpower.

Trapping, on the other hand, involves higher fixed costs and lower variable costs. The purchase of a trap and bait are the two costly components of trapping, but often the traps do not need to be checked every day and thus manpower is much lower. Things such as success rate, opportunity cost, and time also affect the variable and fixed costs. Changing C_0 and C_1 will allow for the equation to work for both harvesting techniques if warranted. For this model, the fixed and variable costs were chosen to be 10$/acre and 100$/acre, respectively.

The cost function is an increasing hyperbolic function that is dependent on the population of wild hogs and the cost of the control methods used by the farmer. It is an increasing cost function with respect to K_t, because as hogs evade capture, they become increasingly harder to harvest, therefore their "smartness" or learned behaviors can cause a higher cost function (Baldwin 1969). Smartness is not dynamic in that hogs continue to gain smartness, but instead it is static meaning that once there is a control method in place, the hogs will adapt to evade capture and become smart. There are different levels of smartness included in the research to show comparisons. This smartness will make it harder and require more effort for the Farmer when he is using a controlling method as they can remember foraging opportunities or lack thereof (Held et al. 1999).

A modified cost equation using Bhat and Huffacker (2007) below describes the increasing hyperbolic cost function:

$$C(N_t, K_t) = C_0 + \frac{\alpha C_1 K_t}{N_t} \tag{3.3}$$

where C_0 is the fixed cost of trapping or hunting, C_1 is the cost per hog trapped or hunted, and α is an effort level coefficient that is equal to 1 for the baseline comparison in the deterministic model. As hogs become “smart” and adapt to evade capture, the effort level coefficient is increased. This increases the cost of effort by multiplying a higher α with the variable cost, which in turn increases the cost function.

It is important to note that the smartness is not dynamic but includes levels of smartness to compare to the baseline scenario of effort level parameter is equal to 1. Three levels of “smartness” will be compared to see whether it plays a role in harvest or population levels. The term α becomes greater than one in the deterministic case because the hogs are assumed to adapt by evading capture when the farmer is using a control method in a previous period. Here, smartness is not changing as effort is increased, but instead, the effort level parameter is increased when smartness is increased. For example, when α is 1, hogs are assumed to be normal, “non-smart” hogs. When α is equal to 1.1, it can be interpreted as the hogs are 10 percent “smarter” than normal hogs. When α is equal to 1.2, the hogs are 20% “smarter than normal hogs. As effort to control increases to combat the increasing ability of hogs to adapt, the cost to control will increase as well (Mehta et al 2007). This increasing cost function is more realistic to real world cost functions for invasive species and can be found in other studies conducted in Mehta et al. (2007) and Sims and Liu (2016).

Overall, as hogs are controlled increasingly, they become harder to trace and control due to smartness to evade capture. Smartness makes it harder and more costly to control the species population. As smartness increases it is expected that the net benefit will decrease.

A growth function is used and shown below. The growth equation of wild hogs can be seen in the conceptual Equation (3.1) and again in Equation (3.2). This can be described with unmolested (unrestricted) growth and harvest as:

$$g(N_t) = rN_t\left(1 - \frac{N_t}{X}\right) \tag{3.4}$$

where r is the intrinsic growth rate set at 0.584 and X is the carrying capacity set at 15 hogs per acre. Figure 3.2.1 below shows the intrinsic growth function curve where population/acre ranges from 0 to 15 and the growth rate reaches a peak at 2 hogs per acre.

The growth function relies on carrying capacity and resources available. When the population is small, the reproductive rate of hogs is high and will soon reach a carrying capacity. For this model, the highest population growth of hogs present on the field is set at 7.5 hogs/ acre as this is the midpoint between 0 and 15 (the carrying capacity). Once the carrying capacity is reached, the growth rate will start to decline. Once this capacity is reached, resources will become scarce, and the population growth rate will decrease. This fluctuation of high growth when the population is low and low growth when the population is high is the logistic growth function. The logistic growth function can be seen in Figure 3.1.

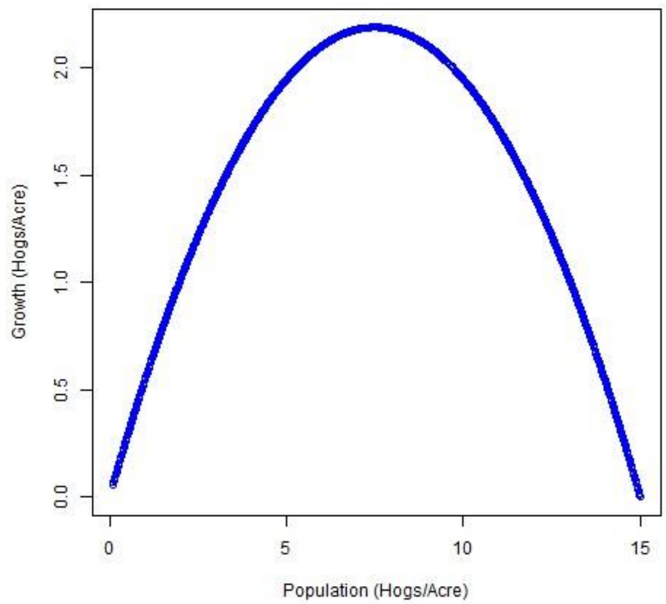


Figure 3.2 Logistic Growth Function depicting the Growth rate of hogs/acre when Population, N_t, ranges from 0 to 15 hogs/acre

To solve Equations (3.1) numerically, the value function iteration in Conrad and Rondeau (2020) is applied. From the equation of motion in Equation (3.1),

$$K_t = N_t + r\left(1 - \frac{N_t}{X}\right) - N_{t+1} \tag{3.5}$$

By replacing above to the profit function

$$\pi_t(N_t, N_{t+1}) = R(1 - dN_t) - C_0 - \frac{\alpha C_1 \left(N_t + rN_t\left(1 - \frac{N_t}{X}\right) - N_{t+1}\right)}{N_t} \tag{3.6}$$

Using Equation (3.2) as the value function iteration, the Bellman equation can use Equation (3.6) to find the result for Equation (3.5) by plugging the result from the Bellman equation into Equation (3.5). This is how K_t is calculated. Letting $N_{t+1} = N'$, the Bellman equation for Equation (3.2) can be defined as:

$$V(N) = \max_{N'}[\pi_t(N, N') + \rho V(N')] \tag{3.7}$$

The numerical solver can be seen described in detail in Appendix A.

The routine should converge, and the final vector should contain the maximum values for net benefit showcased in the value function. This model was run three separate times to compare the α levels and their corresponding harvest and growth functions.

3.3 Stochastic Model

The stochastic model allows for the use of the probability parameter in a Markov transition matrix. In the deterministic model, there was no probability parameter, so the hogs were assumed to always be present. For the stochastic model, these probabilities change in either direction and can be seen later on.

Probability of hogs to be present or absent in the field is determined by whether there is a control method in the previous period or not. When there is a control method in the previous period, the hogs will adapt to evade capture and become “smart”. Hogs are less likely to be present on the field in the next time period due to the control method.

When p is at .5 level, there is an equal chance of the hogs being present on the field or in the natural habitat. As the control methods are enforced, the level of p decreases in result to the hog's ability to evade capture or "smartness". When α is equal to 1, the effort level is the same for the next time period. Effort level increases due to α increasing and the ability to hunt or trap hogs becomes more difficult. It is important to note that when p=1, meaning hogs are 100 percent likely to be present on the field, this gives the same model and results for the deterministic case. The stochastic model can be described as:

$$\max_{K_t} \sum_{t=0}^{\infty} \rho^t \, \pi(N_t, K_t, \alpha | S_t) =$$

$$\max_{K_t} \sum_{t=0}^{\infty} \rho^t \begin{cases} R(1 - dN_t) - \left(C_0 + \dfrac{\alpha C_1 K_t}{N_t}\right) & when\ S_t = 1 \\ R - C_0 & when\ S_t = 0 \end{cases} \tag{3.8}$$

$$s.t.\ N_{t+1} - N_t = r\left(1 - \frac{N_t}{X}\right) - K_t$$

$$\pi_t \geq 0 \text{ , } N_t \geq 0 \text{ , and } K_t \geq 0$$

Descriptions for all other variables and parameters are the same as the deterministic model. This cost function is the same as the one in the deterministic case and can be seen in Equation (3.3).

A few different scenarios arise and allow for a comparison between changing effort levels and probability. The stochastic model will allow for a comparison of α at levels 1, and 1.1 at various levels of probability. The presence probability parameter is designed so that if hogs are present the probability will be p and the probability for no hogs to be present is $1 - p$. For example, S3 in Table 3.2 below, the probability is set at 0.6 so the probability of no hogs would be 0.4.

Table 3.2 Scenarios for Stochastic Model

	p (probability)	α (Effort parameter)
Baseline scenario	0.5	1
S2	0.5	1.1
S3	0.4	1
S4	0.4	1.1
S5	0.1	1
S6	0.1	1.1

These probabilities in the numerical solution explained later have been set up so that there are four probability options. These are called transition probabilities. Table 3.3 below describes the scenario of transition probabilities for S3.

Table 3.3 Transition probabilities example

	Present at N_{t+1}	Absent at N_{t+1}
Present at N_t	0.4	0.6
Absent at N_t	0.6	0.4

Transition probabilities allow for a higher or lower percentage of a hog being present or not depending on the previous period (N_{t-1}). The four probabilities are coordinated in pairs. One pair is for the probability of hogs being present on the field given they were on the field in the previous time period. The other set describes the probability if hogs were not present in the last time period.

For example, in the Table 3.2 there is a baseline scenario of .5. The presence probability is split evenly between both outcomes. There is a 50 percent chance the hogs will be present no matter if the hogs were present or absent in the previous period. There is also a 50 percent chance the hogs will be absent, no matter if they were present or not in the previous period. Using the baseline as .5 allows for the elimination of knowing whether the hogs were present in the previous time period or not. This is different from the deterministic model as there was a 100 percent certainty that hogs would be present on the field.

In scenario S3, the probability presence consisting of 40 percent probability they are present and 60 percent probability hogs are not present at α level of 1. Because hogs were present in the first period (N_t), they are 40 percent likely to be present on the farm in the next period (N_{t+1}) and 60 percent likely to be absent from the farm in the next time period. The probability for hogs to be present after the time period of harvest decreases because the hogs learn to evade capture and are less likely to return to the field. Similarly, if the hogs were absent in the first time period, they are 60 percent likely to be present on the farm in the next time period and 40 percent likely to be absent in the next time period.

The numerical solver and the incorporation of the transition matrix probabilities for the stochastic model can be seen in detail in Appendix A. This model was run for all scenarios in Table 3.2. These scenarios allowed for all the probabilities to be tested along with multiple α levels. This way the probabilities can be compared against each other with the same α level and also the same probabilities against the differing α levels. These can be seen in the Results section.

CHAPTER IV

RESULTS

4.1 Deterministic Model

Table 4.1 includes the descriptions of parameters used in the deterministic empirical model seen in Figure 4.1 and Figure 4.2.

Table 4.1 Parameter Values and Descriptions

Parameter	Description	Value ($/acre)
R	Revenue ($)	100
d	Proportional damage to the farm	0.06
δ	discount rate	0.1
X	carrying capacity	15
r	Intrinsic growth rate	0.584
α	effort level	1
C_1	variable cost of control method	100
C_0	fixed cost of control method	10

The parameters were all taken from the benchmark paper Zivin et al. (2000) except for the variable and fixed cost of control method. In the table, N_t is not listed as it ranges from 0 to 15 but can be described as the number of hogs in the area or population/acre, also known as the carrying capacity. Both α and C_1 may be subject to change in further analysis but for the deterministic model they are at a set value of 1 unless otherwise stated and 100, respectfully. When hunting and trapping are considered, there will be a change in the variable cost and the fixed cost. Trapping will have a higher fixed cost and a lower variable cost, where hunting will have the opposite. This will cause C_1 and C_0 to be of differing values later in the research. There will also be a stochastic parameter p included in different scenarios presented in the next section. This will change the probability of whether hogs are present on the field or not and ultimately play a role in the cost function for each period. The deterministic model is calculated under the scenario of hogs being present on the field in each time period and α is set at 1. The discount rate, δ, is used in the table to calculate the discount factor, ρ, as:

$$\rho = \frac{1}{1+\delta} \tag{4.1}$$

Thus, δ is the discount rate and ρ is the discount factor used in the model.

From the deterministic model, the value function can be created and ranges across the span of N_t. These values stem from the span of N_t seen in the deterministic programming. The value function and varying N_t values are seen in Figure 4.1.

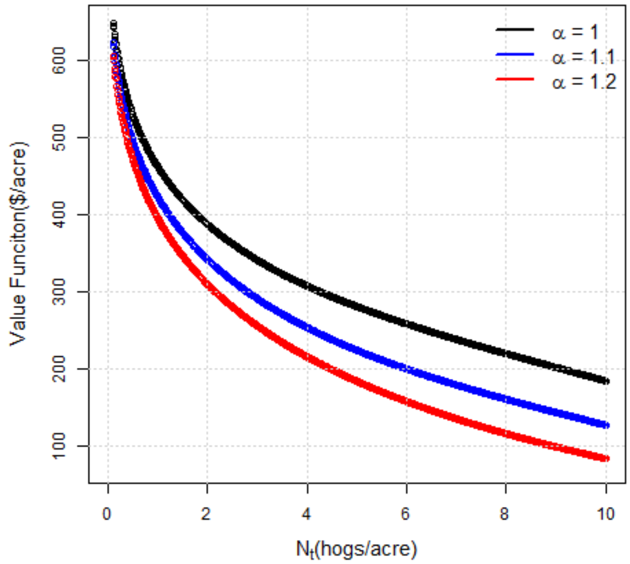


Figure 4.1 Value Function for the Deterministic Model across the span of N_t

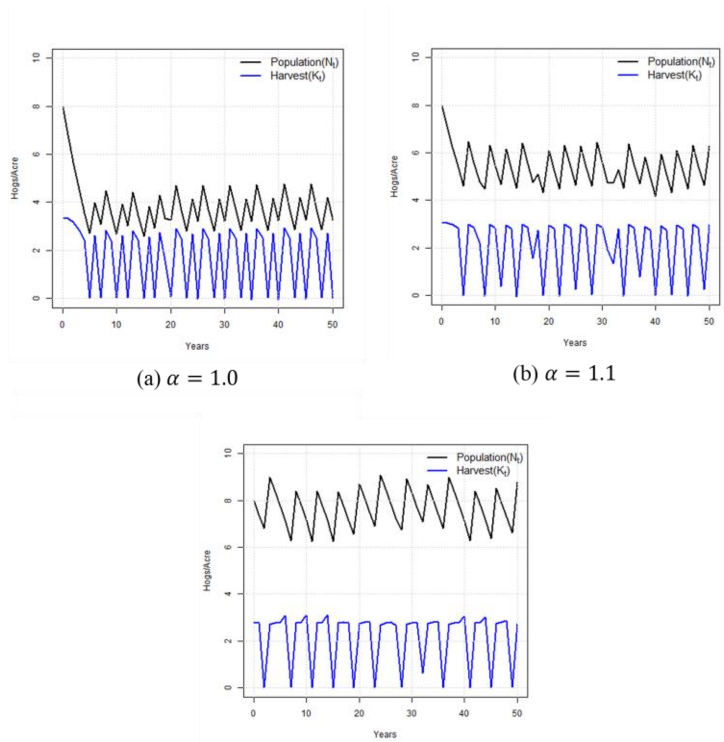
In the figure above, it is easy to see some key components. First, the value function is decreasing as hog population per acre increases. There are small changes in the lines including a decreasing of max profit. Lastly, as α increases, the value function decreases quicker when population increases.

The smartness of hogs is increasing, causing a less effecting harvesting technology and therefore, more money spent on harvesting. This in turn lowers the value function. For this function, the value function equation derived from the deterministic model was used along with α set at 1. From Figure 4.1, it is noticeable how the function is downward sloping and convex for all three cases. The value function is decreasing as the population of hogs is increasing for each level of α. This is expected as the presence of hogs causes damages to the farm and the Farmer also incurs a cost from harvesting the hogs. The convexity results from the increasing marginal effort level cost per hog when harvesting a lower number of hogs than a larger number. The value function is negatively correlated with the population of hogs, which is why the slope is negative and the first derivative is negative. The second derivative is positive meaning that as the population increases, the value function decreases at an increasing rate, i.e., increasing marginal effort level. The marginal cost of effort changes given the level of population and therefore it is not a straight-line value function. When there is a small number of hogs present on the field, generally there is more effort level exerted to harvest the hogs than when there is a large number of hogs present on the field.

The value function, population levels, and steady state level of hogs/acre will change given the different scenarios that are presented in the next section. The scenarios allow for differing levels of effort level (α) and for varying levels of presence possibility (p). The variable cost level of C_1 can also be increased or decreased and can cause a change in the graph. It will not be changed for these scenarios C_1 is set to be the same throughout this model.

From the equation explained in the deterministic case, Figure 4.2 below shows the growth and harvest function and how they relate to each other. The deterministic case focused on the given parameter of effort (a higher means the smarter) and no stochastic presence (meaning hogs show up every growing season). The graph depicts three different levels of α and theyre corresponding growth and harvest functions. To note, these levels are not due to smartness growing or changing but simply to show different starting points and how varying smartness levels will create varying results.

More specifically, Figure 4.2 depicts the increasing levels of α (1, 1.1, 1.2) and how they impact the growth and harvest function. The harvest function in units is measured as hogs/acre and the growth function is measured as population/acre. Per the legend, the growth function is the black line indicating the population levels of the hogs as time and harvest move. The blue line indicates the harvest function and how it corresponds to the growth function over time. It should be noted that this is a discrete function and moves discontinuously. The lines are connections of the high and low dots and are used for ease of reading and understanding the graph. The time ranges from year 1 to year 50 and allows for 50 time periods. The initial stock level was chosen arbitrarily to be 8. Changing the initial stock level will also give you differing results but will not be changed for the sake of this paper.



(a) $\alpha = 1.0$

(b) $\alpha = 1.1$

(c) $\alpha = 1.2$

Figure 4.2 Deterministic Harvest and Growth Function across Time

Key Points from the figures: Black line indicates the growth function. Blue line indicates the harvest function. As α increases, the functions move further apart showing how smartness affects population and harvest.

In (a) of Figure 4.2, the growth function and harvest function sit closely on the line of 3 hogs per acre with the growth function being slightly above. At this level, the harvest and growth functions are almost equal in (a) of Figure 4.2, meaning that the hog population would stay very low due to the similar levels of growth and harvest.

As the effort level is increased due to the increased "smartness", the growth function becomes flatter with shallow, less frequent oscillations. The oscillations show the growth of the population/acre from one period to the next. Shallow oscillations mean smaller changes in the population/acre of the wild hogs in both increasing and decreasing directions. The harvest function has fewer oscillations but stays at the same rate of change with the highest point around 3 hogs/acre. The oscillations towards the top even plateau for a few periods indicating longer periods of harvesting but no change in success rate. Despite these longer periods of harvest in (c) of Figure 4.2, the population of hogs is still much higher than the corresponding harvest levels.

The smartness of the hogs allows them to evade capture more readily and provides a higher population level. This proposition can be made by realizing the increasing cost function plays a major role. Generally, as the hogs become smart, the cost function increases because more effort is needed to harvest the hogs. As the cost rises, farmers become more reluctant to continue harvesting. One of the conditions of the model was the Farmer could not have zero profit. The Farmer will choose not to harvest if there is a chance it will drop his profit below zero. The hog population then has time to recover to a higher population level and eventually become a pest to the farmer again.

At its highest point in (c) of Figure 4.2, the population level is around 9 hogs/ acre. The low points of the harvest function indicate there is no harvest and similarly the high points of the harvest function indicate there is some form of harvesting happening, either hunting or trapping, in that time period. The low growth periods correspond with the high harvest periods as expected.

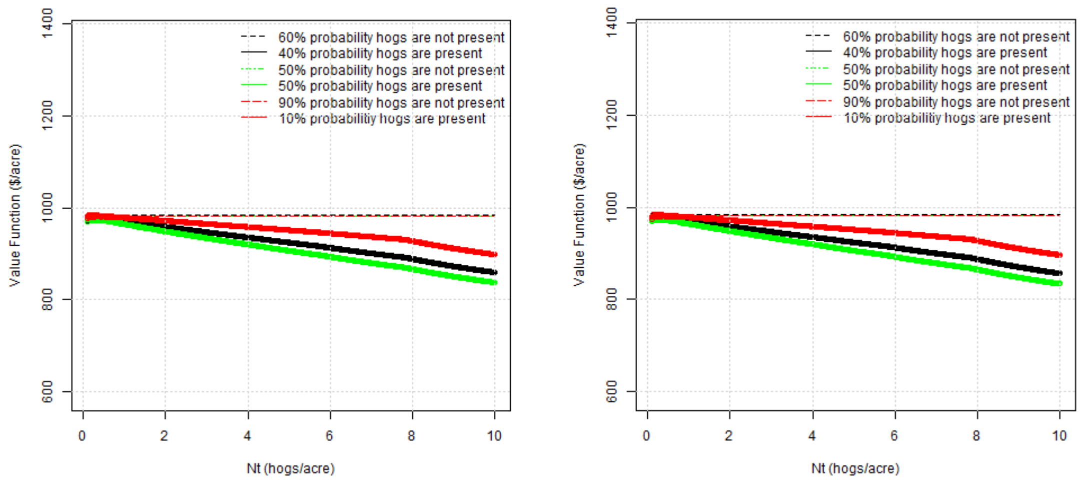
The Harvest and Population graphs seem to have a similar pattern to their oscillating high and low points. As the growth function increases, the harvest function also increases. When hogs have a higher population level, there is reason to harvest more. When smartness is increased, the growth function moves further from the harvest function each time it is increased i.e., from α equals 1 to α equals 1.1. Despite the harvest function increasing, the population increases at a higher rate when smartness is increased, because the hogs are adapting to evade capture. The population in (a), (b), and (c) of Figure 4.2 measures at 4, 6, and 8 hogs/acre despite the harvest level changing. Population levels were kept and obtained for the duration of the harvest levels meaning the hogs are adapting and evading capture. This could be a reason to why the harvest rate never goes above 4 hogs/acre. The results from Zivin et al. (2000) are at a much lower rate of 0.7/hogs per acre for trapping only but for hunting only the population was at 7 hogs/acre. The differentiating steady-state levels are due to the addition of adaptation and presence probability. The extensions of the model allow for a different steady-state to be obtained and shows how adaptation and probability of presence on the field affect harvest and population levels.

4.2 Stochastic Model

The stochastic model uses the same parameters as those found in Table 4.1 in the previous section. As the deterministic case had a value function which showed the highest profit during each time period, the stochastic model has one as well. The main difference is that there are two value functions for each level of α. One value function for the probability the hogs will be present on the field and one value function for the probability the hogs will not be present on the field. Table 3.2 show the scenarios used for this model. It is important to note that the probability of hogs being present 100 percent of the time would garner the same results as the deterministic case.

The results often showed the probability of the hogs not being on the field as a constant. This did not change when the probabilities were changed not did it alter when the αs were increased. Because of this, multiple levels of probability for the hogs being present will be shown along with one level of hogs not being present to show the contrast of the value functions. The two graphs side by side show there is no change in the probability of no hogs both between αs and between probabilities.

As the probability of hogs being present on the field in the next time period increases, the value function decreases. The α is set at 1 for all levels of probability in the first graph of the figure. In the graph on the right, the α is set at 1.1. The probability of no hogs is the same for all probabilities and both αs. They are still represented in the dashed lines on both graphs in the figure to show that they are the same for all. No matter the probability, the value function will remain the same any time there are no hogs present. The value functions with varying N_t and probabilities are shown in Figure 4.3 below.



α = 1 α = 1.1

Figure 4.3 Value function across varying N_t and multiple presence probabilities.

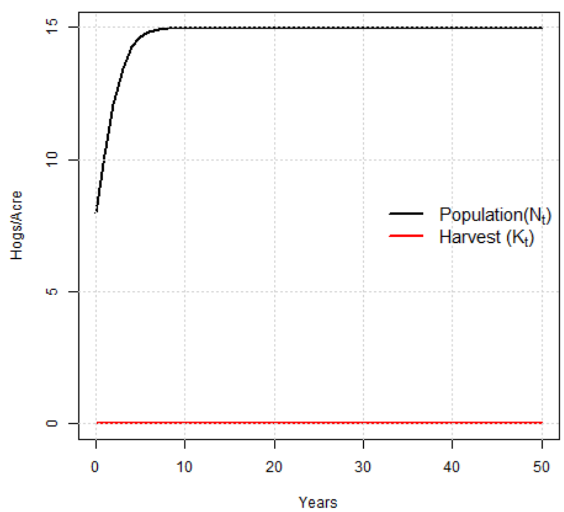
As the figure shows, there are three different pairs of presence probabilities. The value function with the probability of no hogs does not change even if the probability of hogs being present increases. As the probability of hogs being present in the next time period increases, the value function decreases. In the first duo of red lines, there is a 10 percent chance that hogs will be present on the field in the next time period. This is the highest the value function will be besides the value function for the probabilities where no hogs are present. When the hogs have a 40 percent chance of being present on the field, the value function decreases. This is the same for the 50 percent chance probability as well. This is expected as the higher a probability of hogs being present on the field, the higher probability of damage to the farm. There is also a higher cost for harvesting if there is a prediction of more hogs.

Having different presence probabilities (i.e., 10%, 40%, 50%) to compare gives a more realistic perspective as there is not always a guarantee for hogs to be present or absent as described in the deterministic model. In the real world, Farmers have no clue if hogs will be present this year or the next year. There is always a probability of the hogs being present, but this too can waver given there is or is not a harvesting period that happened previously.

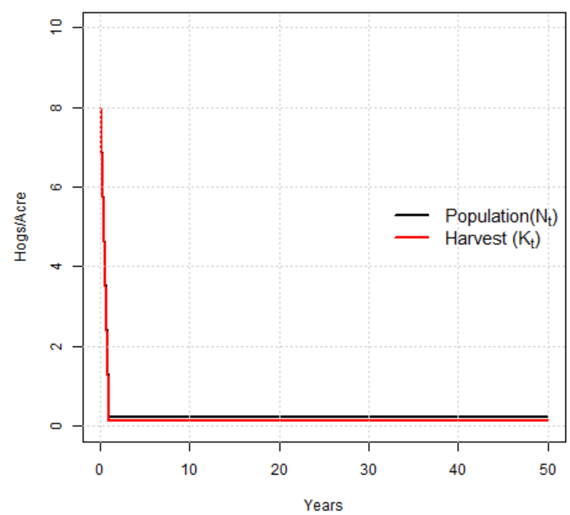
As mentioned throughout the paper, hogs are smart and will remember where a harvest occurs. If they are present during a harvest year (t), they are less likely to visit the field in the next period $(t + 1)$ than if there was no harvest in t. This is why the transition probabilities were used. It allows for all four options of the Farmers harvesting or not and the hogs being present previously or not.

There are two different ways these value functions can be compared. One way was shown earlier, comparing between different probability presence levels with a constant α. For example, comparing 40 percent probability and 50 percent probability with α at 1. The other way is comparing a same level of probability across varying α. For example, the 40 percent probability presence with α at 1 can be compared to 40 percent probability presence with α at 1.1. The comparisons are less dramatic as the change between value functions with different α's is not as drastic as the change in value functions with different probabilities across a constant α. This small difference can be seen in the figure above by comparing the left side to the right side. There is almost no difference in the two α's when comparing the individual probabilities. The main difference is between the varying probabilities set at the same level of α.

Similar to the deterministic model, the graph below depicts the differing levels of α (1 and 1.1) and how they change the growth and harvest function. The harvest function in units is measured as hogs/acre and the growth function is measured as population/acre. Because there are now probabilities and two value functions, there are also two different population and harvest graphs. Figure 4.4 depicts the population and harvest for the probability of hogs and the other depicts the population and harvest when there are no hogs present. Per the legend, the growth function is the black line indicating the pattern of growth in the hogs as time and harvest move. The red line indicates the harvest function and how it corresponds to the growth function over time. It should be noted that this is a discrete function and moves discontinuously. The initial stock level was chosen arbitrarily to be 8. Changing the initial stock level will also give you differing results but will not be changed for the sake of this paper.



(a) When hogs are present



(b) When hogs are absent

Figure 4.4 Population and Harvest levels for both present and absent hogs at α = 1 and presence probability of 40%.

The black line indicates the population level for each case of hogs present on the field or not. In the graph on the left, the hogs had a 40% chance of being present and on the right, they have a 60% chance of being absent from the field. The red line indicates the harvest levels for each scenario. When hogs are present the population reaches a max of 15 hogs/acre and the harvest levels reaches between 1 and 0 hogs/acre. When hogs are not present, both population and harvest are 0.

It is reasonable that the graph of hogs not present would simply run the stock level of 8 straight to 0. If there are no hogs present, the population cannot grow and there is nothing to harvest. When the hogs are present, the harvest level stays far below the population level and therefore it is less likely to decrease the population when there is an uncertainty of whether the hogs will be present or not. There is far less fluctuation than that seen in the deterministic model which could be from splitting the equation in two different probability levels.

The harvest level and population level were run with the higher α of 1.1 but there was no change in either graph. The explanation could be probabilities and smartness themselves do not change the growth and harvest levels, but the two equations allow for the highest population and harvest and the lowest population and harvest that yields the highest expected value for profit maximization.

The way this stochastic model is formulated, hogs' presence is a random event, and they are likely to show up less frequently as α increases. Farmer A is assumed to harvest hogs as his "expected" value of overall profit is maximized. If hogs do become present on the field, the population is too high for the Farmer to effectively harvest. Because the population is so high, the damages would cause too much loss of profit for the Farmer to consider harvesting because the cost would only incur more loss of profit for the Farmer and inevitably, he would choose not to harvest. Thus, the hogs' population will continue to grow and the reason harvest in (a) of Figure 4.4 is just above 0. This can be extended in a later model using a new cost structure.

CHAPTER V

CONCLUSION

Wild hogs are a nuisance in much of the United States and cause major damage to crop fields in agricultural regions. Their high reproductive rate and ability to evade control measures makes reducing the population inefficient and problematic for managers. When there is a period of harvest, the hogs have the ability to remember and are less likely to be captured the same way. This can cause problems because the hogs can potentially return to the field during the next time period. From this research, the adaptability of hogs is crucial to understand in order to get the highest net benefit for the Farmer.

Overall, the suggested model sufficiently replicated the expected results. Increasing smartness of wild hogs drastically decreases the value function a Farmer can achieve. The deterministic model with varying smartness proves a higher value function corresponds with a lower smartness level. This is expected as hogs cause more damage and require a higher cost level when the hogs can easily evade capture. If they are able to evade capture, they can continue to return to the property if they so choose. This will cause the farmer to expend more cost to eradicate them from the field. There is also a connection between the smartness parameter and the population and harvest. When the smartness parameter is increased, the population also increases faster than that of a lower smartness level.

The stochastic model introduced a probability parameter. This allowed for there to be a probability attached to whether hogs would be present on the field or not. The findings were similar to the expected outcome in that the higher the probability of hogs to be present on the field, the lower the value functions for the Farmer. If there were no hogs present, the value function was at its highest compared to the value function of probabilities they would be present. The probabilities also affected the population and harvest. When hogs are present, the population was well above the harvest and thus population decrease would never be an option. This could be recalculated using a different cost function that could allow for a higher harvest, but this would need to be added as an extension to the model.

When hogs are not present, there is no population or harvest. Utilizing the different probabilities of presence helps set up a more realistic spilt of the chance for hogs to be present or absent and thus the net benefit will not be as high as the deterministic model. Overall, the results were very similar to what was expected of the change in smartness and probability of presence of hogs.

There are many more extensions that can be made to this model including success rate, a two-farmer model, and externalities. These extensions would make the model more realistic as most farmers border other farms. Success rate can be added into the cost function and provide another means to calculate smartness. As success rate increases, hogs are less likely to show on the field and the population would decrease at a faster rate.

Because the one Farmer model eliminates all possibilities of a neighbor, there is no chance of free riding. This meaning there is no chance of one Farmer controlling while his neighbor is not controlling and reaping the benefits of no hogs. No other net benefit is addressed and therefore this one Farmer model has no externality or social planner issue. This can be extended by simply using a two Farmer model.

The externalities from using a two-farmer model would help explain the continued increase of the population. For example, in the real world, one Farmer who is controlling for wild hogs would border another Farmer who may not control. The Farmer that is controlling has a higher cost function and is therefore taking on more costs than the Farmer not controlling. The second Farmer is now either reaping the benefit of his neighbor using a control method or has more hogs than before. In either scenario the hogs have a second option to reside, and the population never decreases despite the effort from the first Farmer.

This model focuses mainly on the net benefit of the single farmer given that hogs are highly adaptable. Because this is just a one farmer model, policy implications cannot be made from these specific results. By extending the model and introducing a two-farmer model, certain policy implications can be made. Because hogs are smart and have a working memory, it is imperative to have an incentive to control in many fields. One policy option could be reducing the cost of harvesting through a subsidy. This is currently being implemented with the USDA Feral Swine and Control Pilot Program. It would help generate more revenue for the farmer through grants and allows for wild swine removal and restoration to the property. By extending the model to two farmers, externalities could then be reassessed and encouraging a communal control technique would potentially help decrease the population. This could also help address the social planner’s dilemma as no one person is gaining or losing the benefit of controlling on their field.

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APPENDIX A

NUMERICAL SOLVER FOR BOTH MODELS

A.1 Deterministic Solver

To build the numerical solver for the deterministic model, the following steps are applied. N_t is an evenly gridded sequence spanned from a maximum value of N (N_{min}) to a minimum value of N (N_{max}). These steps should lend you the value function using this equation.

1) Create a vector containing minimum and maximum bounds and the number of values to be sampled, i.e., define a grid of 1,000 evenly gridded $N \in (N_{min}, N_{max})$ and the same grids for N'.
2) Create a matrix for all combinations of $N \times N'$, calculate the $\pi_t(N, N')$. This will calculate all possible levels of profit obtained from having a population level of N_t defined by the sequence span of N_t. Using a max operator will ensure the harvest levels will be strictly positive. Each row of this matrix will correspond to a specific level of N_t where the columns represent the specific levels of N_{t+1}.
3) Set the initial 0-vector for the value function, $V_0 = [0, \cdots, 0]'$. Create a column vector of value 1 of length N_t. This will be used in the iterative procedure.
4) Through the operator to get $V_1 = \max_{N'}[\pi_t(N, N') + \rho V_0]$, choose the (N, N') combinations for all N. The first iteration will generate simple the maximum π_t.
    a. Create an intermediate value function (TV_0), having N_t and harvesting from the hog population so that N_{t+1} is the stock level in the next period. This value function makes it so that only the highest possible values are kept from having N_t. All other values are discarded. To do this use the matrix created in the first stage and add it to the discount factor multiplied by the row vector and column vector from the first stage. This will create an $N_t \times N_{t+1}$ matrix where the element of

each row is the discounted value of the corresponding stock N_{t+1}. When a max operator is applied to this new matrix, it finds the highest value in each row (highest possible profit from having a level of N_t and choosing a level of N_{t+1}.

b. Then, subtract the value function from the intermediate value function (TV_0-V_0). This will calculate how much the value function has changed between this iteration and the previous one for each stock level in the matrix. A max operator can be used to the absolute value of these elements to get the largest change across all stock levels. This can also be known as the error of this iteration.

c. Update V_0 from section one to the new value of TV_0 .

d. Create a command that increases the iteration counter by one. This signals that is it time to return to the top of the loop to test error and iterations to determine whether another iteration should be run.

5) Repeat the process to find the $|V_1 - V_0| \leq \varepsilon$, where $\varepsilon = 10^{-8}$.

6) Using the found V_1 and its N and N' combinations, the K is recovered using Equation (3.2.4). Print the number of iteration and the final error should be recorded.

A.2 Stochastic Solver

To build the numerical solver for the stochastic case, the setup is the same as the deterministic model. The main difference is the two value functions now needed for the differing probabilities. This means the Second stage of the deterministic model will have double everything. There will be two equations used, two matrices created, two intermediate value functions and two value functions. The first equation will be used for the probability of hogs to be present on the field. The second equation will be for the probability hogs will not be on the field. This will be the case for all the doubles. The probabilities are located in the loop created in step 4 of the deterministic model.

From the deterministic model, the main changes can be seen below:

1) Create a vector containing minimum and maximum bounds and the number of values to be sampled, i.e., define a grid of 1,000 evenly gridded $N \in (N_{min}, N_{max})$ and the same grids for N'. This should be the same as the deterministic case.

2) Create two matrices for all combinations of $N \times N'$, calculate the $\pi_t(N, N')$. These will be similar to the deterministic stage, but one matrix will correspond with one value function for a total of two matrices.

3) Set two initial 0-vector for the value function, $V_{0p} = [0, \cdots, 0]'$ and $V_{0a} = [0, \cdots, 0]'$. Here, the first (V_{0p}) corresponds to if hogs are present on the field and the second (V_{0a}) corresponds to if the hogs are absent on the field. Create a single column vector of value 1 of length N_t.

4) Through the operator to get $V_1 = \max_{N'}\left[\pi_t(N, N') + p\rho V_{0p} + (1-p)\rho V_{0a}\right]$, choose the (N, N') combinations for all N. The first iteration will generate simple the maximum π_t.

    a. Create two intermediate value functions (TV_{0p} and TV_{0a}), having N_t and harvesting from the hog population so that N_{t+1} is the stock level in the next period. Similar to step 3, the first corresponds to the probability of hogs being present and the second correspond to the probability of hogs being absent. This is seen in the equation above described as p and $(1-p)$, respectively.

    b. Then, subtract each value function from their respective intermediate value function (TV_{0p}-V_{0p}), for example.

c. Update each V_0 from section one to the new value of TV_0 , respectively.

5) Repeat the process to find the $|V_1 - V_0| \leq \varepsilon$, where $\varepsilon = 10^{-8}$.

6) Using the found V_1 and its respective N and N' combinations, the K is recovered for each value function using Equation (3.2.4). Print the number of iteration and the final error should be recorded.